Pass-Through in a Multiproduct World

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Abstract

The standard partial equilibrium formula for pass-through substantially mismeasures incidence in the presence of demand or supply interdependencies. We study general equilibrium tax incidence in a perfectly competitive, multiproduct setting. If only one product is taxed, the general equilibrium incidence will always be greater on the consumer than suggested by the standard incidence formula. If the tax changes on multiple related commodities, while maintaining perfect competition, a necessary condition for overshifting is that the related commodities are substitutes. Pass-through greater than one-hundred percent is not sufficient to infer market structure. When empirically estimating pass-through, pass-through estimates capture the direct effect of the tax on the market, the indirect feedback effects resulting from price and tax changes in other markets and taxation of inputs to production. Empirically applying our theory to estimate pass-through in alcohol markets, we show that demand interdependencies and simultaneous tax changes on related products are important.

JEL: H22, H25, L10, L50, D50

Keywords: pass-through, tax incidence, multi-market, cascading, market structure

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"What do our results [overshifting] say about the markets for the commodities that exhibit overshifting? They do not imply that any particular model of market structure is correct, but they are inconsistent with perfect competition." -Besley and Rosen (1999)

1 Introduction

Tax incidence, and more generally the pass-through of cost shocks, is of fundamental importance in economics.[1] As is well-known, the economic incidence of a tax need not be equivalent to its statutory incidence because behavioral responses influence prices. Traditional models of tax incidence derive the incidence on the consumer assuming that the taxed commodity has no close complements or substitutes or, alternatively, is a small share of a consumer's budget. Under these assumptions, in the presence of perfect competition, the burden borne by the consumer is given by the standard partial equilibrium formula – the elasticity of supply divided by the elasticity of supply minus the elasticity of demand. Then, the incidence of the tax on the consumer is bounded between zero and one-hundred percent of the tax. Therefore, in the presence of perfect competition, overshifting of the tax to the consumer is *impossible* under this textbook formula (for surveys, see Fullerton and Metcalf (2002), Kotlikoff and Summers (1987), and Mieszkowski (1969)).[2] However, a large theoretical literature acknowledges that these bounds need not hold in the presence of imperfect competition:

1. Edgeworth (1925) and Hotelling (1932) show that if a monopolist faces a tax on tickets, it may reduce the prices on first- and second-class tickets.[3] This phenomenon, where post-tax consumer prices fall, is Edgeworth's Taxation Paradox. In these models, the incidence on consumers is not bounded below by zero.

2. In the presence of imperfect competition or monopolies (Delipalla and Keen 1992; Anderson et al. 2001; Hamilton 2008; Weyl and Fabinger 2013), the incidence of the tax depends on how firms respond to each other. In the presence of an oligopoly market structure, producers strategically interact with each other and

[1] We define tax incidence to be the change in consumer and producer prices resulting from the tax. We do not discuss changes in consumer surplus or lifetime incidence (Poterba 1989).

[2] Also see the literature on salience and tax incidence (Chetty et al. 2009).

[3] The original article written by Edgeworth in the 1800s has been lost; this citation is to a reprint. Coase (1946) provides a graphical treatment of the paradox.

> the incidence of the tax therefore depends on whether the oligopolist believes that competitor firms will match a price increase or, instead, increase market share. As a result, overshifting is possible. In these models, the incidence on the consumer is not bounded above by one. As well, in a simple model of monopoly-markup pricing, overshifting occurs if marginal costs are constant.

Following a large theoretical literature that derives the effect of government policy changes on equilibrium prices, a large empirical literature has developed. As noted in the quote above, most empirical articles that find overshifting speculate that such a result is *inconsistent* with perfect competition. Indeed, numerous studies of the effect of sales and excise taxes on prices have found overshifting of the tax to the consumer (Poterba 1996; Besley and Rosen 1999; Kenkel 2005). As can be seen in Besley and Rosen (1999), these authors found statistically-significant evidence of overshifting in numerous markets some of which might be thought to be perfectly competitive – bananas, bread, crisco, milk, monopoly (the board game), shampoo, soda, and underwear. Recently, it has been argued that under certain conditions, pass-through greater than 100% can be used as a test of market power (Pless and van Benthem 2019).

We present an alternative explanation for these results that presents additional conditions for using pass-through to test for market power. In particular, we develop a theoretical framework in which it is shown that overshifting and Edgeworth's Taxation Paradox (more generally, any shock to marginal cost) are possible, *even if markets are perfectly competitive and the products being studied are a small fraction of consumers' budgets*. Our results generalize with minor modification when market shares of the taxed commodity are large, including in the presence of pre-existing tax distortions. These possibilities arise because of the presence of multiple complementary or substitute products,[4] that are all sold in perfectly competitive markets.

Our model draws inspiration from the empirical literature on tax pass-through: we focus on products that are a small share of consumers' budgets and allow for one (as in an excise tax) or many (as in a sales or Value Added Tax) of these products to be subject to tax. Given we have multiple products, our model is general equilibrium in the sense that the tax can affect prices of any other commodity markets through

[4] To our knowledge, only one other study focuses on multiproduct incidence with perfect competition (Benedek et al. 2015). This paper is primarily an empirical paper. The theoretical motivation features only two goods and therefore will not consider the wide-array of cases that we consider. The paper also does not address issues related to overshifting and abstracts from income effects. Hamilton (2008) considers a oligopoly model where consumers make multiproduct transactions at the store.

demand-side factors. This path of analysis is different from much of the prior general equilibrium literature that has instead focused on the supply side (factor side) by documenting that workers or capital may bear more than one hundred percent of a tax (Harberger 1962; Mieszkowski 1967).[5] Instead, we focus on the demand side of a multi-sector model by allowing for complementarity and substitutability across taxed and untaxed commodities. Our model has three unique features:

1. Products, although a small share of consumer expenditures, are related through non-zero cross-price elasticities.

2. Taxes are allowed to change on a specific product or on multiple products. Some products do not experience tax changes.

3. Some "intermediate" inputs may be subject to the tax, resulting in tax cascading.

In this context, we show that the standard partial equilibrium formula substantially mismeasures the true tax incidence even when the tax affects a single small market as in the case of an excise tax. When taxes affect multiple markets, the case for many of the products studied empirically, the deviations from partial equilibrium are possibly even larger. Our formula for tax incidence nests the partial equilibrium formula and shows that if only one product is taxed, the true general equilibrium incidence will be greater on the consumer than suggested by the standard incidence formula.[6] In particular, we show that, with one taxed commodity, the partial equilibrium formula is a lower bound for the true general equilibrium incidence on consumers.[7] The intuition is that if the products are substitutes, a tax in one market increases the price in the second market that, in turn, increases demand and prices in the first market. If complements, a tax in one market decreases the price in the second market. This, in turn, increases

[5]Our result also has some similarity to the incidence of the property tax in an open economy where some of the incidence may be borne by parties located in other jurisdictions (Zodrow and Mieszkowski 1986). Even if the effect on equilibrium prices is "negligible" because a product or jurisdiction is small, such a result is misleading. In particular, even if the price changes are small, they be spread over a a large number of households and jurisdictions. For this reason, even if each individual price change is small, the aggregate effect need not be small (Wildasin 1988).

[6]Kotlikoff and Summers (1987) argue that partial equilibrium analysis is appropriate if "the product in question have a market that is small relative to the entire economy." In contrast, we show that even if the taxed product is a small relative to the rest of the economy, if there is another product with which it has a non-zero cross-price elasticity, the partial equilibrium formula will not hold.

[7]This result has a parallel to Goulder and Williams (2003) that shows that under general equilibrium "the simple 'excess-burden triangle' formula substantially underestimates the excess burden of commodity taxes, in some cases by a factor of 10 or more."

demand and prices in the first market. In the case of a general sales tax, this is not true; the model shows that the general equilibrium incidence could either be a higher or lower incidence on consumers than that found using the standard incidence formula. Here, the sign of the cross-price elasticities is critical. In a setting that is realistic for many empirical studies, we show that overshifting is possible and Edgeworth's taxation paradox can arise. A necessary condition for overshifting to arise is that the tax must be on multiple commodities and the commodities must be substitutes. Analogously, a necessary condition for Edgeworth's taxation paradox, "undershifting," is that the tax is on multiple commodities and the commodities must be complements. Finally, we show that when a product uses taxed business-to-business transactions as inputs, the effective tax change borne by the consumer can exceed 100 percent of the statutory tax change. This arises because a tax change on a product that also changes on its inputs will result in an effective tax change that is greater than the statutory tax rate change; some of the tax change on the inputs may be passed on the the consumer.

Our model implies that if products have any substitutes or complements, researchers must be careful to structure empirical models to identify incidence effects. We conclude that empirical studies of pass-through identify multiple effects: (1) a direct effect of the tax change on the taxed product, (2) an indirect effect of the tax resulting from demand shifts due to complementarity and substitutability, (3) an indirect effect on demand due to simultaneous changes in taxes in related markets, and (4) indirect effects of input prices as a result of broad-based taxes that tax business inputs.

These general equilibrium effects merit attention because complements and substitutes among products are extremely common (Harding and Lovenheim 2017). For example, different types of alcohol may have some degree of substitutability with each other and with other types of non-alcoholic drinks; these beverages may also be complementary to leisure items. Food for home consumption and restaurant meals may also be related. Other goods, such as airline tickets, are related through network effects.

