Evaluating Tax Harmonization

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ABSTRACT

A second order Taylor approximation implies that tax harmonization advances government objectives only if tax competition reduces average tax rates by more than the standard deviation of observed rates. In 2020 the mean world corporate tax rate was 25.9\%, and the standard deviation 4.5\%, so if there is an efficient harmonized tax rate, it must exceed 30.4\%. An efficient minimum tax rate equals the average tax rate among those constrained by the minimum plus the average effect of tax competition. Hence there are dominated regions: in the 2020 data, there is no effect of tax competition for which a world minimum corporate tax rate between 4\% and 27\% would efficiently advance collective objectives.

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1. **Introduction.**

Concern over the effects of tax competition increasingly prompts calls for tax harmonization, minimum tax rules, or other agreements that would limit competition and reduce tax diversity. The most prominent and important recent example is the worldwide corporate minimum tax proposed by the OECD (2021) and approved in concept by more than 100 countries. Other longstanding efforts include tax coordination initiatives by the European Union and minimum tax proposals for subnational jurisdictions such as U.S. states. These initiatives and others reflect ongoing interest in coordinated responses to tax competition pressures.

Tax coordination can address downward pressure from tax competition, but does so at the cost of requiring governments to adhere to collective rules that may be insensitive to differences in the situations and needs of individual jurisdictions. Minimum tax regimes are more flexible than complete harmonization, but nonetheless impose binding constraints on countries that otherwise would choose low tax rates. Furthermore, effective enforcement of tax harmonization or a minimum tax agreement may require rules preventing governments from differentiating their taxation in ways that they would otherwise choose to do, such as by offering favorable taxation of highly valued economic activities or those located in economically depressed regions.

There are many reasons why business tax rates differ. Industrial compositions and economic activity levels affect perceived costs of business taxation and the relative attractiveness of alternatives such as personal income taxes and VATs. Income distributions and the likely incidence of business taxation also influence choices among tax alternatives. The political appeal of taxing business income differs widely, including among countries with similar economies and income distributions but different national politics. And countries differ in the extent to which their tax choices are influenced by international competition. As a result of these and other considerations, there is considerable dispersion in the rates at which countries tax business income.

The purpose of this paper is to use observed tax choices to evaluate the properties of tax harmonization alternatives. A second-order Taylor approximation yields the simple rule that tax rate harmonization advances collective government objectives only if tax competition reduces tax rates by more than the standard deviation of observed tax rates. This rule captures the reality
that the diversity of economic and political considerations that determine tax rates in the absence of coordination makes it impossible for a single harmonized tax rate to conform to every government’s desired tax policy – and the standard deviation measure reflects the second order nature of the cost of deviating from preferred tax rates. Given the multiplicity of preferred tax rates, costs of deviating from preferred rates, and perceived costs of tax competition, it is striking that the criterion for objective-enhancing tax harmonization takes the form of a simple standard deviation.

The standard deviation rule emerges from comparing uncoordinated taxation to efficient tax harmonization. The efficient harmonized rate is itself the sum of the average observed tax rate and the average amount by which tax competition depresses rates. Since tax harmonization maximizes collective objectives only if tax competition reduces tax rates by more than their observed standard deviation, it follows that an efficient harmonized tax rate must exceed the average observed tax rate plus the standard deviation of observed tax rates. In 2020, the mean corporate tax rate weighted by GDP was 25.9%, and the standard deviation 4.5%, so if there is an objective-maximizing harmonized corporate tax, its rate must lie above 30.4%.

Minimum tax regimes share features of tax harmonization while avoiding some of the costs of enforced conformity for the portion of the sample that prefers tax rates above the required minimum. As a result, in a setting in which tax competition systematically reduces tax rates, there is very likely to be a minimum tax rate that advances collective objectives. Furthermore, for any given harmonized tax regime, there exists a minimum tax alternative that more readily advances collective objectives.

Minimum tax rates directly affect only those who would otherwise choose rates below the mandated minimum. An efficient minimum tax rate equates the benefits to all counties of a slightly higher world average tax rate with the costs imposed on countries whose tax rates are constrained by the minimum. Applying the second order approximation, an efficient minimum tax rate equals the average tax rate of constrained countries plus the average effect of tax competition. Multiple points may satisfy this condition, since a significantly higher minimum tax rate brings many more countries within its ambit, thereby increasing its impact on world average tax rates, and readily generating additional points at which the benefits of slightly higher
world average tax rates might equal the costs borne by countries forced to increase theirs. This potential multiplicity of locally efficient minimum rates means that there can be dominated regions over which no minimum tax rate is consistent with maximizing collective objectives, regardless of the effect of tax competition on tax rates.

The distribution of world corporate tax rates in 2020 is such that there is a wide range of dominated minimum tax rates. Using GDP as a measure of willingness to pay, there is no tax competition scenario in which a world minimum tax rate between 4% and 27% is consistent with maximizing collective objectives. If tax competition otherwise depresses average tax rates by less than 4%, then a minimum tax rate below 4% advances collective objectives, whereas if tax competition depresses average tax rates by 4% or more, then a minimum tax rate of 27% or higher maximizes collective objectives. Alternative measures of willingness to pay produce similar ranges of dominated minimum tax rates.

While it is convenient to treat countries and states as though they impose scalar tax rates on all business income, the reality is that every jurisdiction has its own definition of business income, and activities within the same jurisdiction can be taxed at widely differing rates. The impact of a minimum tax rule or other potential harmonization measure depends, therefore, on exactly how the reform measure would treat these within- and between-country differences. One possibility is that international tax harmonization or minimum taxation would simply require countries to modify their statutory tax rates without changing any of their other tax provisions – and the tax rate analysis directly addresses this scenario. If instead countries would be required to modify every aspect of their tax systems, then a more comprehensive analysis would be required, one that incorporates the additional costs that countries incur, from the standpoint of their national objectives, in complying with a requirement that they tax each of their business activities in a common fashion.

Minimum tax rules and other tax harmonization measures have the potential to address important concerns about the effects of tax competition. While harmonization measures affect opportunities for tax avoidance, the fundamental function of tax harmonization or minimum taxation lies in their impact on competition. Countries could, if they wish, adopt strong
unilateral measures to protect their tax bases,\textsuperscript{1} including those contained in the OECD (2021) blueprint – but those otherwise inclined are deterred from doing so on a unilateral basis out of concern for their anticompetitive effects, reactions from other countries, and the domestic politics of deviating from world norms. Since the competitive setting significantly influences policy choices, it is important to analyze tax harmonization and minimum taxation in the context of tax competition.

2. Tax Harmonization and Government Objectives.

This section considers a setting in which each country’s government chooses its corporate tax rate while balancing economic and political considerations that include not only the economic costs of different taxes, and desired distribution of tax burdens between business and individual taxes, but also competition with other governments. These economic and political preferences can be summarized by a function of a country $i$’s own tax rate and the tax rates of other countries, or equivalently, a function $O_i(\tau_i, d_i)$ of country $i$’s own tax rate $\tau_i$ and the difference $d_i = \tau_i - \bar{\tau}$ between country $i$’s tax rate and the weighted average tax rate of all $n$ countries $\bar{\tau} = \sum \tau_i v_i$, with $\sum v_i = 1$. The weights used to construct $\bar{\tau}$ reflect the relative importance of the tax rates of different countries, which might for example be proportional to GDP or other measures of the volume of taxed activities. Importantly, the relevant weighted average tax rate is taken to be the same for all countries, a specification that entails common weights and excludes the possibility that governments compare their tax rates to others chosen on idiosyncratic bases such as geographic or characteristic proximity. For analytical convenience, $O_i(\tau_i, d_i)$ is taken to be continuous and twice continuously differentiable in its arguments, with higher values of $O_i(\tau_i, d_i)$ corresponding to greater satisfaction of government objectives.

2.1. An approximation.

It is useful to consider the tax rate that maximizes country $i$’s objectives in the absence of international tax differences, and to denote this tax rate by $\tau_i^*$. The tax rate $\tau_i^*$ is that which the government of country $i$ would choose to maximize its objectives if it knew that it were a

\textsuperscript{1} As argued, for example, by Dharmapala (2021).
Stackelberg leader that all other countries would follow exactly. In this sense, $\tau^*_i$ is the tax rate that country $i$ would choose in the absence of international competition, and reflects domestic considerations such as desire for economic development, preferences over the distribution of tax burdens, and government revenue needs.

In practice, most countries do not impose tax rates that they would select in the absence of international competition; and tax rates certainly differ. Country $i$’s objective level $O_i(\tau^*_i, d_i)$ can be evaluated using a Taylor expansion around $O_i(\tau^*_i, 0)$, the second-order approximation of which is

\begin{equation}
O_i(\tau_i, d_i) \approx O_i(\tau^*_i, 0) + (\tau_i - \tau^*_i) y_{\omega i} - (\tau_i - \tau^*_i)^2 y_{\gamma i} - (\tau_i - \bar{\tau}) y_{\gamma 2i} - (\tau_i - \tau^*_i)(\tau_i - \bar{\tau}) y_{\gamma 4i},
\end{equation}

with $y_{\omega i} = \frac{\partial O_i(\tau^*_i, 0)}{\partial \tau_i}$, $y_{\gamma i} = -\frac{1}{2} \frac{\partial^2 O_i(\tau^*_i, 0)}{\partial \tau_i^2}$, $y_{\gamma 2i} = \frac{-\partial O_i(\tau^*_i, 0)}{\partial d_i}$, $y_{\gamma 3i} = \frac{-1}{2} \frac{\partial^2 O_i(\tau^*_i, 0)}{\partial d_i^2}$, and

\[ y_{\gamma 4i} = \frac{-\partial^3 O_i(\tau^*_i, 0)}{\partial \tau_i \partial d_i}. \]

Since $\tau^*_i$ is the objective-maximizing tax rate in the absence of tax differences, it follows that $\frac{\partial O_i(\tau^*_i, 0)}{\partial \tau_i} = y_{\omega i} = 0$; and since $\tau^*_i$ corresponds to a maximum it must be the case that $-\frac{1}{2} \frac{\partial^2 O_i(\tau^*_i, 0)}{\partial \tau_i^2} = y_{\gamma i} > 0$. The sign of $y_{\gamma 2i}$ depends on how country $i$ evaluates differences in world average tax rates, holding its own tax rate constant – if, as is commonly assumed to be the case in models of tax competition, a country feels that it is costly to have a tax rate exceeding the world average, and beneficial to have one below the world average, then $y_{\gamma 2i} > 0$. Alternatively, a country may feel that it benefits from the opportunities created by lower foreign tax rates, and is hurt by higher foreign taxes, in which case $y_{\gamma 2i} < 0$; and the sign of $y_{\gamma 2i}$ may differ between countries. Similarly, models of tax competition commonly assume that there are convex costs of deviating from world average tax rates, which implies that $y_{\gamma 3i} > 0$; but it is also entirely possible
that \( \gamma_{3i} < 0 \), particularly for countries with lower than average tax rates. Tax competition theory currently has little to say about the sign of \( \gamma_{4i} \). It is reasonable to expect the coefficients \( \gamma_{1i} \), \( \gamma_{2i} \), \( \gamma_{3i} \), and \( \gamma_{4i} \) all to be positive, though with declining certainty: it is clear that \( \gamma_{1i} > 0 \), and likely that \( \gamma_{2i} > 0 \), whereas the signs of \( \gamma_{3i} \) and \( \gamma_{4i} \) are less certain.

The second-order Taylor expansion in (1) approximates a country’s objectives. This focuses the analysis in a way that facilitates drawing useful inferences, but does so at the cost of restricting the validity of the findings to settings in which the approximation does not mislead. In many cases the first- and second-order terms in (1) will capture the salient features of tax rate differences; and there is little if any empirical evidence that higher-order terms significantly influence country objectives or tax rate determination.