Using data on beer and wine prices and alcohol excise taxes, we implement an empirical incidence analysis in the spirit of Besley and Rosen (1999). We show that although alcoholic beverage taxes differ for wine, spirits and alcohol, when states increase the tax on one product, they often increase the taxes on one or both of the other products. Given these products have a high degree of interdependence, this provides a unique chance for us to demonstrate the relevance of our theory. When only accounting for beer tax changes, the price of beer rises almost one-for-one with the change in the beer excise tax. However, when accounting for all three taxes, the direct effect of beer taxes on beer prices is substantially lower. In particular, we identify economically meaningful and statistically significant cross-effects: changes in wine and spirit taxes increase the price of beer consistent with alcoholic beverages being substitutes. This empirical example suggest that both elements of our theory, multiple related products and tax changes on multiple products, are important.

While we are investigating the general equilibrium impacts of a tax increase as a "shock" to a market, there are other non-tax shocks that would have similar effects. Our results generalize to any shock that causes an increase in the marginal cost curve. For example, the incidence of regulatory policies in a market or the magnitude of any cost pass-through within an industry may depend critically on the cross-market relationships with other products. Thus our model has applications in fields other than public finance, including for estimation of the incidence of cost shocks in industrial organization, trade and labor economics and for capitalization studies in urban economics. For pass-through using of cost shocks Fabra and Reguant (2014) find full pass through and Miller et al. (2017) find overshifting; the caveats of our theoretical model apply to these types of shocks. Furthermore, our result has parallels to the effect of policies with many sectors: Sachs et al. (2019) show that when the government raises a tax rate, the reform affects wages in that tax bracket, but also the wages of other workers that are complements in production, then resulting in feedback effects. Thus, it is apparent that incidence is an important part of welfare analysis, but pass-through more generally is central to studying price discrimination, platforms, merger analysis, minimum wages with multiple sectors and tariffs, among other applications to which we caution against inferring market power. Our model implies that a pass-through of cost shocks greater than 100% is not (necessarily) connected to imperfect competition.

Our theoretical model has several important implications for policy, including the effects of taxes on consumption of unhealthy products (soda, cigarettes, alcohol). The reduced-form estimates remain very relevant for many policy questions, such as how much a tax on an unhealthy good will increase its price. Nonetheless, even for that question our analysis is relevant, because like more structural estimates generally, our analysis addresses the external validity and generalizability of the reduced-form evidence. Policymakers tend to jump to the conclusion that reduced-form evidence shows that excise taxes on unhealthy goods are over-shifted to prices. But our analysis suggests that what is true for one good might not be true for another good due to the patterns of substitution and complementarities. More generally, we conclude:

1. Empirical studies like Poterba (1996) and Besley and Rosen (1999) are methodologically correct and estimate the true reduced-form incidence parameter. However, the interpretation of the results is more nuanced than the standard textbook formula would suggest. Researchers cannot infer the relative patterns of the own-price supply and demand elasticities from pass-through estimates.

2. While these empirical studies provide correct reduced-form incidence estimates, these estimates are often for the incidence of a broad-based tax (VAT or sales) and not the incidence of a tax on a narrower set of goods. Then, these estimates are likely to be a biased estimate of the incidence of a narrower-based tax.

3. That overshifting is inconsistent with perfect competition is not true. Overshifting may be consistent with perfect competition if products are substitutes and the taxes studied are broad-based, i.e., affecting multiple products. Beyond the example of tax incidence, cost shocks across related industries could generate overshifting if the cost-shock in the related industry is not considered. Researchers should be cautious about inferring market structure, as recently advocated, from incidence estimates unless demand or supply interdependencies can be ruled out.

4. Researchers utilizing other commodity markets as a control group when determining the incidence of taxation, may mismeasure the true incidence of the tax. Thus, researchers might consider augmenting the standard difference-in-difference design with data in other states or countries, which allows for comparisons of a tax change on a product in one state with that product in another state.

5. When determining the fraction of the tax borne by the consumer, researchers should use the effective tax rate on a product rather than the statutory tax rate. Given tax cascading, the effective tax rate on a product with taxed business-to-business inputs will always be greater than the statutory tax. Failure to use the effective tax rate may result in finding overshifting when it does not exist.

2 A Multiproduct Model of Pass-through

We wish to show the distinctions between partial and general equilibrium tax incidence with multiple products in the simplest model that we believe highlights the distinctions.

While a general model would have an arbitrary (and large) number of commodities (n) we apply the composite commodity theorem (Hicks 1939). Then, from Hicks (1939):

> This principle [the composite commodity theorem] is of quite general application. A collection of physical things can always be treated as if they were divisible into units of a single commodity so long as their relative prices can be assumed to be unchanged...

To simplify, let there be three goods (commodities) for consumption, x_i, $i = 1, 2, 3$ and inelastically-supplied labor (leisure), L, with its price (w) normalized to unity.[8] Then, the demands for the commodities are given by $x_i^D(\mathbf{q}, y_i)$ where $\mathbf{q} = [q_1, q_2, q_3]$ is a vector of tax-inclusive prices with $q_i = p_i + \tau_i$, $\forall i$ where the gross price (q_i) equals the net price (p_i) plus a unit tax (τ_i).[9] We assume the composite commodity, x_3, never experiences tax changes, that is, $d\tau_3 = 0$. The term y_i is income for consumers of good i, to be defined.

Let $c_i(x_i^s)$ denote the amount of labor needed to produce x_i^s units of commodity i where $c_i'(x_i^s) > 0$ and $c_i''(x_i^s) > 0$ with the endowment of labor denoted by $\overline{L}$.[10] The supply of good i is implicitly defined by $p_i = c_i'(x_i^s(p_i))$, $\forall i$.[11] We simplify production, to be relaxed later, by assuming that there is no joint production and that none of the commodities are used in the production of another. In Section 2.4 we address the implications of joint production and "cascading" taxes, the taxation on both a final product and its inputs, on incidence and in Appendix A.5 we discuss a more general case in which we allow supply of a commodity to depend on the prices of other commodities as well as own price. Total income is defined to be $y = w\overline{L} + \sum_{i=1}^{3}[p_i x_i - c_i(x_i) + \tau_i x_i]$

[8] As will become clear, that labor is inelastically supplied does not affect our results under the assumption that both markets x_1 and x_2 are small, the assumption under which our analysis is done.

[9] The distinction between an ad valorem and a unit tax does not affect our result.

[10] We depart from the classic model (Harberger 1962) by including only one input in production. However, we could easily expand and allow the commodities to be produced by both capital and labor with $x_i^s = f_i(K_i, L_i)$ where K_i is capital used in its production. Then the cost function would be $c_i(r, x_i^s)$ where r is the price of capital and the price of labor continues to be the numéraire. However, as we focus on taxes in "small" markets, there will be no effect of any commodity tax on the price of capital as the commodity will have a small share of the market of capital and capital is freely mobile.

[11] We assume that the supply of the commodity is not perfectly elastic as we are interested in determining how the incidence between consumers and producers. One means of motivating a supply curve that is not perfectly elastic is to consider a commodity-specific input used in production. If this were the case we would have $x_i^D(q_1, q_2, q_3, y) = x_i^s(p_i, p_i^I)$ and $x_i^I(p_i, p_i^I) = I_i(p_i^I)$ where p_i^I is the price of the input, $x_i^I(p_i, p_i^I)$ is the derived demand for the input, and $I_i(p_i^I)$ is its supply. If the supply of the input is not perfectly elastic, the supply of x_i will not be either. As there is a one-to-one correspondence between the price of the commodity, p_i, and the price of the input p_i^I we can suppress the price of the input in our analysis. As a result, we may have profits in perfect competition.

where the tax is rebated to the consumers.[12] As we are considering aggregate demand, we allow, initially, for the income of consumers of the different commodities, y_i to differ. The equilibrium conditions are:

$$E_i^D(\mathbf{q}, y_i) \equiv x_i^D(\mathbf{q}, y_i) - x_i^S(p_i) = 0, \ i = 1, 2, 3, \tag{1}$$

where $E_i^D(\mathbf{q}, y_i)$ is the excess demand function for commodity i. As demonstrated by the Sonnenschein-Mantel-Debreu theorem, the excess demand function for an economy is not restricted by the usual rationality restrictions placed on individual demands.[13] This being the case, we provide more structure by considering the simple one consumer/producer case. Specifically, we assume that the budget share in consumption by the single consumer of commodity i, $B_i = \frac{q_i x_i}{y}$, equals her ownership share of the commodity. As well, she receives all the tax revenue. Then totally differentiating (1) with respect to τ_1 and τ_2 yields

$$\hat{p}_i = \left[\begin{array}{c} -\left(\widetilde{\eta}_{i1}\hat{\tau}_1 + \widetilde{\eta}_{i2}\hat{\tau}_2\right)\left[\left(\widetilde{\eta}_{kk} - \mu_k\right)\left(\widetilde{\eta}_{jj} - \mu_j\right) - \widetilde{\eta}_{kj}\widetilde{\eta}_{jk}\right] \\ +\left(\widetilde{\eta}_{j1}\hat{\tau}_1 + \widetilde{\eta}_{j2}\hat{\tau}_2\right)\left[\widetilde{\eta}_{ij}\left(\widetilde{\eta}_{kk} - \mu_k\right) - \widetilde{\eta}_{kj}\widetilde{\eta}_{ik}\right] \\ +\left(\widetilde{\eta}_{k1}\hat{\tau}_1 + \widetilde{\eta}_{k2}\hat{\tau}_2\right)\left[\widetilde{\eta}_{ij}\widetilde{\eta}_{jk} + \widetilde{\eta}_{ik}\left(\widetilde{\eta}_{jj} - \mu_j\right)\right] \end{array} \right] |H|^{-1}, \quad \begin{array}{c} i, j, k = 1, 2, 3; \\ i \neq j \neq k, \end{array} \tag{2a}$$

$$|H| = \begin{array}{c} \left(\widetilde{\eta}_{11} - \mu_1\right)\left[\left(\widetilde{\eta}_{22} - \mu_2\right)\left(\widetilde{\eta}_{33} - \mu_3\right) - \widetilde{\eta}_{23}\widetilde{\eta}_{32}\right] + \widetilde{\eta}_{21}\left[\widetilde{\eta}_{12}\left(\widetilde{\eta}_{33} - \mu_3\right) - \widetilde{\eta}_{32}\widetilde{\eta}_{13}\right] \\ +\widetilde{\eta}_{31}\left[\widetilde{\eta}_{12}\widetilde{\eta}_{23} - \widetilde{\eta}_{13}\left(\widetilde{\eta}_{22} - \mu_2\right)\right]. \end{array} \tag{2b}$$

where $\widetilde{\eta}_{ij} = \eta_{ij} + \delta_i \sum_{k=1}^{3} \widetilde{\tau}_k \beta_k \varepsilon_{kj}$, η_{ij} is the compensated price elasticity of commodity x_i with respect to q_j, $\varepsilon_{ij} = \frac{\partial x_i^D}{\partial q_j}\frac{q_j}{x_i}$, is the Marshallian price elasticity of demand, $\mu_i = \frac{\partial x_i^S}{\partial p_i}\frac{p_i}{x_i}$ is the elasticity of supply, $\delta_i = \frac{\partial x_i}{\partial y_i}\frac{y_i}{x_i}$ is the income elasticity of demand, $\beta_k = \frac{q_k x_k}{y}$ is the budget share of x_k, $\hat{p}_i = \frac{dp_i}{q_i}$, and $\hat{\tau}_i = \frac{d\tau_i}{q_i}$. Formal derivation of (2a)-(2b) are in Appendix A.1.

[12] Musgrave and Musgrave (1989) define three views of incidence: absolute, differential, and budget incidence. Rebating revenue to the consumers allows the budget to adjust to a tax perturbation. This avoids the difficulties of absolute incidence where public expenditures are held constant and allows us to focus on the appropriate case of budget incidence.

[13] The basic point of the theorem is that we can place almost no restrictions on market demand functions in general. Of course, a corollary is that we cannot assume that cross-effects are small. This provides further justification for our model.

2.1 Taxation in a Single Small Market

Frequent justification for use of the partial equilibrium formula, for example in Kotlikoff and Summers (1987), is that a market is small relative to other markets. If this is true, the budget share of x_i, B_i, is also near zero. The relationships between cross-price elasticities can be expressed in terms of market share with $\varepsilon_{ji} = \frac{q_i x_i}{q_j x_j}\varepsilon_{ij}$ and $\eta_{ji} = \frac{q_i x_i}{q_j x_j}\eta_{ij}$. While changes in the price in the large market (x_j) should have an impact on purchases in a small market (x_i), $|\varepsilon_{ij}|, |\eta_{ij}| > 0$, any change in price in the small market will have little impact (in percentage terms) in the large market, $\varepsilon_{ji}, \eta_{ji} \approx 0$. Then letting x_1 be the small market, that is, $\frac{q_1 x_1}{q_j x_j} \approx 0,\ j = 2, 3$; this implies that $\widetilde{\eta}_{i1} \to \eta_{i1}$ and we can work with compensated elasticities for these elements in (2a) and (2b). As in the prior literature, to focus on the impact of a tax in a single small market, let $\hat{\tau}_2 = 0$, and substitute $\eta_{j1} = \frac{q_1 x_1}{q_j x_j}\eta_{1j}$ in (2a):

$$\hat{p}_1 = \begin{bmatrix} -\eta_{11}\hat{\tau}_1\left[\left(\widetilde{\eta}_{22}-\mu_2\right)\left(\widetilde{\eta}_{33}-\mu_3\right)-\widetilde{\eta}_{23}\widetilde{\eta}_{32}\right] \\ +\frac{q_1x_1}{q_2x_2}\eta_{12}\hat{\tau}_1\left[\widetilde{\eta}_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\widetilde{\eta}_{31}\widetilde{\eta}_{32}\right] \\ -\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1\left[\widetilde{\eta}_{12}\widetilde{\eta}_{23}-\widetilde{\eta}_{13}\left(\widetilde{\eta}_{22}-\mu_2\right)\right] \end{bmatrix} |H|^{-1}, \tag{3a}$$

$$|H| = \begin{matrix} \left(\eta_{11}-\mu_1\right)\left[\left(\widetilde{\eta}_{22}-\mu_2\right)\left(\widetilde{\eta}_{33}-\mu_3\right)-\widetilde{\eta}_{23}\widetilde{\eta}_{32}\right]-\frac{q_1x_1}{q_2x_2}\eta_{12}\left[\widetilde{\eta}_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\widetilde{\eta}_{31}\widetilde{\eta}_{32}\right] \\ +\frac{q_1x_1}{q_3x_3}\eta_{13}\left[\widetilde{\eta}_{12}\widetilde{\eta}_{23}-\widetilde{\eta}_{13}\left(\widetilde{\eta}_{22}-\mu_2\right)\right] \end{matrix} \tag{3b}$$

Then considering the limit as $\frac{q_1x_1}{q_2x_2}$ and $\frac{q_1x_1}{q_3x_3}$ approach zero, the budget shares, B_1 and B_2 also approach zero. Apply this to (3a) and (3b), and as in Appendix A.2, yields

$$\hat{p}_1 = \frac{\eta_{11}}{\mu_1 - \eta_{11}}\hat{\tau}_1 \equiv \rho_1\hat{\tau}_1 \leq 0 \quad \text{and} \tag{4a}$$

$$\hat{p}_2 = \hat{p}_3 = 0 \tag{4b}$$

As (4a) clearly shows, the standard partial equilibrium incidence formula (Kotlikoff and Summers 1987) is obtained in this case with no effect on prices in the other two markets. However, the assumption needed to derive this is not that the commodity needs to be a small market, as in the prior literature, but rather that it needs to be small (pairwise) relative to *all other* commodities (x_2, x_3) given that x_2 cannot be included in the composite commodity.

Alternatively, rather than assuming "small" markets, that is, markets which have budget shares approaching zero, our results can be obtained if we assume that x_2 and x_3

have perfectly elastic supplies $(\mu_i \to \infty,\ i = 2,\ 3)$. While the increase in q_1 will change demand for x_2 and x_3, because of the perfectly elastic supplies of x_2 and x_3 the prices of the two commodities are unchanged. This being the case, there are no "feedback" effects of price changes in these markets on the demand for x_1. If we do not assume small markets and we are considering a tax increase from a positive tax rate $(\tau_1 > 0)$ there are income effects and the effect of a tax change on $p_1$is given by

$$\hat{p}_1 = \frac{\widetilde{\eta}_{11}}{\mu_1 - \widetilde{\eta}_{11}} \hat{\tau}_1 \equiv \rho_1 \hat{\tau}_1 \leq 0 \tag{5}$$

where $\widetilde{\eta}_{11} = \eta_{11} + \delta_1 \widetilde{\tau}_1 \beta_1 \varepsilon_{11}$ following our earlier definition of $\widetilde{\eta}_{ij}$. However, if there are no pre-existing taxes then $\hat{p}_1$ can be expressed as in (4a) using compensated elasticities. To be clear, all of our subsequent results apply with compensated elasticities under two conditions: either small markets (including in the presence of pre-existing taxes) or no pre-existing taxes and a perfectly elastic supply for x_2 and x_3.

Proposition 1. *With perfect competition and related commodities (non-zero cross-price elasticities), sufficient conditions for the standard partial equilibrium incidence formula,* (4a)*, to apply to commodity x_i are that, only the tax on x_i changes and one of the following must hold:*

(1) expenditures on x_i are small relative to all other commodity groupings or,

(2) the supplies of all other commodities $x_j,\ j \neq i$ are perfectly elastic.

Proposition 1 requires only one commodity experience a tax change which rules out the application of the partial equilibrium formula in the presence of broad-based consumption tax changes. An alternative to the sufficient condition in Proposition 1 is that cross-price elasticities are equal to zero when markets are small.