2.2. Individual tax rate choice.

If countries choose tax rates that advance their own objectives, and equation (1) accurately represents these objectives, then it should be the case that their tax rates maximize (1). Taking this to be the case,\(^2\) and assuming that countries ignore their own effects on the tax rates of others and the world average tax rate, it follows that countries perceive the effects of small changes in their own tax rates to be

\[
\frac{\partial O_i(\tau_i, d_i)}{\partial \tau_i} + \frac{\partial O_i(\tau_i, d_i)}{\partial d_i} = 2\gamma_{1i}(\tau_i^* - \tau_i) - \gamma_{2i} + 2\gamma_{3i}(\bar{\tau} - \tau_i) + \gamma_{4i}(\bar{\tau} + \tau_i^* - 2\tau_i). \tag{2}
\]

Setting (2) equal to zero yields the implied objective-maximizing tax rate

\[
\tau_i = \frac{\left(\gamma_{1i} + \gamma_{4i}\right)\tau_i^* - \gamma_{2i} + \left(\gamma_{3i} + \gamma_{4i}\right)\bar{\tau}}{\gamma_{1i} + \gamma_{3i} + \gamma_{4i}}, \tag{3}
\]

---

\(^2\) While the linearity of differentiation implies that the derivative of a function equals the derivative of its Taylor expansion, there are circumstances in which a second-order Taylor expansion closely approximates the value of a function without the derivative of the second-order expansion closely approximating the function’s derivative. The derivation of (3) assumes that restricting attention to the first- and second-order expansion terms produces valid approximations not only for the value of the \( O_i(\tau_i, d_i) \) function but also for its derivative.
which corresponds to a maximum only if 
\[
\left[ \frac{\partial^2 O_i(\tau_i, d_i)}{\partial \tau_i^2} + \frac{\partial^2 O_i(\tau_i, d_i)}{\partial d_i^2} + 2 \frac{\partial^2 O_i(\tau_i, d_i)}{\partial \tau_i \partial d_i} \right] < 0,
\]
with
\[
(4) \quad \frac{\partial^2 O_i(\tau_i, d_i)}{\partial \tau_i^2} + \frac{\partial^2 O_i(\tau_i, d_i)}{\partial d_i^2} + 2 \frac{\partial^2 O_i(\tau_i, d_i)}{\partial \tau_i \partial d_i} = -2(\gamma_{ii} + \gamma_{3i} + \gamma_{4i}).
\]

With the second-order condition for maximization implying that the denominator of the right side of (3) is positive, the comparative statics associated with terms in the numerator of (3) are largely intuitive. The parameter \(\gamma_{2i}\) captures the perceived cost of differences between a country’s tax rate and the world average, and as a result, higher values of \(\gamma_{2i}\) are associated with lower tax rates. It follows from the first term in the numerator of (3) that higher values of \(\tau_i^*\), the objective-maximizing tax rate in the absence of international tax differences, are associated with higher observed tax rates, and thus \(\frac{\partial \tau_i^*}{\partial \tau_i} > 0\), as long as \(\gamma_{ii} + \gamma_{4i} > 0\). The strategic element of international tax setting appears in the third term of the numerator, where a positive value of \(\left(\gamma_{3i} + \frac{\gamma_{4i}}{2}\right)\) implies that tax rates are strategic complements, with \(\frac{\partial \tau_i}{\partial \tau_i} > 0\), and a negative value would imply that they are strategic substitutes. While strategic complementarity – a country reacting to tax cuts elsewhere by reducing its own tax rate – is a common feature of tax competition models, it is far from guaranteed to be the case, and indeed there are important cases in which tax rates will be strategic substitutes. Furthermore, a system consisting of countries all with the same properties as \(i\) is stable only if \(\frac{\partial \tau_i}{\partial \tau_i} < 1\), which implies that \(\gamma_{ii} + \gamma_{4i} > 0\) and therefore \(\frac{\partial \tau_i}{\partial \tau_i} > 0\). Another property of (3) is that \(\frac{\partial \tau_i}{\partial \tau_i} + \frac{\partial \tau_i}{\partial \bar{\tau}} = 1\), so \(\frac{\partial \tau_i}{\partial \tau_i} = 1 - \frac{\partial \tau_i}{\partial \bar{\tau}}\). Finally, Equation (3) also carries the implication that
\[
(5) \quad \tau_i^* = \tau_i + \frac{(\tau_i - \bar{\tau}) \left(\gamma_{3i} + \frac{\gamma_{4i}}{2}\right) + \frac{\gamma_{2i}}{2}}{\gamma_{ii} + \frac{\gamma_{4i}}{2}}.
\]
Equations (1) and (5) together imply that if country $i$ chooses its tax rate to maximize $O_i(\tau_i, d_i)$, then its objective level can be approximated by

$$O_i(\tau, d) \approx O_i(\tau^*, 0) - \tau^*_i \gamma_{ii} - \tau^*_i \gamma_{ii} + 2\tau^*_i \gamma_{ii} + 2 \left( \frac{\tau_i - \bar{\tau}}{(\gamma_{ii} + \gamma_{ii})^2} \right) \gamma_{ii}$$

(6)

$$- (\tau_i - \bar{\tau}) \gamma_{ii} - (\tau_i - \bar{\tau})^2 \gamma_{ii}.$$

Collecting terms and simplifying, (6) implies that

$$O_i(\tau, d) \approx O_i(\tau^*, 0) - \tau^*_i \gamma_{ii} + (\tau_i - \bar{\tau})^2 \left( \gamma_{ii} + \gamma_{ii} + \gamma_{ii} \right) - \bar{\tau}^2 \gamma_{ii} + 2\tau_i \gamma_{ii}$$

(7)

$$+ 2 \frac{\bar{\tau}(\tau_i - \bar{\tau}) \left( \gamma_{ii} + \gamma_{ii} \right)}{\gamma_{ii} + \gamma_{ii}} \gamma_{ii} + \frac{\gamma_{ii} \gamma_{ii}}{\gamma_{ii} + \gamma_{ii}}.$$

2.3. **Aggregate objective satisfaction.**

One consequence of country differences in preferred tax rates and perceived costs of deviating from the world average tax rate is that any harmonization effort is apt to further the objectives of some while thwarting the objectives of others. An overall assessment of the consistency of tax harmonization with national objectives therefore requires a method of aggregating outcome assessments from the standpoint of national governments. If tax rate preferences are embedded in broader objective functions $F_i \left[ O_i(\tau_i, d_i) + y_i \right]$, with $y_i$ a transferable commodity such as money, then $O_i(\tau_i, d_i)$ can be interpreted as willingness to pay for tax outcomes. With accompanying transfers of $y$, tax harmonization that increases the sum of $O_i(\tau_i, d_i)$ can be designed to further the objectives of every country. In the absence of such transfers, a natural aggregation is to take a weighted sum of national objectives, with weights $w_i$ reflecting collective assessment of the relative importance of advancing the objectives of different governments. Denoting this weighted sum by $S$, it follows that
$$S = \sum O_i(\tau_i, d_i)w_i,$$

with \(\sum w_i = 1\), and \(w_i = 1/n\) if transfers of \(y\) are used to offset the distributional effects of collective tax measures.

2.4. Efficient tax harmonization.

An important alternative to independent tax setting is for all countries to harmonize their taxes at a common rate. Harmonized taxes at rate \(\tau_h\) yield aggregate objective satisfaction of

$$H \approx \sum O_i(\tau_{i*}, 0)w_i - \sum (\tau_{i*} - \tau_h)^2 \gamma_{i*}w_i.$$

The first order condition corresponding to maximizing (9) implies that the objective-maximizing harmonized tax rate \(\tau_{h*}\) is

$$\tau_{h*} \equiv \frac{\sum \tau_{i*} \gamma_{i*}w_i}{\sum \gamma_{i*}w_i}.$$

Equation (10) offers the entirely reasonable implication that the objective-maximizing harmonized tax rate is the weighted average of the tax rates that maximize individual country objectives in the absence of competition, with weights \(\gamma_{i*}w_i\).

If governments adopt (10) in harmonizing their tax rates, collective objectives are given by

$$H^* \approx \sum O_i(\tau_{i*}, 0)w_i - \sum \tau_{i*}^2 \gamma_{i*}w_i + \left[\frac{\sum \tau_{i*} \gamma_{i*}w_i}{\sum \gamma_{i*}w_i}\right]^2.$$

In evaluating (11), it is useful to define

$$\Delta \equiv \frac{\sum (\tau_{i*} - \tau_i) \gamma_{i*}w_i}{\sum \gamma_{i*}w_i}.$$
as the average extent to which tax competition reduces tax rates, with weights given by \( \gamma_i w_i \).

Applying this definition, \( \sum \tau_i^* \gamma_i w_i = \Delta \sum \gamma_i w_i + \sum \tau_i \gamma_i w_i \), and (11) implies

\[
H^* \approx \sum O_i \left( \tau_i^*, 0 \right) w_i - \sum \tau_i^2 \gamma_i w_i + \left[ \Delta + \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right]^2 \sum \gamma_i w_i.
\]

The definition of \( \Delta \) in (12), together with the formula in (5) and the aggregation rule in (8), means that (7) implies that, in the absence of harmonization,

\[
S \approx \sum O_i \left( \tau_i, 0 \right) w_i - \sum \tau_i^2 \gamma_i w_i + \sum \left( \tau_i - \bar{\tau} \right)^2 \left( \gamma_i + \gamma_{3i} + \gamma_{4i} \right) w_i
\]

\[
- \bar{\tau}^2 \sum \gamma_i w_i + 2 \bar{\tau} \sum \tau_i \gamma_i w_i + 2 \Delta \bar{\tau} \sum \gamma_i w_i.
\]

Using (13) and (14) to identify the effect of efficient tax harmonization on aggregate objective satisfaction,

\[
H^* - S \approx \left[ \Delta + \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} - \bar{\tau} \right]^2 \sum \gamma_i w_i - \sum \left( \tau_i - \bar{\tau} \right)^2 \left( \gamma_i + \gamma_{3i} + \gamma_{4i} \right) w_i.
\]

It follows from (15) that

\[
\frac{H^* - S}{\sum \gamma_i w_i} \approx \Delta^2 - \sum \left( \tau_i - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right)^2 \frac{\gamma_i w_i}{\sum \gamma_i w_i}
\]

\[
- \sum \left( \tau_i - \bar{\tau} \right)^2 \frac{\left( \gamma_{3i} + \gamma_{4i} \right) w_i}{\sum \gamma_i w_i} - 2 \Delta \left[ \bar{\tau} - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right].
\]

2.5 Implications.

Since \( \sum \gamma_i w_i > 0 \), efficient tax harmonization advances collective objectives if and only if the right side of (16) is positive. The first term on the right side of (16) is the square of the effect of tax competition on average tax rates, and the second term is the weighted variance of \( \tau_i \), with weights given by \( \frac{\gamma_i w_i}{\sum \gamma_i w_i} \). If \( \gamma_{3i} = \gamma_{4i} = 0 \), so there is no strategic interaction in tax
setting, and \( v_i = \frac{\gamma_i w_i}{\sum \gamma_i w_i} \), which makes the tax rate average calculated using objective-related weights \( \gamma_i w_i \) equal to the tax rate average relevant for country comparisons, then the third and fourth terms are both zero, and (16) is positive if the weighted variance of observed tax rates is less than the squared effect of tax competition on rates. Expressed differently, tax harmonization advances collective objectives if tax competition reduces average tax rates by more than the standard deviation of observed tax rates.

The standard deviation rule captures important aspects of the impact of tax harmonization. Tax harmonization is costly from the standpoint of achieving the objectives of governments with preferred tax rates that differ substantially from the harmonized rate. The aggregate cost of tax harmonization depends on the distribution of \( \tau_i^* \), which is unknown, though reflected in the distribution of observed tax rates – and that is why the variance term appears in (16). It remains the case that the effect of harmonization also depends on the values of \( \gamma_{3i} \), \( \gamma_{3i} \), and \( \gamma_{4i} \), which are likewise unknown, though it is nonetheless striking that the criterion for tax harmonization to advance collective objectives takes as simple a form as it does in (16).