2.2 Taxation with Two Small Markets

Now consider the incidence of a tax when a market may be one of a few small, related markets. Rather than x_1 being small relative to both x_2 and x_3, let both x_1 and x_2 be small relative to x_3 or $\frac{q_1 x_1}{q_3 x_3} \approx 0$ and $\frac{q_2 x_2}{q_3 x_3} \approx 0$.[14] As well, we potentially allow for changes

[14]We make the assumption of two small markets following the prior literature. However, if the market for x_2 were not small, the formula for the impact of a general sales tax (or VAT) on the price of x_1 would be similar to those we derive except replacing $\widetilde{\eta}_{i2}$ for η_{i2}.

in the tax rates on both x_1 and x_2. Substituting $\eta_{31} = \frac{q_1 x_1}{q_3 x_3}\eta_{13}$ and $\eta_{32} = \frac{q_2 x_2}{q_3 x_3}\eta_{23}$ gives:

$$\hat{p}_i = \left[\begin{array}{c} -\left(\eta_{i1}\hat{\tau}_1 + \eta_{i2}\hat{\tau}_2\right)\left[\left(\widetilde{\eta}_{33} - \mu_3\right)\left(\eta_{jj} - \mu_j\right) - \frac{q_j x_j}{q_3 x_3}\eta_{j3}\widetilde{\eta}_{j3}\right] \\ -\left(\eta_{j1}\hat{\tau}_1 + \eta_{j2}\hat{\tau}_2\right)\left[\eta_{ij}\left(\widetilde{\eta}_{33} - \mu_3\right) + \frac{q_j x_j}{q_3 x_3}\eta_{j3}\widetilde{\eta}_{i3}\right] \\ -\left(\frac{q_1 x_1}{q_3 x_3}\eta_{13}\hat{\tau}_1 + \frac{q_2 x_2}{q_3 x_3}\eta_{23}\hat{\tau}_2\right)\left[\eta_{12}\widetilde{\eta}_{j3} + \widetilde{\eta}_{i3}\left(\eta_{jj} - \mu_j\right)\right] \end{array} \right] |H|^{-1}, \quad \begin{array}{c} i,j = 1,2; \\ i \neq j, \end{array} \tag{6a}$$

$$|H| = \begin{array}{c} \left(\eta_{11} - \mu_1\right)\left[\left(\widetilde{\eta}_{33} - \mu_3\right)\left(\eta_{22} - \mu_2\right) - \frac{q_2 x_2}{q_3 x_3}\eta_{23}\widetilde{\eta}_{23}\right] - \eta_{21}\left[\eta_{12}\left(\widetilde{\eta}_{33} - \mu_3\right) - \frac{q_2 x_2}{q_3 x_3}\eta_{23}\widetilde{\eta}_{13}\right] \\ + \frac{q_1 x_1}{q_3 x_3}\eta_{13}\left[\eta_{12}\widetilde{\eta}_{23} - \left(\eta_{22} - \mu_2\right)\widetilde{\eta}_{13}\right]. \end{array} \tag{6b}$$

Evaluating (6a) using (6b) at $\frac{q_1 x_1}{q_3 x_3} = \frac{q_2 x_2}{q_3 x_3} = 0$ gives

$$\hat{p}_i = \underset{\text{(A)}}{\rho_i \hat{\tau}_i} + \underset{\text{(B)}}{\left(1 + \rho_i\right)\frac{\eta_{ji}\eta_{ij}}{|\widetilde{H}|}\hat{\tau}_i} + \underset{\text{(C)}}{\frac{\eta_{ij}\mu_j}{|\widetilde{H}|}\hat{\tau}_j}, \quad i,j = 1,2;\ i \neq j \text{ and} \tag{7a}$$

$$\hat{p}_3 = 0 \tag{7b}$$

where $\rho_i = \frac{\eta_{ii}}{\mu_i - \eta_{ii}} \leq 0,\ i = 1,2$ and $|\widetilde{H}| = \left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12} > 0$ for the stability condition to hold.[15] In addition to the determinant, $|\widetilde{H}|$, being positive, the elasticities must satisfy the conditions $\eta_{i1} + \eta_{i2} + \eta_{i3} = 0,\ i = 1,2,\ \eta_{11} \leq 0,\ \eta_{22} \leq 0,$ $\eta_{12}\eta_{21} \geq 0$. As well, the second order condition must hold: $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$.

In (7a), term (A) is simply the partial equilibrium incidence as derived in (4a), while term (B) is the additional "general equilibrium" impact arising from the cross-market effects. Part (C) is the effect of a (simultaneous) increase in taxes in a related market. If $\hat{\tau}_1$ and $\hat{\tau}_2$ are both non-zero, the tax reform could be thought of as a sales tax change, whereas if one of these terms is zero, we might think of this as a specific tax, such as an alcohol excise tax.

The gross price to the consumers is simply $q_i = p_i + \tau_i$ so that the change in the gross price is simply $\hat{q}_i = \hat{p}_i + \hat{\tau}_i,\ i = 1, 2$. Then from (7a) we obtain

$$\hat{q}_i = \underset{\text{(A)}}{\left(1 + \rho_i\right)\hat{\tau}_i} + \underset{\text{(B)}}{\left(1 + \rho_i\right)\frac{\eta_{ji}\eta_{ij}}{|\widetilde{H}|}\hat{\tau}_i} + \underset{\text{(C)}}{\frac{\eta_{ij}\mu_j}{|\widetilde{H}|}\hat{\tau}_j}, \quad i,j = 1,2;\ i \neq j \tag{8}$$

where the interpretations of terms (A) - (C) are the same as for (7a).

Again, as an alternative to the assumption of small markets, in this case for both x_1 and x_2, we can derive the same results with a perfectly elastic supply of x_3 without

[15]Expression (7a) is derived in Appendix A.3.

the assumption of small markets. As when we consider a tax in a single market and elastic supplies for the other commodities, there are income effects when we start with existing taxes ($\tau_1 > 0,\ \tau_2 > 0$). In this case in (7a) and (8) we replace $\eta_{ij},\ j = 1, 2$ with $\widetilde{\eta}_{ij} = \eta_{ij} + \delta_i \sum_{k=1}^{2} \widetilde{\tau}_k \beta_k \varepsilon_{kj}$.

The intuition of terms (A) - (C) in (7a) and (8) can be displayed in a series of supply and demand diagrams. Quite interesting is that the results can be presented graphically in two different ways. Recall that demand is given above by $x_i^D(\mathbf{q}, y_i)$. Under the assumptions above, that is, the small markets assumption, this can be rewritten as $x_i^D(q_1, q_2)$. Then, we can express a "partial equilibrium" demand curve, $D_1 \equiv x_1^D(q_1, \overline{q}_2)$ where $\overline{q}_2$ holds the price in the other market fixed, which implies that changes in the consumer price of x_2 shift the demand curve for x_1. As an alternative, we can define a "general equilibrium" demand curve $\widetilde{D}_1 \equiv \widetilde{x}_1(q_1, \tau_2) \equiv x_1(q_1,\ p_2(q_1) + \tau_2)$, which takes into account the relation between the consumer price of x_1 and the consumer price of x_2.[16] Notice that tax changes in market 2 would shift this demand curve but that price changes in market 2 induce a movement along the demand curve. Then as $\hat{q}_2 = -\frac{\eta_{21}}{(\eta_{22} - \mu_2)} \hat{q}_1$, the price elasticity of this demand curve is given by $\eta_{11}^* = \eta_{11} - \frac{\eta_{12}\eta_{21}}{(\eta_{22} - \mu_2)} > \eta_{11}$, which means that this demand curve is less elastic than the demand curve $x_1^D(q_1, \overline{q}_2)$. For $\widetilde{x}_1(q_1, \tau_2)$ to be positively sloped, that is $\eta_{11}^* > 0$, requires $(\eta_{22}\eta_{11} - \eta_{12}\eta_{21}) - \eta_{11}\mu_2 < 0$ which is a violation of the second order condition for expenditure minimization. Then $\widetilde{x}_1(q_1, \tau_2)$ cannot be positively-sloped and it is not possible for more than one hundred percent of the tax to be borne by the consumer.

We initially present the intuition for the case where the tax rate on the first good (x_1) increases, but does not increase on the second good (x_2). First we focus on an analysis using the partial equilibrium demand curves. As we are only considering a tax on commodity x_1, terms (A) and (B) only appear in the expression for $\hat{q}_1$, while term (C) only appears in $\hat{q}_2$. Figure 1a supposes these two goods are substitutes while Figure 2b considers complementary goods. The imposition of the tax shifts the supply curve for x_1 yielding the standard tax wedge between supply and demand, increasing consumer prices (to q_1^p, term (A)) and decreasing producer prices (to p_1^p, term (A)). With substitute goods, this higher price will increase demand for the untaxed substitute commodity x_2 and, as a result, raise its price (to $\widetilde{q}_2$, term (C)). The higher price of

[16]This demand curve formulation is similar to Lee and Saez (2012), who write "Both the demand$D_1(w_1)$ and supply $S_1(w_1)$ curves in the low-skilled labor market are defined assuming that the market clears in the high-skilled labor market..." So, in their model, the linkages between the two markets are incorporated into the demand-supply diagram for market 1.

the untaxed substitutes results in feedback effects in the first market – an increase in demand for the taxed commodity, increasing its price even more (to $\widetilde{q}_1$, term (B)). This price increase shifts some of the incidence from the producer to the consumer. The story is similar in the case of complements with the only difference being that the higher price of the taxed commodity decreases demand for its untaxed complement, thereby reducing the price of the untaxed commodity (to $\widetilde{q}_2$, term (C)). But, because the two goods are complements, the lower price of the untaxed good increases demand for the taxed commodity. Again, consumer prices in the taxed market rise more. This intuition makes it clear: any partial equilibrium analysis of an excise tax on a single product that ignores complementarity or substitutability provides a lower bound on the true general equilibrium incidence. Regardless of whether products are complements or substitutes, the feedback in the first market is always positive.