Nonzero values of \( \gamma_{3i} \) or \( \gamma_{4i} \) modify the implications of (16). The third term on the right side of (16) is the interaction between squared deviations from mean tax rates and the \( \gamma_{3i} \) and \( \gamma_{4i} \) terms that appear in strategic interactions. If the \( \gamma_{3i} \) and \( \gamma_{4i} \) terms are positive, so that tax rates are strategic complements, then since squared deviations are also necessarily positive, it follows that \( \Delta \) must exceed the weighted standard deviation of tax rates in order for (16) to be positive. Countries choose tax rates that balance the costs of deviating from their preferred rates against the costs of deviating from the world average tax rate. Consequently, higher values of \( \gamma_{3i} \) and \( \gamma_{4i} \), which increase the cost of deviating from the world average tax rate, imply that any observed deviation from \( \bar{\tau} \) must be associated with a more costly deviation from the preferred rate and therefore higher costs of tax harmonization. If instead the \( \gamma_{3i} \) and \( \gamma_{4i} \) terms are negative, so tax rates are strategic substitutes, then (16) implies that there are lower costs of tax harmonization for any given observed tax rate variance.
Any differences between mean tax rates calculated using \( \nu_i \), the weight attached to country \( i \)'s tax rate in producing a world average for comparison purposes, and \( \frac{\gamma_i w_i}{\sum \gamma_i w_i} \), the collective assessment weight attached to deviations of country \( i \)'s tax rate from its preferred rate, also influence the implications of (16). This is evident from solving (16) for values of \( \Delta \) for which \( H^* - S = 0 \). Applying the quadratic formula to (16), and for convenience denoting

\[
\left( \frac{1}{\sigma^2} \right) \left( \sum \gamma_i w_i \right)^2 \sum \left( \tau - \bar{\tau} \right)^2 \left( \gamma_i + \gamma_i w_i \right) \sum \gamma_i w_i \] 

by \( \sigma^2 \), any solution \( \tilde{\Delta} \) must satisfy

\[
\tilde{\Delta} = \left[ \bar{\tau} - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right] \pm \sqrt{\sigma^2 + \left[ \bar{\tau} - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right]^2}.
\]

Restricting attention to the larger root, \( \tilde{\Delta} \) increases more than one-for-one with

\[
\left[ \bar{\tau} - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right].
\]

Since both \( \nu_i \) and \( \gamma_i w_i \) are likely to increase directly with the scale of country \( i \)'s economy, the average tax rates calculated using these weights may differ little if at all. Big countries with extensive business activity can be expected to have large \( \nu_i \) and \( \gamma_i w_i \) weights, but if the \( \nu_i \) weights were even more heavily concentrated among the highest-tax large countries, then the \( \left[ \bar{\tau} - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right] \) terms in (17) will be positive, requiring a larger value of \( \Delta \) for harmonization to advance collective objectives. If instead the \( \nu_i \) weights were less concentrated among high-tax-rate countries than are the collective objective weights, then the opposite would be the case.

The standard deviation rule carries an important implication for the range of potential objective-maximizing harmonized tax rates. From (10) and (12), the objective-maximizing harmonized tax rate is the sum of the average observed tax rate and the average effect of tax competition

\[
\tau_h^* = \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} + \Delta.
\]
If \( v_i = \frac{y_i w_i}{\sum y_i w_i} \) and \( (\gamma_{3i} + \gamma_{4i}) \) is nonnegative, so tax rates are not strategic substitutes, then (16) implies that in order for tax harmonization to advance government objectives it is necessary for \( \Delta \) to exceed the standard deviation of tax rates. It follows from (18) that an objective-maximizing harmonized rate must exceed the average tax rate plus the standard deviation of tax rates, both of which are observable, and which therefore identify a lower bound of the range of potential efficient harmonized tax rates.

2.6. Interpretation.

How can it be that the rule for efficient tax harmonization takes so simple a form as (16)? Equation (16) derives from (15), the terms of which correspond to a two-step decomposition, depicted in Figure 1, in which all tax rates are first unified at \( \bar{\tau} \), then subsequently increased to \( \tau_h^* \). Since the first step does not change \( \bar{\tau} \), its impact on country \( i \) objective satisfaction is given by the integral of \[ \left[ \frac{\partial Q_i(\tau_i, d_i)}{\partial \tau_i} + \frac{\partial Q_i(\tau_i, d_i)}{\partial d_i} \right] \] over the range from \( \tau_i \) to \( \bar{\tau} \). If \( \tau_i \) is an optimizing choice, then the derivative of country \( i \)'s objective level with respect to \( \tau_i \) is zero in the neighborhood of \( \tau_i \), though from (4) the relevant second derivative is nonzero and given by \(-2(\gamma_{3i} + \gamma_{4i})\). Since this second derivative is unchanging, it follows that the effect on country \( i \) welfare of replacing \( \tau_i \) with \( \bar{\tau} \) is given by \[ \left( \frac{1}{2} \right)(-2)(\gamma_{3i} + \gamma_{4i})(\bar{\tau} - \tau_i)^2 \], as in the Harberger triangle and analogous second order approximations to deadweight loss. The weighted sum of these losses appears as the second term on the right side of (15).

The second step of the decomposition is the movement of unified tax rates from \( \bar{\tau} \) to \( \tau_h^* \), which is most intuitively analyzed by starting at \( \tau_h^* \) and reducing tax rates to \( \bar{\tau} \). Since \( \tau_h^* \) maximizes aggregate objective satisfaction given by (9), the derivative of aggregate objectives with respect to \( \tau_h^* \) is zero at \( \tau_h^* \), but the second derivative is \(-2\sum \gamma_{3i}\). It follows that the effect on aggregate objectives of reducing the harmonized tax rate from \( \tau_h^* \) to \( \bar{\tau} \) equals

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\(^3\) Harberger (1964, 1971); Auerbach (1985); Hines (1998).
\[ \frac{1}{2} \left(-2 \sum \gamma_i \right) \left( \bar{\tau} - \tau_h^* \right)^2. \]

Applying (18) and reversing direction, this implies that the gain from increasing the harmonized rate from \( \bar{\tau} \) to \( \tau_h^* \) is given by \( \left( \Delta + \frac{\sum \tau_i \gamma_i \Delta_i}{\sum \gamma_i \Delta_i} - \bar{\tau} \right)^2 \sum \gamma_i \Delta_i \), which is the first term on the right side of (15).

As depicted in Figure 1, it is possible to analyze the potential move from independent tax setting to harmonization at \( \tau_h^* \) by comparing both to the hypothetical alternative of harmonizing taxes at \( \bar{\tau} \). Since independent tax setting is individually efficient, any loss in moving to \( \bar{\tau} \) is of second order; and the Taylor approximation in (1) makes this second order effect constant over its range, which is why the aggregate impact appears as a variance. The same process is at work in comparing \( \tau_h^* \) to \( \bar{\tau} \): there is no first order effect, and the Taylor approximation ensures that the second order effect of reducing harmonized tax rates is constant, so the loss depends on a squared term. It is the assumed constancy of these second order effects, and not the specific Taylor approximation in (1), that is responsible for the form that (16) takes.

If competitive tax reductions impact outcomes, then neither tax harmonization nor independent tax setting maximizes collective objectives, except in very special cases. Tax harmonization is insufficiently sensitive to individual country preferences; and individual tax choice fails to incorporate effects on others. This is clear for small potential changes: starting from harmonization at \( \tau_h^* \), there is scope to increase aggregate objective satisfaction by increasing some tax rates and reducing others while leaving the average unchanged. Appendix A considers the properties of tax rates that maximize (8) over the unrestricted choice of \( \tau_i \). These taxes correspond neither to (16), which characterizes efficient tax harmonization, nor to (3), which characterizes individual tax rate choice. Tax rates that maximize (8) are differentiated and either all higher or all lower than those that countries choose independently.

2.7. Extensions.

The specification of a country’s objective as \( O_i(\tau_i, d_i) \) imposes that the relevant feature of the tax rates of other countries is their weighted average. While this is a standard formulation
in tax competition models,\textsuperscript{4} it is possible that countries instead care about pairwise comparisons of their tax rates to those of others, which requires considering $O_i(\tau_i, \mathbf{d}_i)$, with $\mathbf{d}_i$ a vector of differences between country $i$'s tax rate and those of every other country. Given the large number of countries in the world, a second-order Taylor approximation to an objective function that incorporates pairwise comparisons would have tens of thousands of unobserved parameters, rendering it largely infeasible to analyze. A restricted version of this model is given by

$$O_i(\tau_i, \mathbf{d}_i) \approx O_i(\tau^*_i, \mathbf{0}) - (\tau_i - \tau^*_i)^2 \gamma_{ii} - \sum_j (\tau_i - \tau_j) v_j \gamma_{2i} - \sum_j (\tau_i - \tau'_j) (\tau_i - \tau_j) v_j \gamma_{4i},$$

which limits consideration to cases in which the preference coefficients on all pairwise comparisons are the same for any given country, though may differ between them.

As shown in Appendix B, the model described by (19) produces implied choices of $\tau_i$ that are the same as those in (3), and objective satisfaction levels under harmonization that are the same as in (9), but with independent tax setting produces collective objective satisfaction that differs slightly from (14). As a result, the comparison between harmonization and independent tax setting is modified by replacing $-\sum (\tau_i - \bar{\tau})^2 \sum \gamma_{3i} w_i$ on the right side of (16) with

$$-\sum (\tau_i - \bar{\tau})^2 \left[ \frac{\sum \gamma_{3i} w_i - v_i}{\sum \gamma_{3i} w_i} \right] \sum \gamma_{3i} w_i.$$  This modification generally has the effect of reducing the impact of the $\gamma_{3i}$ terms,\textsuperscript{5} which dampens any effect of strategic complementarity or strategic substitutability on the comparison between harmonization and independent tax setting. Consequently, if tax rates are strategic complements because $\gamma_{3i} > 0$, then the model in which countries care about pairwise tax rate comparisons requires that the average effect of tax competition exceed the standard deviation of tax rates by somewhat less than otherwise in order for harmonization to advance collective objective satisfaction.

\textsuperscript{4} Keen and Konrad (2013) offer an analytical review of this literature.
One of the important features of (16) is that it arises from imposing (10), the objective-maximizing harmonized tax rate $\tau_h^*$. Adoption of $\tau_h^*$ as a harmonized rate requires exact knowledge of aggregate desired tax rates in the absence of competition, or equivalently $\Delta$, the effect of tax competition on aggregate tax rates. To the extent that there is uncertainty over the value of $\Delta$, then tax harmonization is apt to produce an outcome that is less consistent with collective objectives than appears in equation (10). For example, if instead of adopting $\tau_h^*$ as the harmonized rate, governments instead were to adopt $\tau_h^* + \varepsilon_h$, then as shown in Appendix C, the effect is to replace $\Delta^2$ in (16) with $\left(\Delta^2 - \varepsilon_h^2\right)$. Even unbiased estimates of $\Delta$ that are used to determine $\tau_h^*$ will have positive expected values of $\varepsilon_h^2$, thereby reducing expected satisfaction levels under tax harmonization and requiring downward adjustments to $\Delta^2$ in (16).


In order to apply (16) it is necessary to specify the weights used to calculate the variance and other terms in the expression, an exercise complicated by the reality that these weights are unknown. If collective decision makers attach equal weight to costs imposed on different countries, either because they anticipate making transfers to offset distributional consequences, or for other reasons, then $w_i$ is the same for all, and the remaining weights capture relative willingness to pay to avoid disfavored tax rates. If willingness to pay is proportional to taxable business income and therefore GDP, then the first term on the right side of (16) is the square of the effect of tax competition on GDP-weighted average tax rates, and the second term is the GDP-weighted tax rate variance. Using GDP weights to proxy for relies on the assumption that countries with similar taxable business incomes find it equally costly to deviate from their preferred business tax rates. While this is entirely plausible, it need not be the case, since economic and political conditions differ, as a result of which some

\footnote{Notably, the impact of the $\gamma_{s_i}$ terms becomes zero if $v_i = \frac{\gamma_{u_i}w_i}{\sum \gamma_{u_i}w_i}$ and $\gamma_{s_i} = \gamma_{s_i}v_i, \forall i$.}
countries may feel more strongly than others about taxing at their preferred rates. In the absence of detailed information on country preferences, GDP weights are reasonable choices, capturing the obvious effects of economic scale on the consequences of taxes and therefore the amounts that countries are likely willing to pay.

Table 1 presents means and standard deviations of statutory corporate tax rates around the world, using data for 2020 reported by the Tax Foundation. The data indicate that, for the 224 countries and territories for which the Tax Foundation report data, the unweighted mean tax rate in 2020 was 22.58%, with a standard deviation of 9.18%. Instead weighting these figures by population, the mean corporate tax rate was 26.72%, with a standard deviation of 4.60%. GDP data are available for a subset of 178 these countries and territories that generally omits smaller jurisdictions. In this subset, and weighting the calculations by GDP, the mean corporate tax rate was 25.85%, with a standard deviation of 4.54%. It is noteworthy that the population-weighted and GDP-weighted calculations produce very similar standard deviations, both of which suggest that statutory tax rate harmonization has the potential to advance collective objectives only if the effect of tax competition is to reduce (weighted) average tax rates by more than 4.6%. Furthermore, the objective-maximizing harmonized tax rate exceeds 30.4% in the case of GDP weights and exceeds 31.3% in the case of population weights.

The figures in Table 1 carry implications for the effect of strategic tax setting behavior on the potential for objective-enhancing tax harmonization. If \( \gamma_i = \gamma_3 \gamma_i \) and \( \gamma_{4i} = \gamma_4 \gamma_i, \forall i \), so that all countries have the same value of \( \frac{d \tau_i}{d \bar{\tau}} \), and \( \bar{\tau} = \frac{\sum \tau_i \gamma_{4i} w_i}{\sum \gamma_{4i} w_i} \), then tax harmonization advances collective objectives if and only if tax competition reduces average tax rates by more than the product of \( \sqrt{1 + \gamma_3 + \gamma_4} \) and the standard deviation of observed tax rates. If \( \gamma_3 = 1 \) and \( \gamma_4 = 0.2 \) for all countries, which from (3) would imply that \( \frac{d \tau_i}{d \bar{\tau}} = 0.5 \), then since \( \sqrt{1 + \gamma_3 + \gamma_4} = 1.48 \), it follows that with GDP weights statutory tax rate harmonization has the potential to advance collective objectives only if the effect of tax competition is to reduce average tax rates by more than 6.7% – and the objective-maximizing harmonized rate is 32.6% or higher. This is a

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significant upward adjustment to the required effect of tax competition, albeit based on the assumption that tax rates react very strongly to world averages. If instead $\gamma_3 = 0.4$ and $\gamma_4 = 0.1$, so that $\frac{d\tau_i}{d\tau} = 0.3$ and $\sqrt{1 + \gamma_3 + \gamma_4} = 1.22$, then tax harmonization has the potential to advance collective objectives only if the effect of tax competition is to reduce average tax rates by more than 5.49%, a more modest adjustment.

While the statutory corporate tax rate is a very important component of the effective corporate tax burden, rules concerning income inclusions, the availability of tax credits and deductions, and other aspects of tax base definitions also play important roles. Consequently, an analysis of statutory corporate tax rates offers an incomplete picture of relative tax burdens – though is informative about the effects of harmonizing statutory corporate tax rates. In practice, corporate tax rate changes tend to be accompanied by tax base changes (Kawano and Slemrod, 2016), which is why international agreements to harmonize taxes are likely to include restrictions to any offsetting tax base changes that countries might otherwise be inclined to adopt.


Minimum required tax rates are important alternatives to complete tax harmonization. Minimum taxes partition the world into two endogenous groups: countries in group A, for whom the required minimum tax rate does not impose a binding constraint, and countries in group B, for whom it does. If $\tau_m$ is the minimum tax rate, then under a minimum tax regime every country in group B imposes that tax rate. Countries in group A impose tax rates $\hat{\tau}_i$ that are not directly affected by the minimum tax requirement but nonetheless may differ from their currently observed tax rates, since minimum taxes change world average tax rates, which then may influence the tax rates that countries in group A choose.

4.1. Minimum tax features.

Countries in group A have objective satisfaction levels of

$$ O_i(\hat{\tau}_i, \hat{\tau}_i - \tau_m) \approx O_i(\tau^*_i, 0) - (\hat{\tau}_i - \tau^*_i)^2 \gamma_{2i} - (\hat{\tau}_i - \tau_m)^2 \gamma_{3i} - (\hat{\tau}_i - \tau^*_i)(\hat{\tau}_i - \tau_m) \gamma_{4i}, \nonumber $$

(20)
in which

\[ (21) \quad \bar{\tau}_m = \sum_A \hat{\tau}_i + \tau_m \sum_B \nu_i \]

is the average tax rate under the minimum tax regime, and (3) implies that

\[ (22) \quad \hat{\tau}_i = \tau_i + \frac{(\bar{\tau}_m - \bar{\tau}) \left( \gamma_{3i} + \frac{\gamma_{4i}}{2} \right)}{(\gamma_{1i} + \gamma_{3i} + \gamma_{4i})}. \]

Countries in group A are those for which \( \hat{\tau}_i > \tau_m \). Since \( \hat{\tau}_i \) is a function of the unknown parameters \( \gamma_{3i} \) and \( \frac{\gamma_{4i}}{2} \), it is impossible to infer from \( \tau_i \) alone which countries would fall into groups A and B, since even a low tax rate country might respond to \( \bar{\tau}_m > \bar{\tau} \) by so increasing its tax rate that it would fall in group A. Consequently, it is necessary to restrict the range of possible strategic interactions in order to apply the theory to tax rate data. This section proceeds by assuming that all countries have the same value of \( \frac{d\tau_i}{d\bar{\tau}} \), and specifically that \( \gamma_{3i} = \gamma_3 \gamma_{1i} \) and \( \gamma_{4i} = \gamma_4 \gamma_{1i}, \forall i \). Under these circumstances, it follows from (21) and (22) that

\[ (23) \quad \bar{\tau}_m = \bar{\tau} + \frac{\tau_m \sum_B \nu_i - \sum_B \bar{\tau}_i \nu_i}{1 - \frac{(\gamma_3 + \frac{1}{2})}{(1 + \gamma_3 + \gamma_4)} \sum_A \nu_i}. \]

With common values of the strategic interaction terms \( \gamma_3 \) and \( \frac{\gamma_4}{2} \), every minimum tax rate \( \tau_m \) has an associated critical value \( \tau_c \) for which all countries with \( \tau_i \leq \tau_c \) will be constrained by the minimum tax and therefore in group B, whereas those with \( \tau_i > \tau_c \) are unconstrained and in group A. From (3), this critical value satisfies

\[ \tau_m = \tau_c + \frac{(\bar{\tau}_m - \bar{\tau}) \left( \gamma_3 + \frac{\gamma_4}{2} \right)}{(1 + \gamma_3 + \gamma_4)}; \]

applying (23) yields
(24) \[
\tau_m = \tau_c + \left(\frac{\gamma_3 + \frac{\gamma_4}{2}}{1 + \frac{\gamma_4}{2}}\right) \left[\tau_c \sum_B V_i - \sum_B \tau_i V_i\right],
\]

which also implies that

(25) \[
\overline{\tau}_m = \overline{\tau} + \left(\frac{1 + \gamma_3 + \frac{\gamma_4}{2}}{1 + \frac{\gamma_4}{2}}\right) \left[\tau_c \sum_B V_i - \sum_B \tau_i V_i\right].
\]

Countries in group B have objective satisfaction levels of

(26) \[
O_i(\tau_m, \tau_m - \overline{\tau}_m) \approx O_i(\hat{\tau}_i, 0) - (\tau_m - \overline{\tau})^2 \gamma_{1i} - (\tau_m - \overline{\tau}) \gamma_{2i} - (\tau_m - \overline{\tau})^2 \gamma_{3i} - (\tau_m - \overline{\tau})(\tau_m - \overline{\tau}) \gamma_{4i}.
\]

Denoting aggregate objective satisfaction under a minimum tax by \(M\), and applying (20) and (26) to (8), it follows that

(27) \[
M = \sum_A O_i(\hat{\tau}_i, \tau_m - \overline{\tau}_m) w_i + \sum_B O_i(\tau_m, \tau_m - \overline{\tau}_m) w_i.
\]

Minimum taxes affect the satisfaction levels of group B countries by directly constraining their tax choices, and affects the satisfaction levels of all countries by influencing the world average tax rate. It simplifies the analysis to consider these two effects separately.

4.2. Efficient minimum tax rates.

Differentiating \(M\) with respect to \(\tau_m\), and imposing (24) and (26), it follows that

(28) \[
\frac{1}{2} \frac{\partial M}{\partial \tau_m} \approx \left(1 + \frac{\gamma_4}{2}\right) \sum_B \tau_i \gamma_{1i} w_i + \left(\gamma_3 + \frac{\gamma_4}{2}\right) \sum_B (\tau_i - \overline{\tau}) \gamma_{1i} w_i
\]

\[
- \left(1 + \gamma_3 + \frac{\gamma_4}{2}\right) \sum_B \gamma_{1i} w_i + \left(\gamma_3 + \frac{\gamma_4}{2}\right) \sum_B \overline{\tau}_m \gamma_{1i} w_i.
\]

Imposing (25), (28) implies that
Equation (29) is analogous to the second term on the right side of (15) in the context of tax harmonization, and arises from the same source, which is the second-order loss from requiring a country to impose taxes at other than their preferred rates. It is noteworthy that on the right side of (29) the relevant tax rate constraint is $\tau_c$, not $\bar{\tau}_m$, reflecting that the higher $\bar{\tau}_m$ associated with a minimum tax itself changes desired tax rates and therefore the extent to which the minimum tax constraint binds.

Differentiating $M$ with respect to $\bar{\tau}_m$,

\[
\frac{1}{2} \frac{\partial M}{\partial \bar{\tau}_m} \approx \left( \gamma_3 + \frac{\gamma_4}{2} \right) \sum_i \bar{\tau}_i \gamma_i w_i + \left( \gamma_3 + \frac{\gamma_4}{2} \right) \tau_c \sum_i \gamma_i w_i - \gamma_3 \bar{\tau}_m \sum_i \gamma_i w_i - \frac{\gamma_4}{2} \sum_i \bar{\tau}_i \gamma_i w_i.
\]

Using (17) to replace $\sum_i \tau^*_i \gamma_i w_i$ with $\Delta \sum_i \gamma_i w_i + \sum_i \bar{\tau}_i \gamma_i w_i$, and imposing (24) and (25), (30) implies that

\[
\frac{1}{2} \frac{\partial M}{\partial \bar{\tau}_m} \approx \Delta \sum_i \gamma_i w_i - \frac{\gamma_4}{2} \sum_i (\bar{\tau}_i - \bar{\tau}_m) \gamma_i w_i + \left( \gamma_3 + \frac{\gamma_4}{2} \right) \left[ \tau_c \sum_i \gamma_i w_i - \sum_i \bar{\tau}_i \gamma_i w_i \right]
\]

\[
+ \left[ \frac{\gamma_4}{2} - \gamma_3 \right] \sum_i \gamma_i w_i \left\{ \tau_c \sum_i V_i - \sum_i \bar{\tau}_i V_i \right\}.
\]

Equations (24) and (25) together imply that

\[
\bar{\tau}_m = \bar{\tau}_m \left[ \frac{1 + \frac{\gamma_4}{2} + \left( \gamma_3 + \frac{\gamma_4}{2} \right) \sum_i V_i}{(1 + \gamma_3 + \gamma_4) \sum_i V_i} - \left( \gamma_3 + \frac{\gamma_4}{2} \right) \left( \frac{1}{1 + \frac{\gamma_4}{2}} \right) \sum_i \bar{\tau}_i V_i \right],
\]

from which it follows that
\[
\frac{\partial \tau_m}{\partial \tau_m} = \left[ 1 + \frac{\gamma_4}{2} + \left( \frac{\gamma_3 + \gamma_4}{2} \right) \sum_B v_i \right] \left( \frac{1 + \gamma_4}{1 + \gamma_3 + \gamma_4} \right) \sum_B v_i .
\]

Consequently,

\[
\frac{1}{2} \left[ \frac{\partial M}{\partial \tau_m} + \frac{\partial M}{\partial \tau_m} \frac{\partial \tau_m}{\partial \tau_m} \right] \approx \Delta \sum \gamma_i w_i - \frac{\gamma_4}{2} \sum_i \left( \tau_i - \bar{\tau} \right) \gamma_i w_i - \left[ \tau_c \sum_B \gamma_i w_i - \sum_B \tau_i \gamma_i w_i \right] \left( \frac{1 + \gamma_4}{2} \right) \sum_B v_i .