We can now compare the figure using the partial equilibrium demand curve as D_i with a representation using the general equilibrium demand curve $\widetilde{D}_i$. In Figure 2 we show both the "partial equilibrium" demand curve D_1 and this "general equilibrium" demand curve $\widetilde{D}_1$ for the case of substitutes. As can be seen in the figure, the general equilibrium demand curve captures the direct shift of the supply curve due to the tax and the shift in D_1 (from D_1^0 to $D_1^{\prime}$) due to increase in q_2, i.e, this demand curve would be steeper than the ordinary demand curve because it captures both terms (A) and (B) in a single step. However, in market 2, the general equilibrium demand curve must shift because the tax changed in the other market. Intuitively, as the price for x_1 goes up, this reduces demand for x_1, the demand for x_2 rises, which raises its price, which then dampens the reduction in demand for x_1.

In Appendix A.7 we consider an example with three "small" markets with tax changes in addition to a composite commodity. We also consider an example with an arbitrary (n) number of small markets with tax changes. While each individual market is small, the aggregate need not be; this allows to consider a broad-based sales or value-added tax change. For both of these examples we provide general expressions for tax incidence when taxes are increased in the n multiple markets. In the 3-small market example, we show that if related commodities x_1 and x_2 do not have a relationship with the other commodity (x_3) also experiencing a tax change, that is, there are zero cross-price elasticities between x_1 and x_3 and between x_2 and x_3 ($\eta_{13} = \eta_{31} = \eta_{23} = \eta_{32} = 0$), then the incidence for x_i, $i = 1, 2$ is still determined by (7a). It is also shown in Appendix A.7 that this result generalizes to the n-commodity case with tax changes on all other unrelated groups of commodities having no effects on the tax incidence of the related products. Even if many commodities are subject to tax changes as under a sales tax, only the subset of related commodities (i.e., those with non-zero cross-price elasticities) affect the tax incidence of a commodity.

2.2.1 Multi-market Incidence

In the partial equilibrium framework, a tax increase in one market does not affect consumer prices in other markets. As shown above, this is not the case in our model. Given that many products' prices might be affected by a tax change in a single market, how do we consider a more general incidence formula that incorporates prices changes in multiple markets? For example, consider the extreme case of (nearly) perfect complements. It is inappropriate to consider only the impact of a tax on tennis racquets on the price of racquets without considering its impact on the price of tennis balls.

Continuing with the assumption of two "small" related commodities (x_1, x_2) with non-zero cross-price elasticities we develop a *total* incidence measure incorporating both markets. Let the indirect utility function be given by $V(q_1, q_2, q_3, y)$. Then differentiating $V(q_1, q_2, q_3, y)$ with respect to τ_1 and applying Roy's Identity gives:

$$\frac{dV}{d\tau_1} = -q_1 x_1 \frac{\partial V}{\partial y}\left(\hat{q}_1 + \frac{q_2 x_2}{q_1 x_1}\hat{q}_2 + \frac{q_3 x_3}{q_1 x_1}\hat{q}_3 + dy\right). \tag{9}$$

where dy is the change in income resulting from changes in profits and tax revenue. While invoking a small market argument for x_1 and x_2 implies that $\hat{q}_3 \to 0$ as $\frac{q_1 x_1}{q_3 x_3} \to 0$ and $\frac{q_2 x_2}{q_3 x_3} \to 0$ it does not imply that $\frac{q_3 x_3}{q_1 x_1}\hat{q}_3 \to 0$. Just as the partial equilibrium incidence measure for a single market ignores the weighted impact on the prices of other goods (x_3), to focus on the incidence effects in the markets that are being taxed, we focus on the related markets, x_1 and x_2, and not on any impacts in the general price level (x_3). Further, as we are not attempting to measure the welfare impact of the taxes but rather the impact of price changes, we do not include the impacts on tax revenue and profits (dy) in our incidence measure – this is also the same assumption as in the partial equilibrium case. Of course, alternatively, if we have small markets or begin from a starting point of no existing taxes, there are no income effects. Then using (8) for both $\hat{q}_1$ and $\hat{q}_2$ and, as incidence is measured in changes in prices and not quantity, dividing (9) by $-\frac{\partial V}{\partial y} q_1 x_1$ gives the multi-market incidence for a change in τ_1:

$$I^q_{\tau_1} = \left[(1+\rho_1)\left(1+\frac{\eta_{12}\eta_{21}}{|\widetilde{H}|}\right) + \frac{q_2 x_2}{q_1 x_1}\frac{\mu_1\eta_{21}}{|\widetilde{H}|}\right]\hat{\tau}_1. \tag{10}$$

To evaluate the incidence on producers, let profits be their measure of welfare. Then differentiating profits in both markets with respect to τ_1 and applying Shephard's lemma yields

$$\frac{d\left[\pi_1+\pi_2\right]}{d\tau_1} = x_1\frac{dp_1}{d\tau_1} + x_2\frac{dp_2}{d\tau_1} \tag{11}$$

Then using (7a) to substitute for $\frac{dp_1}{d\tau_1}$ and $\frac{dp_2}{d\tau_1}$ in (11) and dividing by $q_1 x_1$ yields

$$I^p_{\tau_1} = \left[\ \rho_1 \quad + (1+\rho_1)\,\tfrac{\eta_{12}\eta_{21}}{|\widetilde{H}|} + \tfrac{q_2 x_2}{q_1 x_1}\tfrac{\mu_1\eta_{21}}{|\widetilde{H}|}\ \right]\hat{\tau}_1. \tag{12}$$

The total multi-market incidence is a budget share weighted average of the incidence across the two taxes markets. Having derived this formula, we now return to consider the incidence in each single market as this is the primary focus of empirical studies.

2.2.2 Partial and General Equilibrium Differences in Incidence

As seen in equation (7a), the partial equilibrium formula for incidence does not hold if the taxed market has a non-zero cross-price elasticity of demand with another small market. But, we might reasonably ask how much of a difference does using the general equilibrium formula make?

First consider the case with $\hat{\tau}_2 = 0$. From inspection of term (B) in (7a), it is apparent that the cross-market effect always mutes the decrease in p_1. The magnitude of the difference in the general and partial equilibrium measures of incidence can be expressed in percentage terms. Using (7a) gives

$$\frac{\hat{p}^G_i|_{\hat{\tau}_j=0} - \rho_i}{\rho_i} = \frac{\mu_i}{\eta_{ii}}\frac{\eta_{12}\eta_{21}}{|\widetilde{H}|} \leq 0,\ i=1,2,\ j\neq i \tag{13a}$$

where $\hat{p}^G_i$ is the incidence measure in (7a) with $\hat{\tau}_j = 0$. With a single tax, the producer incidence is always closer to zero in general equilibrium.

Additionally, we are interested in the incidence of increases in broad-based taxes, such as a sales or VAT. We consider the change relative to the partial equilibrium measure and (7a) with $\hat{\tau}_1 = \hat{\tau}_2$,

$$\frac{\hat{p}_i^G|_{\hat{\tau}_i=\hat{\tau}_j} - \rho_i}{\rho_i} = \frac{\eta_{ij}}{\eta_{ii}} \frac{(\mu_i \eta_{ji} + \mu_j (\mu_i - \eta_{ii}))}{|\widetilde{H}|} \lesseqgtr 0,\ \ i,j = 1,2;\ \ i \neq j. \tag{13b}$$

As can be seen in (13b), the sign of the difference in the two measures is ambiguous and depends, among other parameters, on whether the two commodities are substitutes or complements. The simulation analysis in Section 3 provides some indication of the magnitude of these differences.

Proposition 2. *With perfect competition, two related products and*

(1) when a single commodity's tax changes, the partial equilibrium formula (4a) *always overstates, in absolute value, producer incidence and understates consumer incidence compared to the general equilibrium incidence;*

(2) when both commodities' taxes change, the partial equilibrium formula (4a) *always overstates [understates], in absolute value, producer incidence and understates [overstates] consumer incidence compared to the general equilibrium incidence if the commodities are Hicksian substitutes [complements].*

2.3 Overshifting & Undershifting

The simple partial equilibrium model with competitive markets predicts that the incidence of a tax increase falls between 0 and 100 percent for both consumers and producers. As discussed, Besley and Rosen (1999), among others, finds "overshifting," more than 100 percent of the incidence of a sales tax borne by consumers of a number of items subject to sales taxation. They argue that this might be considered evidence of imperfectly competitive industries and a "markup" pricing strategy. Here we relax the partial equilibrium assumption but maintain the assumption of perfect competition to see whether and under what conditions "overshifting"might be generated. We also consider the possibility that consumer prices might fall with perfect competition in response to the tax, something first demonstrated as Edgeworth's Paradox in the context of imperfect competition. In this section, we derive analytical results; in Section 3, we provide some simulations that provide some indication of the range of elasticities for which overshifting or undershifting can be obtained.