\]

\[
+ \left[ \tau_c \sum_B v_i - \sum_B \tau_i v_i \right] \left[ \frac{\gamma_4}{2} \left( \frac{\gamma_3 + \gamma_4}{2} \right) \right] - \gamma_3 \sum_i \gamma_i w_i
\]

At an objective-maximizing minimum tax rate, denoted \( \tau_m^* \), with corresponding critical tax rate \( \tau_c^* \), the right side of (33) is zero.\(^7\) In interpreting (33) it is useful to consider special cases. If \( \gamma_3 = \gamma_4 = 0 \), so there is no strategic interaction in tax setting, then from (24), \( \tau_m = \tau_c \), and setting (33) equal to zero implies that

\[
\tau_m^* = \tau_c^* = \frac{\sum_B \tau_i \gamma_i w_i}{\sum_B \gamma_i w_i} + \Delta \frac{\sum_B v_i / \sum_i v_i}{\sum_B \gamma_i w_i / \sum_i \gamma_i w_i} .
\]

If \( v_i = \frac{\gamma_i w_i}{\sum \gamma_i w_i} \), then (34) simplifies to

\[
\tau_m^* = \tau_c^* = \frac{\sum_B \tau_i \gamma_i w_i}{\sum_B \gamma_i w_i} + \Delta .
\]

Equation (35) indicates that the objective-maximizing minimum tax rate is the sum of the average tax rate of constrained countries and the amount by which competition reduces average tax rates. Of course, the set of group B countries whose tax rates are constrained by the

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\(^7\) Since (32) implies that there is a monotonic relationship between \( \tau_m \) and \( \tau_m^* \), maximizing \( M \) over the choice of \( \tau_m \) is equivalent to maximizing \( M \) over the choice of \( \tau_m^* \).
minimum tax rule is itself a function of the minimum tax rate; but to find \( \tau_m^* \), it is simply necessary to use tax rate data to search for values of \( \tau_m \) for which \( \tau_m - \frac{\sum_B \tau_i' y_i w_i}{\sum_B y_i w_i} = \Delta \).

An important feature of (35) is that the relevant value of \( \Delta \) in (35) is that for all countries, not just the affected group B whose tax rates would be constrained by the minimum rate. This makes the rule easy to apply, and captures two distinct effects of a minimum tax rate, the first of which is to harmonize the tax rates of countries in group B. Restricting attention to the collective objectives of group B would, applying (18), entail setting \( \tau_m^* \) equal to the average tax rate of group B countries plus the average amount by which competition reduces group B tax rates. But since a minimum tax also affects countries in group A by increasing the average tax rate, the values that group A countries attach to having competitive tax rates also matter for the objective-maximizing tax rate. Appendix D decomposes these effects and shows how they are reflected in the single value \( \Delta \).

If \( \nu_i \neq \frac{y_i w_i}{\sum y_i w_i} \), then (34) indicates that the extent to which \( \Delta \) affects \( \tau_m^* \) depends on the ratio \( \frac{\sum_B \nu_i / \sum_B y_i w_i}{\sum_B y_i w_i} \), the numerator of which is the relative weight on group B countries in calculating the world average tax rate, and the denominator of which is the relative weight on group B tax rate preferences in collective objectives. If this ratio is well below one, then higher values of \( \Delta \) have little impact on \( \tau_m^* \), reflecting that increasing the tax rates of group B countries hardly changes \( \tau_m \), whereas it adversely impacts the objectives of group B countries. If instead this ratio significantly exceeds one, then \( \Delta \) has an outsized impact on \( \tau_m^* \), since collective decisions attach little weight to costs imposed on group B countries compared to potential gains to all from having higher average tax rates.

If tax rates are strategic complements or substitutes, so \( \gamma_3 \) and \( \gamma_4 \) are nonzero, but \( \nu_i = \frac{y_i w_i}{\sum y_i w_i} \), then (33) implies that the objective-maximizing critical tax rate is
Equation (36) differs from (35) in that the denominator of the second term on the right side is

\[ 1 + \frac{\frac{\gamma_3 + \gamma_4}{2}}{1 + \frac{\gamma_4}{2}} \sum_{\beta} \gamma_{i\beta} \bar{w}_i + \gamma_4 \left( 1 - \frac{\sum_{\beta} \gamma_{i\beta} \bar{w}_i}{\sum \gamma_{i\beta} \bar{w}_i} \right) \]

Positive values of \( \gamma_3 \) and \( \gamma_4 \), which make tax rates strategic complements, dampen the effect of \( \Delta \) on \( \tau^*_c \); the opposite happens if \( \gamma_3 \) and \( \gamma_4 \) are both negative, in which case tax rates are strategic substitutes. Equation (36) can be applied to tax rate data by searching for points at which

\[ \left[ \tau_c - \frac{\sum_{\beta} \tau_{i\beta} \bar{w}_i}{\sum \gamma_{i\beta} \bar{w}_i} \right] \left[ 1 + \frac{\frac{\gamma_3 + \gamma_4}{2}}{1 + \frac{\gamma_4}{2}} \sum_{\beta} \gamma_{i\beta} \bar{w}_i + \gamma_4 \left( 1 - \frac{\sum_{\beta} \gamma_{i\beta} \bar{w}_i}{\sum \gamma_{i\beta} \bar{w}_i} \right) \right] \]

equals \( \Delta \). Using the implied critical values, and applying (24) to (36), the resulting efficient minimum tax rate is

\[ \tau^*_m = \frac{\sum_{\beta} \tau_{i\beta} \bar{w}_i}{\sum \gamma_{i\beta} \bar{w}_i} + \Delta \left( 1 - \frac{\gamma_4 \left( 1 - \frac{\sum_{\beta} \gamma_{i\beta} \bar{w}_i}{\sum \gamma_{i\beta} \bar{w}_i} \right)}{1 + \frac{\frac{\gamma_3 + \gamma_4}{2}}{1 + \frac{\gamma_4}{2}} \sum_{\beta} \gamma_{i\beta} \bar{w}_i + \gamma_4 \left( 1 - \frac{\sum_{\beta} \gamma_{i\beta} \bar{w}_i}{\sum \gamma_{i\beta} \bar{w}_i} \right) \right) \]

Equation (37) differs from (35) in two respects. The first is that, since \( \tau^*_m \neq \tau^*_c \), the group B countries whose average tax rate is the first term on the right side of (37) are not those with tax rates below \( \tau^*_m \), but instead those with tax rates below \( \tau^*_c \). The second respect is that the second term on the right side of (37) includes an adjustment to \( \Delta \). The \( \gamma_4 \) term that appears in (37) is
the ratio of the coefficient in equation (1) on the interaction between the deviation of actual and desired tax rates and deviation of a country’s tax rate from the world average to the coefficient on the squared deviation of a country’s tax rate from its desired rate. It is reasonable to expect the perceived marginal cost of deviating from a preferred tax rate to increase much more with deviations from preferred rates than with deviations from world averages, in which case the magnitude of \( \gamma_{1i} \) will significantly exceed that of \( \gamma_{4i} \), and make equation (37) closely approximate \( \tau^*_m = \frac{\sum_{B} \tau_{W_i}}{\sum_{B} W_j} + \Delta \). Consequently, the primary impact of strategic tax setting behavior on \( \tau^*_m \) arises through its modification to the impact of \( \Delta \) in determining \( \tau^*_c \) in (36).

4.3. Dominated minimum tax rate regions.

Equation (35) carries the implication that if tax competition reduces average tax rates, so \( \Delta > 0 \), and there are no strategic interactions because \( \gamma_{1} = \gamma_{4} = 0 \), then a low rate of minimum taxation increases collective objective attainment compared to a regime with no minimum taxes. At a very low minimum tax rate, \( \left[ \frac{\sum_{B} \tau_{W_i}}{\sum_{B} \gamma_{1i} W_j} - \tau_c \right] \) is close to zero, but \( \Delta \) will not be, so the right side of (33) is positive, and therefore \( dM/d\tau_m > 0 \). Consequently, in a broad range of cases minimum taxes advance collective objectives – and by the same reasoning, do so more effectively than harmonization, since replacing harmonized rates with a minimum tax at the same level affords countries greater choice while also supporting a higher world average tax rate. There remains the question of which minimum tax rates represent efficient choices.

One of the important implications of (33) is that multiple solutions are possible, depending on the distribution of average tax rates in the data. As noted earlier, this multiplicity arises because minimum tax rules bear only on countries whose rates are otherwise below the required minimum, so the only way to increase the tax rates of a group of countries is to have a minimum tax at a rate above that which they would otherwise choose. Mechanically, the average tax rate of group B countries increases with \( \tau_c \) at a rate that may be quite high over certain tax rate ranges. If average tax rates occasionally increase more than one for one with \( \tau_c \),
as will be the case if the distribution of national tax rates is strongly concentrated at certain rates such as 20% and 25%, then $\tau_c - \frac{\sum \gamma_i \tau_i \gamma_i \gamma_i \gamma_i}{\sum \gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}$ will be decreasing in $\tau_c$ over some ranges while increasing over others, as a result of which more than one value of $\tau_c$ may satisfy (33).

Objective functions with both convex and nonconvex regions commonly feature multiple solutions. In this case, the second order Taylor approximation removes one potential source of nonconvexity by ruling out higher order preference terms, but nevertheless the collective objective function may have nonconvex regions due to the unrestricted distribution of $\gamma_i$, and $\gamma_{2i}$ parameters and preferred tax rates $\tau^*_i$. If more than one value of $\tau_c$ satisfies (33) for a given value of $\Delta$, then it is necessary to use (24), (26) and (27) to evaluate which $\tau_c$ most advances collective objectives; Appendix E offers equation (E5), which can be used for this comparison. If there are nonconvex regions of the collective objective function, then the mapping from $\Delta$ to the associated $\tau^*_m$ will be discontinuous, with dominated ranges of minimum tax rates that do not maximize collective objectives at any level of $\Delta$.

5. **Analysis of Minimum Taxes with 2020 Data.**

It is evident from (33) that minimum tax rates that maximize collective objectives are functions both of $\Delta$ and of the distribution of observed tax rates. Consequently, it is possible to use the 2020 world corporate statutory tax rate data to identify the extent to which different possible minimum tax rates may be consistent with maximizing collective objectives.

It is useful to start by considering the case in which $\gamma_3 = \gamma_4 = 0$ and $\nu_i = \frac{\gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}{\sum \gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}$, for which, from (35), $\Delta = \tau_m - \frac{\sum \gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}{\sum \gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}$ characterizes a local objective-maximizing point. Figure 2 plots values of $\frac{\sum \gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}{\sum \gamma_i \gamma_i \gamma_i \gamma_i \gamma_i}$, using GDP weights, for 177 of the countries for which tax rate and
GDP data are available. As might be expected, the locus in Figure 2 exhibits sharp upward jumps at popular tax rates such as 20% and 25%. Figure 3 is the corresponding plot of\[\tau_m - \frac{\sum_i \tau_i Y_{ii} W_i}{\sum_i Y_{ii} W_i},\]again with GDP weights. It is clear from the multiplicity of values of \(\tau_m\) that share the same value of\[\tau_m - \frac{\sum_i \tau_i Y_{ii} W_i}{\sum_i Y_{ii} W_i}\]in Figure 3 that there will be multiple local objective-maximizing points for many values of \(\Delta\) between 2% and 8%. Applying (E5) from Appendix E to identify which of these points maximizes collective objective satisfaction, and omitting those that do not, yields Figure 4.

Figure 4 indicates that the objective-maximizing choice of \(\tau_m\) increases one-for-one with \(\Delta\) over the range 0-3.8%. At \(\Delta = 3.8\) there is a discontinuous jump in the objective-maximizing \(\tau_m\): at \(\Delta = 3.8\), collective objectives are maximized by \(\tau_m = 3.8\), whereas at \(\Delta = 3.81\), collective objectives are maximized by \(\tau_m = 27.33\). There is no value of \(\Delta\) for which minimum tax rates between 3.8% and 27.33% maximize collective objectives. And as Figure 4 also indicates, there is a subsequent noticeable, though smaller, discontinuous jump in the objective-maximizing \(\tau_m\) in the neighborhood of 30%.