2.3.1 Can Tax Increases Be Overshifted?

Focusing on the case of two small markets in Section 2.2, we first consider the possibility of overshifting when only the tax in one market (x_1) increases. As shown in Appendix A.4, using (7a) with $\hat{\tau}_2 = 0$ for $\hat{p}_1 > 0$ requires

$$\begin{aligned} &\eta_{11}\mu_2 - [\eta_{11}\eta_{22} - \eta_{12}\eta_{21}] > 0 \Rightarrow \hat{p}_1 > 0, \\ &\text{s.t. } (\eta_{11} - \mu_1)(\eta_{22} - \mu_2) - \eta_{12}\eta_{21} > 0. \end{aligned} \tag{14a}$$

As $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$ by the fact that the Slutsky matrix is negative semi-definite[17], the condition cannot be satisfied: overshifting is not possible.

While overshifting is not possible with an increase in the tax rate in a single market, increases in sales or VAT tax rates increase taxes in multiple markets. To consider the possibility of overshifting with tax increases in multiple markets, let $\hat{\tau}_1 = \hat{\tau}_2$. Then using (7a) for $\hat{p}_1 > 0$ requires

$$\begin{aligned} &(\eta_{11} + \eta_{12})\mu_2 - (\eta_{11}\eta_{22} - \eta_{12}\eta_{21}) > 0 \Rightarrow \hat{p}_1 > 0. \\ &\text{s.t. } (\eta_{11} - \mu_1)(\eta_{22} - \mu_2) - \eta_{12}\eta_{21} > 0. \end{aligned} \tag{14b}$$

The distinction between the overshifting condition with the tax increase only in market 1 (14a) and equal (percentage) tax increases on both x_1 and x_2 (14b) is the term $\eta_{12}\mu_2$ in (14b). While the second term of (14b), $-(\eta_{11}\eta_{22} - \eta_{12}\eta_{21})$, is negative, the first term, $(\eta_{11} + \eta_{12})\mu_2$ is positive if $\eta_{12} > |\eta_{11}|$, that is, the cross price elasticity of x_1 with respect to q_2 is greater than the absolute value of its own price elasticity. Indeed, Okrent and Alston (2012) and Harding and Lovenheim (2017) show some commodities have cross-price elasticities that are larger than the own-price elasticity.

It is important to note that overshifting is possible in only one of the two markets being taxed. Necessary conditions for both markets to have overshifting are $\eta_{12} > |\eta_{11}|$ and $\eta_{21} > |\eta_{22}|$. However, then the condition $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$ is violated.

When overshifting is or is not possible can also be seen by examining (7a). The numerator of term (B) consists of only demand-side parameters, specifically the product of the cross-price elasticities. In contrast, the numerator of term (C), which gives the impact of the second market, consists of demand-side and supply-side parameters. For term (B) to be of sufficient magnitude to generate overshifting from the single tax, the product of the cross-price elasticities would have to be of a magnitude in

[17] For discussion, see Mas-Colell et al. (1995), p. 69 Proposition 3.G.2.

excess of those ensuring the negative semi-definiteness of the Slutsky matrix, that is, $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$. The elasticity of supply in term (C), faces no such constraint. This supply elasticity allows for a "bigger" increase in the price in the second market, which may generate larger feedback effects in the first market.

2.3.2 The Edgeworth Paradox: Can Tax Increases Be Undershifted?

As with overshifting, we first investigate the possibility that a tax increase in a single market leads to undershifting, that is, $\hat{p}_1 < -1$ and that consumer prices may fall ($\hat{q}_1 < 0$). As shown in Appendix A.4, using (7a) with $\hat{\tau}_2 = 0$ for $\hat{p}_1 < -1$ we obtain

$$\begin{aligned} \mu_1 (\mu_2 - \eta_{22}) < 0 \Rightarrow \hat{p}_1 < -1 \\ \text{s.t. } (\eta_{11} - \mu_1)(\eta_{22} - \mu_2) - \eta_{12}\eta_{21} > 0, \end{aligned} \tag{15a}$$

which, as both μ_1 and $\mu_2 - \eta_{22}$ are clearly positive, cannot be satisfied.

Consider the case with tax increases on both x_1 and x_2. Then using (7a) with $\hat{\tau}_1 = \hat{\tau}_2$ for $\hat{p}_1 < -1$ we have the condition,

$$\begin{aligned} \mu_1 (\mu_2 - \eta_{22}) + \eta_{12}\mu_2 < 0 \Rightarrow \hat{p}_1 < -1 \\ \text{s.t. } (\eta_{11} - \mu_1)(\eta_{22} - \mu_2) - \eta_{12}\eta_{21} > 0. \end{aligned} \tag{15b}$$

The distinction between (15a) and (15b) is the term $\eta_{12}\mu_2$ – the same difference between the overshifting conditions. As with overshifting, while undershifting is not possible when only τ_1 changes, it is possible when both taxes change. While overshifting required that x_1 and x_2 be substitutes, undershifting requires they be complements.

Proposition 3. *With perfect competition and two related commodities:*

(1) the necessary condition for overshifting to arise is that both commodities are taxed and the commodities are Hicksian substitutes. The sufficient conditions for overshifting are given by (14b)*.*

(2) the necessary condition for Edgeworth's paradox to arise is that both commodities are taxed and the commodities are Hicksian complements. The sufficient conditions for Edgeworth's paradox are given by (15b)*.*

Figure 3a illustrates overshifting with taxes on two substitute commodities and Figure 4b shows the case of undershifting with taxes on two complementary commodities. Now, as we consider the case where both commodities are taxed, terms (A), (B), and

(C) may potentially appear in the expressions for $\hat{q}_1$ and $\hat{q}_2$. We focus on the impacts on consumer prices in Figure 3a – showing overshifting – and the impacts of producer prices in Figure 4b – showing undershifting.

In Figure 3a, focusing on commodity x_1 with substitute commodity x_2, we illustrate the total impact of the tax τ_1 – increasing the price to $\widetilde{q}_1$, the sum of the partial equilibrium impact (term (A)) and the impact arising from the increase in demand and price for x_2 (term (B)). As x_2 is also taxed at τ_2, there is a further increase in demand for x_1 (term (C) of (8)). This additional increase in demand raises the price of x_1 to consumers, $\widetilde{\widetilde{q}}_1$, above the consumer price with full incidence, $p_1^o + \tau_1$. Analogous impacts are illustrated for x_2. In Figure A.1 we illustrate over-shifting with taxes on both x_1 and x_2 using the general equilibrium demand curve representation. In this case, the shift out in $\widetilde{D}_1$ reflects the increase in τ_2 when x_1 and x_2 are substitutes.

Next, consider the possibility of undershifting with complementary commodities. To simplify the figure, we assume a perfectly elastic supply for commodity x_2 in Figure 4b, i.e. the increase in the consumer price of x_2 is τ_2. We focus on this case as undershifting in market 1 is most likely to arise because the increase in taxes yields the largest price increase for commodity x_2. Under this assumption, the decrease in the producer price of x_1 is the partial equilibrium impact (term (A)) and the reduction in producer price due to increase in the gross-of-tax price of its complement (term (C)). As shown the net of tax price of x_1, $\widetilde{\widetilde{p}}_1$, falls below the price when producers bear the full incidence of the tax, $p_1^0 - \tau_1$.

2.4 Cascading Taxes

Thus far our emphasis has been on how demand relationships between products cause differences between the general and partial equilibrium incidence. More generally, supply side linkages may also result in similar mechanisms as in our model. We model this as tax cascading, but the conclusions apply more generally than this tax example.

While the sales tax is often regarded as a tax on final consumption goods, in fact, intermediate materials or services are sometimes subject to sales taxation, a phenomena often referred to as "cascading" or "pyramiding" with about two-thirds of sales tax revenues derived from purchases other than those of final consumers (Ring 1989). In particular, tax cascading arises because a significant portion of transactions taxed under the retail sales tax are business to business, that is, "intermediate" goods. Wildasin (2001) gives the example where firms selling taxable products to consumers also purchase taxable inputs from their upstream suppliers. Then, as is the case in many states, if half of sales tax revenue is from final consumption, the price of these taxable consumption goods will reflect the the other half of sales taxes coming from business inputs. As such, for a six percent sales tax, the effective tax on the final consumption product would be 12%. Wildasin (2001) constructs effective tax rates by commodity category using an input-output table, and they range from an effective tax rate of 0% to 12%. Tax rates vary in a haphazard or arbitrary way across commodity groups, not well approximated by a uniform statutory rate on all commodities. While it might be argued that the VAT does not have this problem as the tax is a share of valued added at each stage of production, exemptions under credit-invoice VAT systems and, under subtraction VATs, differential taxation of different products that may be inputs, leads to effective differential taxation of final products due to some firms being exempt from VAT not being able to rebate VAT on inputs or the possibility of VAT evasion.

To highlight the impact of cascading on tax incidence, we offer a slight modification of our three product model. Rather than have x_1 and x_2 be linked by demand, we assume that x_2, in addition to being consumed, is also an input in the production x_1. Production of a unit of x_1 requires $\alpha > 0$ units of x_2, making the marginal cost of a unit of x_1(and therefore p_1) equal to $c_1^{'}(x_1) + \alpha(p_2 + \tau_2)$. Alternatively, define $p_1^n = c_1^{'}(x_1)$, the return to producers of x_1 net of both τ_1 and $p_2 + \tau_2$ making $q_1 = p_1^n + \tau_1 + \alpha(p_2 + \tau_2)$. The demand for x_2 is given by

$$E_2^D(\mathbf{q}, y) = x_2^D(\mathbf{q}, y) + \alpha x_1^D(\mathbf{q}, y) - x_2^S(p_2) \tag{16a}$$

and revise the excess demand for x_1 to be

$$E_1^D(\mathbf{q}, y) = x_1^D(\mathbf{q}, y) - x_1^S(p_1^n) \tag{16b}$$

We isolate the effects of cascading, by assuming that $\eta_{12} = \eta_{21} = 0$ and that both x_1 and x_2 are "small" markets $\left(\frac{q_i x_i}{q_3 x_3} \approx 0,\ i = 1, 2\right)$. As shown in Appendix A.6, differentiating (16a) and (16b) yields

$$\hat{p}_1^n = \rho_1 \hat{\tau}_1 + (1 + \rho_1) \frac{\alpha^2 \eta_{11}^2}{|H^C|} \hat{\tau}_1 + \frac{\alpha \eta_{11} \mu_2}{|H^C|} \hat{\tau}_2 \text{ and} \tag{17a}$$

$$\hat{p}_2 = \frac{\alpha \eta_{11} \mu_1}{|H^C|} \hat{\tau}_1 + \frac{-\eta_{22}(\eta_{11} - \mu_1) + \alpha^2 \eta_{11}^2}{|H^C|} \hat{\tau}_2 \tag{17b}$$

where $\left|H^C\right| = (\eta_{11} - \mu_1)(\eta_{22} - \mu_2) - \alpha^2\eta_{11}^2 > 0$. Note that $\hat{p}_2$, as seen in (17b), captures two effects. The first term is the decrease in p_2 due to the decrease in demand for it as an input in x_1 now that x_1 is taxed. The second term consists of two effects of taxing x_2: the price (to the producer) falls directly due to the tax, a partial equilibrium effect. A compounding effect is a price decrease due to a reduction in quantity demanded of x_1 and, therefore, a reduction in demand for x_2 as an input. Using (17a) and (17b) yields the change in the producer price of x_1 inclusive of the tax on x_2,

$$\hat{p}_1 = \hat{p}_1^n + \alpha(\hat{p}_2 + \hat{\tau}_2) = \underset{\text{(A)}}{\rho_1\hat{\tau}_1} \quad \underset{\text{(B)}}{+\frac{\mu_1\alpha^2\eta_{11}^2}{|H^C|}\left[1 + \frac{1}{(\mu_1 - \eta_{11})}\right]\hat{\tau}_1} \quad \underset{\text{(C)}}{+\frac{\alpha\mu_1\mu_2}{|H^C|}\hat{\tau}_2}, \tag{18a}$$

and, analogously, the change in the consumer price,

$$\hat{q}_1 = \hat{p}_1 + \hat{\tau}_1 = \underset{\text{(A)}}{(1+\rho_1)\hat{\tau}_1} \quad \underset{\text{(B)}}{+\frac{\mu_1\alpha^2\eta_{11}^2}{|H^C|}\left[1 + \frac{1}{(\mu_1 - \eta_{11})}\right]\hat{\tau}_1} \quad \underset{\text{(C)}}{+\frac{\alpha\mu_1\mu_2}{|H^C|}\hat{\tau}_2}. \tag{18b}$$

If the change in the tax is the same on both x_1 and x_2 as the result, for example, of an increase in a general sales tax, (18a) becomes

$$\hat{p}_1 = \left\{\rho_1\left(1 + \frac{\mu_1\alpha^2\eta_{11}}{|H^C|}\right) + \frac{\mu_1\alpha}{|H^C|}\left(\alpha\eta_{11}^2 + \mu_2\right)\right\}\hat{\tau} \tag{19}$$

where $\hat{\tau} \equiv \hat{\tau}_1 = \hat{\tau}_2$, the common (percentage) change in $\hat{\tau}_1$ and $\hat{\tau}_2$.

Figure 4 shows the intuition of tax cascading for the simple case where the supply of the output, x_1, is horizontal. In market 1, the tax there shifts supply up (term A). This reduces the quantity demand for x_1 and, therefore, the demand for its input, x_2, also lowering its price. This lower price of input x_2 reduces marginal cost for x_1 (term B). As both markets are taxed, the tax on x_2 increases the marginal cost for x_1 (term C) but not by the full amount of the tax, $\alpha\tau_2$ because the reduction in demand for x_1 associated with the tax on x_2 reduces the derived demand for x_2 (shifts in demand to D_2'') and the price of x_2. In this example, prices rise by more than the statutory tax rate, but the appropriate benchmark to assess incidence is the effective tax rate. Even though the statutory tax rate is τ_1, the effective tax rate is $\tau_1 + \alpha(\hat{p}_2 + \tau_2)$. If the supply for x_2 is perfectly elastic, the effective tax is $\tau_1 + \alpha\tau_2$.

The formal condition for the possibility that consumer prices rise by more than the statutory tax change ("overshifting") is stated in Appendix A.6; it is relatively easy to show conditions under which "overshifting" occurs.Two obvious cases are when

$\eta_{11} = 0$, demand for x_1 is perfectly inelastic, or when $\mu_1 \to \infty$, the supply for x_1 is perfectly elastic. With $\eta_{11} = 0$, (19) simplifies to

$$lim_{\eta_{11}\to 0}\,\hat{p}_1 = \left[\frac{\mu_1\mu_2\alpha}{-\mu_{11}(\eta_{22}-\mu_{22})-\alpha^2\eta_{11}^2}\right]\hat{\tau} > 0 \tag{20a}$$

and when $\mu_1 \to \infty$, (19) simplifies to

$$lim_{\mu_1\to\infty}\;\hat{p}_1 = \left[\frac{\alpha}{(\mu_{22}-\eta_{22})}\left(\alpha\eta_{11}^2+\mu_2\right)\right]\hat{\tau} > 0. \tag{20b}$$

If a researcher ignored that inputs were taxed, they would observe what looks like "overshifting" in the data, however, this would be because of comparing the price change to the statutory tax change rather than the effective change.

Proposition 4. *With perfect competition and two commodities, one of which is an input to the other commodity, but which are neither substitutes nor complements, the consumer incidence of the tax as a fraction of the statutory tax on the final product can exceed one.*

3 Evidence from Simulations

3.1 Simulations

We report simulations that demonstrate the magnitude of deviations from the partial equilibrium formula and parameter values for which overshifting and Edgeworth's taxation paradox arise. We focus on producer incidence.

First, we demonstrate the importance of multi-market incidence discussed in section 2.2.1. The budget share weighted average of the two price changes is given in table 1 for various elasticity combinations. In the upper panel of the table, we set all own-price elasticities at one and allow the cross-price elasticities to vary. In this setting, the partial equilibrium incidence on producers is one-half. As discussed above, the general equilibrium incidence in the taxed market is closer to zero, regardless of whether the related products are complements or substitutes. However, the multi-market incidence depends critically on whether the products are complements or substitutes. In the presence of substitutes, the multi-market incidence on producers is closer to zero than the partial equilibrium formula. As indicated, if the taxed market is a small budget share relative to the substitute product's budget share, it is possible that the multi-market incidence on producers results in an overall price increase due to prices rising in the untaxed market. In the presence of complements, the multi-market incidence on producers results in prices falling by more than suggested by the partial equilibrium formula. In the lower-panel, we focus on supply elasticities that are large, so that the partial equilibrium formula yields only a very small price effect for producers. Notice, in this setting, the partial equilibrium formula is a reasonable approximation for the general equilibrium effects in the taxed market (the error is between 2% and 6%). However, the multi-market incidence can be off by more than 100% the standard partial equilibrium incidence. Even if the partial equilibrium formula is a reasonable approximation to the incidence in a given market, researchers should be careful to account for the effect of the tax in related markets, especially if consumer purchase both products (like the example of tennis racquets and balls).

In table 2 we show the incidence under our formula and the percent change relative to the partial equilibrium formula. We show a baseline estimate when all own-price elasticities are equal to unity in absolute value; each row of the table perturbs one of these own-price elasticities in the spirit of "comparative statics". The columns show the incidence for various cross-price elasticities. In the first two columns, both cross-price elasticities are equal. In the second set, they differ. When $\eta_{12} = 1.5$ and $\eta_{21} = 0.5$, then market 1 is three times as large as market 2 (recall $\eta_{ji} = \frac{q_i x_i}{q_j x_j}\eta_{ij}$). We see that when two taxes change, the incidence may be over or undershifted to the consumer and the bias from the standard formula can be substantial. The pairs of cross-price elasticities indicate when over and undershifting are most likely to arise. Then, in table 3 we show simulations when the supply elasticity in both markets is large (i.e., the standard case when the supply curve approaches horizontal). We present this case as it is commonly assumed in the literature. Even with relatively horizontal supply curves the bias in the partial equilibrium formula can be large in percentage terms. Finally, in table A.1 we consider inelastic supply in market one with very elastic supply in market 2; the possibility of Edgeworth's Paradox and overshifting is most likely in these cases.