Incorporating strategic interactions appears to affect these results rather little. The four panels in Figure 5 plot values of\[\tau_c - \frac{\sum_i \tau_i Y_{ii} W_i}{\sum_i Y_{ii} W_i} \left[1 + \frac{\gamma_3 + \gamma_4}{2}\left(\frac{1 + \gamma_4}{\sum_i Y_{ii} W_i} + \frac{\gamma_3}{2}\left(\frac{1}{\sum_i Y_{ii} W_i}ight)\right)\right]\]for four different scenarios: (i) in the upper left, \(\gamma_3 = \gamma_4 = 0\), which is the same as in Figure 3; (ii) in the upper right, \(\gamma_3 = 1\) and \(\gamma_4 = 0.2\), which implies that \(d\tau_c/d\bar{\tau} = 0.5\); (iii) in the lower left, \(\gamma_3 = 0.4\) and \(\gamma_4 = 0.1\), which implies that \(d\tau_c/d\bar{\tau} = 0.3\); and (iv) in the lower right, \(\gamma_3 = 0.25\) and \(\gamma_4 = 0\), which implies that \(d\tau_c/d\bar{\tau} = 0.2\). As the figure indicates, all four of these scenarios feature multiple local optima at intermediate ranges of \(\Delta\), and do so with roughly the same patterns. Figure 6 presents four panels that plot the corresponding objective-maximizing

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8 There are data for 178 countries, but the single country with the highest tax rate is outside the range of the figures.
choices of $\tau_m$. As is evident from all four panels, these choices again feature discontinuous jumps over similar ranges of minimum tax rates.

The evidence presented in figures 1-6 uses statutory corporate tax rates weighted by GDP. Figures 7-11 present corresponding figures produced using statutory corporate tax rates weighted by population, applying data for a larger sample of 222 countries. The data in Figure 8 clearly indicate that there are multiple local optima at values of $\Delta$ between 2% and 7.5%. The objective-maximizing choices of $\tau_m$ in Figure 9 again feature large discontinuous jumps, which appear in approximately the same places, and at roughly the same rates, as those for GDP-weighted calculations presented in Figure 4. Incorporating strategic tax rate interactions, as in the calculations depicted in Figures 10 and 11, produces only small changes in the values of implied minimum tax rates and does not change their patterns.

Figures 12-16 present the same calculations using unweighted corporate tax rate data for the same 178 countries for which GDP data are available. The data in Figure 13 imply that there are again multiple local optima, though over a 4%-9% range of $\Delta$ that somewhat differs from the corresponding ranges in the GDP- and population-weighted calculations. Figure 14 indicates that there are multiple discrete jumps in the objective-maximizing choices of $\tau_m$ over much of the range of $\Delta$. The figure indicates that, at low values of $\Delta$, the implied minimum tax rate increases roughly one-for-one with $\Delta$. At $\Delta = 5\%$ the implied minimum tax rate is roughly 7%, which increases to 27% as $\Delta$ rises to 7%. This sharp increase in $\tau_m$ is the product of several large discontinuous jumps, though there exist values of $\Delta$ between 5% and 7% for which minimum tax rates between 7-27% would represent objective-maximizing choices in a framework that assigns equal weights to every country and territory. Figures 15 and 16 display the product of calculations confirming that these patterns persist in the presence of strategic tax interactions, though it is noteworthy that in the scenario with $\gamma_3 = 1$ and $\gamma_4 = 0.2$ there are much less dramatic jumps in $\tau_m$ for values of $\Delta$ between 7% and 9.5.

6. **Tax Rates.**

Tax harmonization and minimum taxation are alternative methods of addressing tax competition, which itself is the inevitable product of deliberate tax policy adoption. Since
national tax policies are typically formed independently, competitive tax rate-setting can become a race to the bottom, producing tax rates that are very low or even zero. There is considerable controversy over the likelihood and course of such a race to the bottom in business tax rates, and a lively possibility that incentives to engage in tax exporting by imposing higher taxes, the burden of which is partially borne by foreigners, could offset or even reverse the race to the bottom. Many workhorse models of tax competition carry the implication that tax rates are strategic complements, though some have the feature that tax rates can be strategic substitutes, with countries reacting to foreign rate reductions by increasing their own tax rates.

Empirical investigation of the role of competition in corporate tax policy determination confronts a limited availability of exogenous changes with which to estimate the magnitudes of any competitive effects. Despite this challenge it is possible to draw important lessons from patterns in the data, the first and most obvious of which is that corporate tax rates are not all zero, thereby firmly rejecting the simplest version of a race to the bottom model. A second clear feature of recent experience is that statutory corporate tax rates have fallen significantly since 1980, which is consistent with countries adjusting their corporate tax systems to competitive pressures in an increasingly globalized world. Smaller countries tend to have lower tax rates.

9 See, for example, Zodrow and Mieszkowski (1986), Wilson (1986), Wildasin (1988), Black and Hoyt (1989), Bucovetsky and Wilson (1991), Bucovetsky (1991), and Baldwin and Krugman (2004). Davies and Eckel (2010), Haufler and Stahler (2013) and Niu (2017) note that if governments have limited tax instruments then with sufficient taxpayer heterogeneity there may not be a Nash equilibrium of any kind in the tax-setting game.


11 Many of these studies are surveyed in Wilson (1999) and Keen and Konrad (2013). Rota-Graziosi (2019) identifies sufficient conditions for the Nash game in tax rates to be supermodular, in which case the Nash equilibrium exists and has the property that tax rates are strategic complements. The Rota-Graziosi paper notes that it is much more straightforward to identify sufficient conditions for supermodularity when the government is assumed to choose tax rates to maximize tax revenue than when the government chooses tax rates to maximize welfare.


13 Konrad (2009) and Kiss (2012) consider the intriguing possibility that minimum tax agreements might actually reduce equilibrium tax rates by changing reaction functions or limiting the ability of countries to punish others for deviating from collusive agreements to maintain high tax rates. Peralta and van Ypersele (2006) and Hebous and Keen (2021) consider settings in which the introduction of a minimum tax, or a minimum tax coupled with a maximum tax, will advance the objectives of all countries.

14 This is documented by Slemrod (2004), Hines (2007), Ali Abbas and Klemm (2013), Keen and Konrad (2013), Azémar, Desbordes, and Wooton (2020), and numerous others.

which is likewise consistent with competition exerting significant pressures on tax rates.\textsuperscript{16} Estimated reaction functions often suggest that tax rates are strategic complements,\textsuperscript{17} though these findings may be sensitive to specifications that, if modified, can yield the opposite conclusion that tax rates are strategic substitutes.\textsuperscript{18}

The tax rates that countries choose provide valuable information on the objectives of their governments. Using this information to evaluate harmonized taxes and minimum tax requirements takes government objectives to be the basis of analysis. Government objectives include not only the criteria that countries use to determine the tax rates that they would choose in the absence of international competition, but also how they evaluate the effects of differences between a country’s tax rate and the world average. Since government objectives can be inconsistent with national welfare, it follows that the implications of tax rate choices for tax harmonization and minimum taxation, while informative about how governments would evaluate these policies, need not directly bear on economic welfare.

\textbf{7. \textit{Conclusion.}}

Countries choose tax policies based on many considerations, including revenue needs, economic conditions, distributional preferences, and prevailing notions of sound fiscal policy. Some governments tailor business taxes to make their countries attractive investment locations; and even countries without explicit tax-based economic development strategies generally try to avoid adopting tax systems that would excessively discourage business activity. There is valuable information about country preferences in the tax rates that they choose.

Tax competition appears to reduce business tax rates to levels below those that countries would otherwise choose. Coordinated action can address the effects of tax competition, but common coordination methods such as tax harmonization or minimum taxation require strict adherence to uniform rules that limit their appeal. As a result, tax harmonization can advance collective objectives only if the standard deviation of tax rates is less than the average effect of

\textsuperscript{16} See Bucovetsky (1991) and Haufler and Wooton (1999).
\textsuperscript{17} See Hayashi and Boadway (2001), Devereux, Lockwood and Redoano (2008), Overesch and Rincke (2011), and Altshuler and Goodspeed (2015); Leibrecht and Hochgatterer (2012) and Devereux and Loretz (2013) survey this literature.
\textsuperscript{18} See, for example, the analysis in Rork (2003), Chirinko and Wilson (2017), and Parchet (2019).
tax competition. Minimum tax rules afford greater flexibility, though here too the average effect of tax competition is of central concern, since it plus the average tax rate of affected countries equals the minimum tax rate that most effectively advances collective objectives. It is evident that a sound understanding of the impact of tax competition is an indispensable element in evaluating tax harmonization alternatives.

This paper analyzes international business taxation, but the second order approximation that is the basis of the analysis applies more generally to any competitive context. This includes subnational taxation and many other government policies with competitive implications, such as environmental and other business regulations, minimum wages, and others. The extent to which harmonizing any of these policies is consistent with advancing collective objectives should be a function of the standard deviation of the policies that jurisdictions choose when left on their own – and common minimum requirements may have the feature that there are broad ranges of dominated rates, as there are with business taxes.
References


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Table 1
World Corporate Tax Rate Means and Standard Deviations, 2020

<table>
<thead>
<tr>
<th>Sample</th>
<th>Weights</th>
<th>$\bar{\tau}$</th>
<th>$\sigma$</th>
<th>$(\bar{\tau} + \sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>224 countries</td>
<td>Unweighted</td>
<td>22.58</td>
<td>9.18</td>
<td>31.76</td>
</tr>
<tr>
<td>224 countries</td>
<td>Population</td>
<td>26.72</td>
<td>4.60</td>
<td>31.32</td>
</tr>
<tr>
<td>178 countries with GDP data</td>
<td>Unweighted</td>
<td>23.86</td>
<td>7.53</td>
<td>31.39</td>
</tr>
<tr>
<td>178 countries with GDP data</td>
<td>GDP</td>
<td>25.85</td>
<td>4.54</td>
<td>30.39</td>
</tr>
</tbody>
</table>

Note to Table 1: the table presents means ($\bar{\tau}$) and standard deviations ($\sigma$) of statutory corporate tax rates in 2020. The top two rows consider data reported by the Tax Foundation for 224 countries and territories, while rows 3-4 consider data for the 178 of these countries and territories for which it is possible to obtain GDP data. Unweighted means and standard deviations appear in rows 1 and 3, while these statistics are weighted by jurisdiction population in the row 2 figures and are weighted by GDP in the row 4 figures.
Figure 1
Decomposing the Criterion for Efficient Tax Harmonization

Independently chosen tax rates

Uniform tax rates

Loss from lower uniform tax rate:

$$-\frac{1}{2} \sum \frac{\partial^2 O_i}{\partial \tau_i^2} \left( \tau_i^*, 0 \right) \left( \bar{\tau} - \tau_i^* \right)^2 w_i =$$

$$\left[ \Delta + \sum \frac{\tau_i \gamma_i w_i}{\sum \gamma_i w_i} - \bar{\tau} \right]^2 \sum \gamma_i w_i$$

Loss from tax uniformity at unchanged average tax rate:

$$\frac{-1}{2} \sum \left[ \frac{\partial^2 O_i}{\partial \tau_i^2} (\tau_i, d_i) + \frac{\partial^2 O_i}{\partial d_i^2} (\tau_i, d_i) + 2 \frac{\partial^2 O_i}{\partial \tau_i \partial d_i} (\tau_i, d_i) \right] (\bar{\tau} - \tau_i)^2 w_i =$$

$$\sum (\tau_i - \bar{\tau})^2 (\gamma_{i1} + \gamma_{i3} + \gamma_{i4}) w_i$$

Figure 1 illustrates that independent tax setting can be evaluated relative to objective-maximizing tax harmonization by comparing both of these alternatives to a third possibility, uniform taxes at the original average tax rate. Replacing independently chosen tax rates with their mean value produces a second-order loss for every country, the aggregate value of which is

$$\sum (\tau_i - \bar{\tau})^2 (\gamma_{i1} + \gamma_{i3} + \gamma_{i4}) w_i.$$  

Replacing uniform tax rates of $$\tau_h^*$$ with uniform tax rates of $$\bar{\tau}$$ produces second-order losses with aggregate value

$$\left( \tau_h^* - \bar{\tau} \right)^2 \sum \gamma_i w_i = \left[ \Delta + \sum \frac{\tau_i \gamma_i w_i}{\sum \gamma_i w_i} - \bar{\tau} \right]^2 \sum \gamma_i w_i.$$
Figure 2
GDP-Weighted Average Statutory Tax Rates, 2020

Note: the figure plots average statutory corporate tax rates $\frac{\sum_{B} \tau_{i} \gamma_{i} w_{i}}{\sum_{B} \gamma_{i} w_{i}}$ of countries with tax rates equal to or less than $\tau_{m}$, with tax rates weighted by GDP.
Note: the figure plots \[
\tau_m - \frac{\sum_{b} \tau_{\gamma_i} w_i}{\sum_{b} \gamma_i w_i}
\] for different possible values of \( \tau_m \) using GDP weights.
Figure 4
Implied Objective-Maximizing Minimum Taxes with GDP Weights, 2020

Note: The figure presents objective-maximizing choices of $\tau_m$ (horizontal axis) corresponding to different values of $\Delta$ (vertical axis).
Figure 5
GDP-Weighted Tax Rate Differences with Strategic Interactions, 2020

Note: The four panels of Figure 5 use 2020 corporate statutory tax rate data, weighted by GDP, to plot values of
\[
\tau_c - \frac{\sum_{i} \gamma_i \tau_i w_i}{\sum_{i} \gamma_i w_i} \left[ 1 + \frac{1}{2} \sum_{i} \gamma_i w_i \right] \left[ 1 - \frac{1}{2} \sum_{i} \gamma_i w_i \right]
\]
for four different strategic tax setting scenarios: (i) in the upper left, \( \gamma_3 = \gamma_4 = 0 \); (ii) in the upper right, \( \gamma_3 = 1 \) and \( \gamma_4 = 0.2 \); (iii) in the lower left, \( \gamma_3 = 0.4 \) and \( \gamma_4 = 0.1 \); and (iv) in the lower right, \( \gamma_3 = 0.25 \) and \( \gamma_4 = 0 \).
Figure 6
Implied Objective-Maximizing Minimum Taxes with Strategic Tax Setting and GDP Weights, 2020

Note: The four panels of Figure 6 use 2020 corporate statutory tax rate data, weighted by GDP, to plot objective-maximizing choices of $\tau_m$ (horizontal axis) corresponding to different values of $\Delta$ (vertical axis) for four different strategic tax setting scenarios: (i) in the upper left, $\gamma_3 = \gamma_4 = 0$; (ii) in the upper right, $\gamma_3 = 1$ and $\gamma_4 = 0.2$; (iii) in the lower left, $\gamma_3 = 0.4$ and $\gamma_4 = 0.1$; and (iv) in the lower right, $\gamma_3 = 0.25$ and $\gamma_4 = 0$. 

Note: 177 countries
Note: the figure plots average statutory corporate tax rates \( \frac{\sum_{i} \gamma_i, w_i}{\sum_{i} \gamma_i, w_i} \) of countries with tax rates equal to or less than \( \tau_m \), with tax rates weighted by population.
Figure 8
Population-Weighted Tax Rate Differences, 2020

Note: the figure plots $\tau_m - \frac{\sum_{\theta} \gamma_{\theta} W_i}{\sum_{\theta} \gamma_{\theta}}$ for different possible values of $\tau_m$ using population weights.
**Figure 9**

Implied Objective-Maximizing Minimum Taxes with Population Weights, 2020

Note: The figure presents objective-maximizing choices of $\tau_m$ (horizontal axis) corresponding to different values of $\Delta$ (vertical axis).
Figure 10
Population-Weighted Tax Rate Differences with Strategic Interactions, 2020

Note: The four panels of Figure 10 use 2020 corporate statutory tax rate data, weighted by population, to plot values of
\[
\tau_c - \frac{\sum_B \gamma_{iB} w_i}{\sum_B \gamma_{iB} w_i} \left[ 1 + \frac{\left( \gamma_3 + \frac{\gamma_4}{2} \right) \frac{\sum_B \gamma_{iB} w_i}{\sum_B \gamma_{iB} w_i}}{1 + \frac{\gamma_4}{2}} \right] \left[ 1 - \frac{\sum_B \gamma_{iB} w_i}{\sum_B \gamma_{iB} w_i} \right]
\]
for four different strategic tax setting scenarios: (i) in the upper left, \( \gamma_3 = \gamma_4 = 0 \); (ii) in the upper right, \( \gamma_3 = 1 \) and \( \gamma_4 = 0.2 \); (iii) in the lower left, \( \gamma_3 = 0.4 \) and \( \gamma_4 = 0.1 \); and (iv) in the lower right, \( \gamma_3 = 0.25 \) and \( \gamma_4 = 0 \).
Figure 11
Implied Objective-Maximizing Minimum Taxes with Strategic Tax Setting and Population Weights, 2020

Note: The four panels of Figure 10 use 2020 corporate statutory tax rate data, weighted by population, to plot objective-maximizing choices of $\tau_m$ (horizontal axis) corresponding to different values of $\Delta$ (vertical axis) for four different strategic tax setting scenarios: (i) in the upper left, $\gamma_3 = \gamma_4 = 0$; (ii) in the upper right, $\gamma_3 = 1$ and $\gamma_4 = 0.2$; (iii) in the lower left, $\gamma_3 = 0.4$ and $\gamma_4 = 0.1$; and (iv) in the lower right, $\gamma_3 = 0.25$ and $\gamma_4 = 0$. 
Note: the figure plots average unweighted statutory corporate tax rates $\frac{\sum_{i,j} \tau_i \gamma_{ij} w_i}{\sum_{i} \gamma_{ij} w_i}$ of countries with tax rates equal to or less than $\tau_m$. 
Figure 13
Unweighted Tax Rate Differences, 2020

Note: the figure plots \( \tau_m \) for different possible values of \( \tau_m \) using unweighted statutory tax rates.
Figure 14
Implied Objective-Maximizing Minimum Taxes with Unweighted Data, 2020

Note: The figure presents objective-maximizing choices of $\tau_m$ (horizontal axis) corresponding to different values of $\Delta$ (vertical axis).
Figure 15

Unweighted Tax Rate Differences with Strategic Interactions, 2020

Note: The four panels of Figure 15 use unweighted 2020 corporate statutory tax rate data to plot values of

\[
\tau_c = \left[ \sum_{i} \gamma_i w_i \right] + \left[ \sum_{i} \gamma_i w_i \right] + \left[ \sum_{i} \gamma_i w_i \right] + \left[ \sum_{i} \gamma_i w_i \right]
\]

for four different strategic tax setting scenarios: (i) in the upper left, \( \gamma_3 = \gamma_4 = 0 \); (ii) in the upper right, \( \gamma_3 = 1 \) and \( \gamma_4 = 0.2 \); (iii) in the lower left, \( \gamma_3 = 0.4 \) and \( \gamma_4 = 0.1 \); and (iv) in the lower right, \( \gamma_3 = 0.25 \) and \( \gamma_4 = 0 \).
Figure 16
Implied Objective-Maximizing Minimum Taxes with Strategic Tax Setting and Unweighted Statutory Corporate Tax Rate Data, 2020

Note: The four panels of Figure 16 use unweighted 2020 corporate statutory tax rate data to plot objective-maximizing choices of $\tau_m$ (horizontal axis) corresponding to different values of $\Delta$ (vertical axis) for four different strategic tax setting scenarios: (i) in the upper left, $\gamma_3 = \gamma_4 = 0$; (ii) in the upper right, $\gamma_3 = 1$ and $\gamma_4 = 0.2$; (iii) in the lower left, $\gamma_3 = 0.4$ and $\gamma_4 = 0.1$; and (iv) in the lower right, $\gamma_3 = 0.25$ and $\gamma_4 = 0$. 
Appendix A

This appendix characterizes tax choices that maximize collective objectives.

If tax competition reduces affects tax rates, then neither tax harmonization nor unfettered tax competition maximizes collective objectives. Maximizing (8) over the unrestricted choice of \( \tau_i \) yields the first order condition

\[
(A1) \quad 2\gamma_{ii}(\tau^*_i - \tilde{\tau}_i)w_i - \gamma_{i2}w_i + 2\gamma_{i3}(\bar{\tau} - \tilde{\tau}_i)w_i + \gamma_{4i}(\bar{\tau} + \tau^*_i - 2\tilde{\tau}_i)w_i + \frac{\partial S}{\partial \bar{\tau}}v_i = 0, \forall i,
\]

in which \( \tilde{\tau}_i \) is the value of \( \tau_i \) that maximizes (8), and \( \frac{\partial S}{\partial \bar{\tau}} \) is given by

\[
(A2) \quad \frac{\partial S}{\partial \bar{\tau}} = \sum \gamma_{i2}w_i + 2\sum (\tilde{\tau}_i - \bar{\tau})\gamma_{i3}w_i + \sum (\tilde{\tau}_i - \tau^*_i)\gamma_{4i}w_i.
\]

Equation (A1) implies that

\[
(A3) \quad \tilde{\tau}_i = \tau_i + \frac{dS}{d\bar{\tau}}v_i \quad \text{for} \quad 2(\gamma_{ii} + \gamma_{i3} + \gamma_{4i})w_i.
\]

Equation (A3) indicates that the tax rates that maximize collective objective satisfaction differ from the rates that countries choose independently; furthermore, these objective-maximizing rates are nonuniform. Equation (A3) indicates that if \( \frac{\partial S}{\partial \bar{\tau}} > 0 \) then objective-maximizing tax rates all exceed the rates that countries choose independently; and the opposite is the case if \( \frac{\partial S}{\partial \bar{\tau}} < 0 \).
Appendix B

This appendix considers the implications of replacing $O_i(\tau_i, d_i)$ with $O_i(\tau_i', d_i)$, and therefore (1) with (19). Expanding equation (19),

\[ O_i(\tau_i, d_i) \approx O_i(\tau_i', 0) - (\tau_i' - \tau_i)^2 \gamma_{ii} - \left[ \tau_i - \sum_j \tau_j v_j \right] \gamma_{2i} \]

\[ - \left[ \tau_i^2 - 2\tau_i \sum_j \tau_j v_j + \sum_j \tau_j^2 v_j \right] \gamma_{3i} - (\tau_i - \tau_i') \left[ \tau_i - \sum_j \tau_j v_j \right] \gamma_{4i} \]  

(B1)

Differentiating (B1) with respect to $\tau_i$ yields the first order condition

\[ \frac{\partial O_i(\tau_i, d_i)}{\partial \tau_i} + \sum_j \frac{\partial O_i(\tau_i, d_i)}{\partial d_j} \approx 2\gamma_{ii}(\tau_i' - \tau_i) - \gamma_{2i} \]

\[ + 2\gamma_{3i} \left( \sum_j \tau_j v_j \right) + \gamma_{4i} \left( \sum_j \tau_j v_j + \tau_i' - 2\tau_i \right) \]

(B2)

Imposing $\sum_j \tau_j v_j = \bar{\tau}$, (B2) is identical to (2), and therefore (B2) implies (3) and (5), so the tax rates that countries choose to maximize (19) are the same as those they choose to maximize (1) – and as a result, tax rate choices cannot distinguish these models.

Equations (19) and (5) together imply that

\[ O_i(\tau_i, d_i) \approx O_i(\tau_i', 0) - (\tau_i' - \tau_i)^2 \gamma_{ii} - \tau_i^2 \gamma_{ii} + 2\tau_i^2 \gamma_{ii} + 2 \]

\[ - \left[ \tau_i - \left( \tau_i' - \bar{\tau} \right) \left( \gamma_{3i} + \frac{\gamma_{4i}}{2} \right) + \frac{\gamma_{2i}}{2} \right] \gamma_{ii} \]

\[ - \left( \tau_i - \bar{\tau} \right) \gamma_{2i} - \left( \tau_i - \bar{\tau} \right)^2 \gamma_{3i} - \sum_j \left( \tau_i - \bar{\tau} \right)^2 v_j \gamma_{3i} \]

\[ + \left( \tau_i - \tau_i' \right) \left( \gamma_{3i} + \frac{\gamma_{4i}}{2} \right) + \frac{\gamma_{2i}}{2} \gamma_{4i} \]

(B3)

Equation (B3) differs from (6) only in the inclusion of the $\sum_j (\tau_j - \bar{\tau})^2 v_j \gamma_{3i}$ term, so

\[ O_i(\tau_i, d_i) \approx O_i(\tau_i, d_i) - \sum_j (\tau_j - \bar{\tau})^2 v_j \gamma_{3i} \]  

(B4)
Equation (B4) implies that using $O_i(\tau_i, d_i)$ in place of $O_i(\tau, d_i)$ changes (16) to

\[
\frac{H^* - S}{\sum \gamma_i w_i} \approx \Delta^2 - \sum \left( \tau_i - \frac{\sum \gamma_i \tau_i w_i}{\sum \gamma_i w_i} \right)^2 \frac{\gamma_i w_i}{\sum \gamma_i w_i} - \sum \left( \tau_i - \bar{\tau} \right)^2 \left( \frac{\gamma_i w_i}{\sum \gamma_i w_i} \right) + \frac{\gamma_i w_i}{\sum \gamma_i w_i} \left( \sum \gamma_i w_i \right) \left( \sum \gamma_i w_i + \gamma_i w_i \right) - 2\Delta \left( \bar{\tau} - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right).
\]

(B5)
Appendix C

This appendix considers the implications of harmonizing taxes at something other than the rate $\tau^*_h$ in (10) that maximizes collective objectives.

If governments impose taxes at a harmonized rate $\tau_h = \tau^*_h + \varepsilon_h$, then from (9) and (10), collective objectives are given by

\[(C1) \quad H^* \approx \sum O_i (\tau^*_i, 0) w_i - \sum \tau^*_i \gamma_i w_i + \frac{\left[ \sum \tau^*_i \gamma_i w_i \right]^2}{\sum \gamma_i w_i} - \sum \gamma_i w_i \left[ \sum \tau^*_i \gamma_i w_i + \varepsilon_h \right]^2.
\]

It follows from (C1) that

\[(C2) \quad H^* \approx \sum O_i (\tau^*_i, 0) w_i - \sum \tau^*_i \gamma_i w_i + \frac{\left[ \sum \tau^*_i \gamma_i w_i \right]^2}{\sum \gamma_i w_i} - \varepsilon_h^2 \sum \gamma_i w_i.
\]

Equation (C2) differs from (11) only in the final term on the right side. Applying (C2) in place of (11), (13) becomes

\[(C3) \quad H^* \approx \sum O_i (\tau^*_i, 0) w_i - \sum \tau^*_i \gamma_i w_i + \left[ \Delta + \frac{\sum \tau^*_i \gamma_i w_i}{\sum \gamma_i w_i} \right]^2 \sum \gamma_i w_i - \varepsilon_h^2 \sum \gamma_i w_i.
\]

Using (C3) instead of (13) to derive (16) yields

\[(C4) \quad \frac{H^* - S}{\sum \gamma_i w_i} \approx \Delta^2 - \varepsilon^2_h - \sum \left( \tau_i - \frac{\sum \tau_i \gamma_i w_i}{\sum \gamma_i w_i} \right)^2 \frac{\gamma_i w_i}{\sum \gamma_i w_i} \sum \gamma_i w_i \left[ (\tau_i - \bar{\tau})^2 \frac{\gamma_i w_i}{\sum \gamma_i w_i} - 2\Delta \frac{\gamma_i w_i}{\sum \gamma_i w_i} \right].
\]

Equation (C4) differs from (16) only in replacing $\Delta^2$ with $\left(\Delta^2 - \varepsilon^2_h\right)$. 
Appendix D

This appendix identifies the components of \( \Delta \) that are responsible for its appearance in the rule for objective-maximizing tax rates in (35).

Section 4.2 of the paper notes that if \( \gamma_3 = \gamma_4 = 0 \), so there are no strategic tax rate interactions, and \( \nu_i = \frac{Y_i w_i}{\sum Y_i w_i} \), then (34) simplifies to (35), reproduced here as

\[
(D1) \quad \tau_m^* = \frac{\tau_0}{\sum B Y_i W_i} + \Delta.
\]

If \( \gamma_3 = \gamma_4 = 0 \), then from (5) and (12), \( \Delta = \frac{\sum B Y_i W_i}{\sum Y_i w_i} \), so

\[
(D2) \quad \Delta = \frac{\sum B Y_i W_i}{\sum B Y_i w_i} + \frac{\sum A Y_i W_i}{\sum A Y_i w_i} \left[ \frac{\sum A Y_i W_i}{\sum A Y_i w_i} - \frac{\sum B Y_i W_i}{\sum B Y_i w_i} \right].
\]

The first term on the right side of (D2) is the average amount by which perceived competition with other countries reduces the tax rates of group B countries. The second term on the right side of (D2) is the product of the collective objective weight on group A countries and the difference between the average effects of tax competition on group A and group B countries. If group A and group B countries do not differ in how they perceive the effects of tax rate comparisons, then this difference is zero, \( \Delta = \frac{\sum B Y_i W_i}{\sum B W_i} \), and the objective-maximizing minimum tax rate in (D1) is the same rate that countries in group B would choose as a harmonized rate to maximize group B collective objectives. If countries in group A attach greater weight to tax comparisons than do countries in group B, then \( \Delta > \frac{\sum B Y_i W_i}{\sum B W_i} \), and a
somewhat higher minimum tax rate will advance collective objectives; the opposite is the case if countries in group A attach less weight to tax comparisons than do countries in group B, and

$$\Delta < \frac{\sum_{g} \frac{y_{2i}}{2} w_{i}}{\sum_{g} w_{i}}.$$
Appendix E

This appendix presents expressions for collective satisfaction levels that can be used to compare multiple minimum tax rates for which the derivative in (33) is zero.

From (20) and (22) it follows that a country in group A has an objective satisfaction level given by

\[
O_i(\hat{\gamma}_i, \hat{\gamma}_i - \bar{\tau}_m) \approx O_i(\gamma_i^*, 0) - \gamma_i^2 \gamma_{yi} - \gamma_i^2 (1 + \gamma_3 + \gamma_4) \gamma_{yi} + 2\gamma_i \bar{\tau}_m (\bar{\gamma}_3 + \frac{\gamma_4}{2}) \gamma_{yi}
\]

\[
= \left(\gamma_3 + \frac{\gamma_4}{2}\right) \left(\bar{\gamma}_3 - \frac{\gamma_{yi}}{\gamma_3 + \gamma_4}\right) + \gamma_{yi} + 2\gamma_i \left(\bar{\tau}_m - \bar{\gamma}_3 + \frac{\gamma_4}{2}\right)
\]

\[
\left[\gamma_{yi} - 2\gamma_i \gamma_{yi} + 2\bar{\gamma}_m (\bar{\tau}_m - \bar{\gamma}_3 + \frac{\gamma_4}{2})\right]
\]

\[
= \left(\tau_i - \bar{\gamma}_3 + \frac{\gamma_4}{2}\right) \left(\gamma_3 + \frac{\gamma_4}{2}\right) + \frac{\gamma_{yi}}{\gamma_3 + \gamma_4}
\]

\[
O_i(\hat{\gamma}_i, \hat{\gamma}_i - \bar{\tau}_m) \approx O_i(\gamma_i^*, 0) - \gamma_i^2 \gamma_{yi} - \gamma_i^2 (1 + \gamma_3 + \gamma_4) \gamma_{yi} + 2\gamma_i \bar{\tau}_m (\bar{\gamma}_3 + \frac{\gamma_4}{2}) \gamma_{yi}
\]

\[
= \left(\gamma_3 + \frac{\gamma_4}{2}\right) \left(\bar{\gamma}_3 - \frac{\gamma_{yi}}{\gamma_3 + \gamma_4}\right) + \gamma_{yi} + 2\gamma_i \left(\bar{\tau}_m - \bar{\gamma}_3 + \frac{\gamma_4}{2}\right)
\]

\[
\left[\gamma_{yi} - 2\gamma_i \gamma_{yi} + 2\bar{\gamma}_m (\bar{\tau}_m - \bar{\gamma}_3 + \frac{\gamma_4}{2})\right]
\]

\[
= \left(\tau_i - \bar{\gamma}_3 + \frac{\gamma_4}{2}\right) \left(\gamma_3 + \frac{\gamma_4}{2}\right) + \frac{\gamma_{yi}}{\gamma_3 + \gamma_4}
\]

(E1)

Collecting terms and simplifying, (E1) implies that, for countries in group A,

\[
O_i(\hat{\gamma}_i, \hat{\gamma}_i - \bar{\tau}_m) \approx O_i(\gamma_i^*, 0) - \gamma_i^2 \gamma_{yi} - \gamma_i^2 (1 + \gamma_3 + \gamma_4) \gamma_{yi} + \bar{\tau}_m \gamma_{yi} + \frac{\gamma_{yi}}{1 + \gamma_3 + \gamma_4}
\]

\[
= \left(\tau_i - \bar{\gamma}_3 + \frac{\gamma_4}{2}\right) \left(\gamma_3 + \frac{\gamma_4}{2}\right) + \frac{\gamma_{yi}}{1 + \gamma_3 + \gamma_4}
\]

(E2)
Equations (22) and (26) together imply that a country in group B has an objective satisfaction level given by

\[ O_i(\tau_m, \tau_m - \bar{\tau}_m) \approx O_i(\tau_i^*, 0) - \tau_i^2 \gamma_{i1} - \tau_m^2 \gamma_{i1} + 2\tau_m \tau_i \gamma_{i1} \]

(E3)

\[ + 2\tau_m \frac{\left(\tau_i - \bar{\tau}_i\right)\left(\gamma_3 + \frac{\gamma_4}{2}\right) + \frac{\gamma_{2i}}{2\gamma_{i1}}}{\frac{1}{1 + \frac{\gamma_4}{2}}} \gamma_{i1} - \left(\tau_m - \bar{\tau}_m\right)\gamma_{2i} - \left(\tau_m - \bar{\tau}_m\right)^2 \gamma_3 \gamma_{i1} \]

\[ - \tau_m \left(\tau_m - \bar{\tau}_m\right)\gamma_4 \gamma_{i1} + \tau_i \left(\tau_m - \bar{\tau}_m\right)\gamma_4 + \gamma_4 \left(\tau_m - \bar{\tau}_m\right) \]

Collecting terms and simplifying, group B countries have satisfaction levels of

\[ O_i(\tau_m, \tau_m - \bar{\tau}_m) \approx O_i(\tau_i^*, 0) - \tau_i^2 \gamma_{i1} - \tau_m^2 \left(1 + \gamma_3 + \gamma_4\right) \gamma_{i1} - \bar{\tau}_m^2 \gamma_3 \gamma_{i1} \]

(E4)

\[ + \bar{\tau}_m \frac{\gamma_2i}{\frac{1}{1 + \frac{\gamma_4}{2}}} + \tau_i \gamma_{i1} \left[2\tau_m \left(1 + \gamma_3 + \gamma_4\right) - \bar{\tau}_m \gamma_4\right] - \bar{\tau}_m \gamma_4 \frac{\left(\tau_i - \bar{\tau}\right)\left(\gamma_3 + \frac{\gamma_4}{2}\right)}{\frac{1}{1 + \frac{\gamma_4}{2}}} \gamma_{i1} \]

It follows from (E2) and (E4) that

\[ M \approx \sum_i O_i(\tau_i^*, 0) w_i - \sum_i \tau_i^2 \gamma_{i1} w_i + 2\Delta \bar{\tau}_m \sum_i \gamma_{i1} w_i + \sum_i \tau_i^2 \left(1 + \gamma_3 + \gamma_4\right) \gamma_{i1} w_i \]

\[ - \bar{\tau}_m^2 \gamma_3 \sum_i \gamma_{i1} w_i + 2 \left(\bar{\tau}_m - \bar{\tau}\right) \left(\gamma_3 + \frac{\gamma_4}{2}\right) \sum_i \gamma_{i1} w_i - \gamma_4 \bar{\tau}_m \sum_i \gamma_{i1} w_i \]

(E5)

\[ + \frac{\left(\gamma_3 + \frac{\gamma_4}{2}\right)^2 \left(\bar{\tau}_m - \bar{\tau}\right)^2}{\left(1 + \gamma_3 + \gamma_4\right)} \sum_i \gamma_{i1} w_i - \tau_m^2 \left(1 + \gamma_3 + \gamma_4\right) \sum_i \gamma_{i1} w_i \]

\[ + 2\tau_m \left(1 + \gamma_3 + \gamma_4\right) \sum_i \gamma_{i1} w_i - \bar{\tau}_m \gamma_4 \frac{\left(\gamma_3 + \frac{\gamma_4}{2}\right)}{\frac{1}{1 + \frac{\gamma_4}{2}}} \sum_i \left(\tau_i - \bar{\tau}\right) \gamma_{i1} w_i \]
Consequently, (E5) can be used to compare multiple values of $\tau_m$ that satisfy the first order condition for maximizing collective objectives. In performing these comparisons, it is helpful that the first two terms on the right side of (E5) do not change with $\tau_m$ and therefore can be ignored; and that the last term on the right side of (E5) equals zero if $\nu_i = \frac{\gamma_{ii} w_i}{\sum\gamma_{ii} w_i}$. 