While the tables are appealing for precise estimates of incidence, they do not allow us to show the precise elasticity regions for over- or undershifting. In figure A.2, we show the producer incidence when both market 1 and 2 are small and only market 1 is taxed. For ease of interpretation, we fix all own-price elasticities at $\mu_1 = \mu_2 = |\eta_{11}| = |\eta_{11}| = 1$ and allow the cross-price elasticities to vary.[18] Three constraints are

[18] As $\eta_{ij} = \frac{q_j x_j}{q_i x_i}\eta_{ji}$, varying the *relative* cross-price elasticities is equivalent to varying *relative* budget

imposed: the second order conditions for expenditure minimization; the condition on the Hessian matrix; and the product of the two (compensated) cross-price elasticities must be positive. Areas where these conditions do not hold are in white. Our general equilibrium formula has producer incidence that is always closer to zero by anywhere between 0 and approximately 20 cents, which is up to a 40% error in the partial equilibrium formula for these parameters.

In figure A.3, we maintain all assumption as above, but now allow for taxes in both markets 1 and 2. We have equal tax changes in both markets, as in the case of the general sales tax. In this figure, the difference between the partial and general equilibrium formula is no longer always positive. Now, the change in the producer price relative to the partial equilibrium formula can either increase or decrease. The figures indicate that this deviation can be substantial. Indeed, the possibility of overshifting can arise if the cross-price elasticity of good 1 with respect to the price of good 2 is sufficiently large. For the given parameterization, Edgeworth's paradox cannot arise.

Finally, in figure A.4, we hold all own-price elasticities the same, except we allow the supply curve elasticity to be very large. In this case, undershifting is more likely to arise. We plot the producer price results for market 1. Again, the deviations from partial equilibrium can be positive or negative and are substantial. We have the possibility that both Edgeworth's paradox and overshifting can arise which is consistent with the intuition in figure 4b.

To generalize the results, we hold three of these elasticities constant and change one by ± 0.50. See figure A.5. This allows us to see how the general equilibrium incidence formula changes, in the spirit of "comparative statics." We focus on the contour plot of areas that show if Edgeworth's paradox or overshifting arise (the incidence is changing smoothly across this figure: increasing toward the darker areas and decreasing toward the lighter areas). As is evident, overshifting is more likely to arise if $|\eta_{11}|$ is small or $|\eta_{22}|$ is large. Edgeworth's paradox is more likely to arise, if μ_1 is small or μ_2 is large.

3.2 Evidence from a Specific Utility Function

3.2.1 An Example of Overshifting

As an alternative approach to obtaining some indication of the conditions when overshifting might occur, we provide an example of a utility function under which overshift-

shares, a point discussed more explicitly shortly.

ing is possible. In this simple three commodity framework, overshifting would require one of the two taxed commodities (x_1, x_2) to be a complement with the untaxed commodity, x_3. A utility function that meets those conditions is

$$U(x_1, x_2, x_3) = (min[\beta_1 x_1, \beta_3 x_3])^{\alpha} x_2^{1-\alpha} \tag{21}$$

where $\beta_i > 0$, $i = 1, 2$. Then x_1 and x_3 are "perfect" complements and x_2 is a substitute with both of them. Appendix A.8 derives the the compensated demand equations, which yield the relevant demand elasticities for determining incidence,

$$\eta_{11} = -\frac{(1-\alpha)\,\beta_3 q_1}{(\beta_3 q_1 + \beta_1 q_3)},\ \eta_{12} = (1-\alpha),\ \eta_{22} = -\alpha,\ \text{and}\ \eta_{21} = \frac{\alpha \beta_3 q_1}{(\beta_3 q_1 + \beta_1 q_3)}. \tag{22}$$

Then it is easy to show that

$$\eta_{12} - |\eta_{11}| = (1-\alpha) - \left| -\frac{(1-\alpha)\,\beta_3 q_1}{(\beta_3 q_1 + \beta_1 q_3)} \right| = (1-\alpha)\,\frac{\beta_1 q_3}{(\beta_3 q_1 + \beta_1 q_3)} > 0 \tag{23}$$

and that $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} = 0$.

We provide simple numerical simulations of the potential magnitude of overshifting that can arise from this utility function. By no means are we attempting to provide an indication of the range of overshifting that might be possible. While relative prices of the commodities will affect the elasticities and therefore potential overshifting, we normalize all three commodity prices to unity in this example. We choose $\alpha = 0.95$ making x_2 approximately five percent of expenditures and choose β_1 and β_3 to have values such that x_1 is also approximately five percent of expenditures.[19] This gives $\eta_{11} = -0.003$, $\eta_{22} = -0.95$, $\eta_{12} = 0.05$, and $\eta_{21} = 0.06$. We then vary μ_1 between three values [0.25, 0.5 , 1] and μ_2 between three values [1, 2, 10]. Then, we focus on how variation in the supply elasticities, μ_1 and μ_2 affect tax incidence.

Panel A of Table 4 confirms what was shown in (23) – overshifting will occur with all non-zero (and non-negative) values of the parameters of the utility function. While the extent of overshifting is relatively small, from 101 to 109 percent of the tax, we can see that it increases with the elasticity of supply for x_2 and decreases in the elasticity of supply for x_1. As show by (7a), there is relatively little impact from the cross-market "feedback" effects of τ_1 (term B) but significant impacts of τ_2 (term C).

[19] If $\frac{\beta_1}{\beta_3} = 16$ when $\alpha = 0.95$ we have approximately five percent of expenditures on both x_1 and x_2.

3.2.2 An Example of Undershifting

As an example of undershifting we use the same utility function, but have x_1 and x_2 be complements and x_3 be a substitute with both x_1 and x_2:

$$U(x_1, x_2, x_3) = (min[\beta_1 x_1, \beta_2 x_2])^{\alpha} x_3^{1-\alpha}. \tag{24}$$

In this case, the compensated elasticities associated with this utility function are

$$\eta_{11} = \frac{(\alpha - 1)\beta_2 q_1}{(\beta_2 q_1 + \beta_1 q_2)},\ \eta_{22} = \frac{(\alpha - 1)\beta_1 q_2}{(\beta_2 q_1 + \beta_1 q_2)},\ \eta_{12} = \frac{(\alpha - 1)\beta_1 q_2}{(\beta_3 q_1 + \beta_1 q_3)},\ \text{and}\ \eta_{21} = \frac{(\alpha - 1)\beta_2 q_1}{(\beta_3 q_1 + \beta_1 q_3)} \tag{25}$$

Then, as shown in (15b) while $\eta_{12} < 0$ is a necessary condition for undershifting, the role of the supply elasticities, particularly the elasticity of supply of x_1, μ_1, are critical. In Panel B of Table 4 we parameterize (24) so that both x_1 and x_2 are five percent of expenditures.[20] This gives $\eta_{11} = \eta_{22} = \eta_{12} = \eta_{21} = -0.45$. In this case, we again vary μ_1 between three values [0.25, 0.5, 1] and μ_2 between three values [1, 2, 10]. As can be seen in the table (bold cells), undershifting occurs when $\mu_1 = 0.25$ regardless of the value of μ_2. However, the extent of undershifting increases in μ_2 reaching a decrease in price to the supplier of 126% when $\mu_1 = 0.25$ and $\mu_2 = 10$.

4 An Empirical Example: Taxes on Alcohol

As an empirical example of our theory, we study the alcoholic beverage industry. In the United States, state governments differentially tax beer, wine and spirits using a combination of excise taxes. These taxes are included in the posted price and are salient for consumers. Although all three excise taxes are governed by different statutes, state governments often change all three taxes at the same time. For example, in 85% of the instances when a state changes its excise tax rate on wine, the spirit and beer rates also change. Table 5 summarizes these contemporaneous comovements of tax rates. This provides some initial evidence that multiple taxes often change simultaneously beyond the sales tax or VAT applications discussed previously. This combined with beer, wine, and spirits being related products, provides an important, albeit, selective example.

Rather than develop new empirical methods or data, we test our theory using the methods outlined in Besley and Rosen (1999) modified for excise taxation. To do this,

[20] We set $\beta_1 = \beta_2 = 1$ and $\alpha = 0.1$ in (24) with prices normalized to unity.

we use similar data as Besley and Rosen (1999). Pricing data, from 1982 to 2018, comes from the American Chamber of Commerce Researchers Association (ACCRA) price index, which tracks among other items, the price of a six-pack of beer, a bottle of wine, and a bottle of liquor. These prices are tracked at the city level by Chamber of Commerce units that respond to the survey. Researchers report the retail prices of the specified items when responding to the survey, so these prices include the excise tax but exclude any sales tax. Tax data comes from the Tax Policy Center and is supplemented with other sources. We do not include any local excise taxes so that we can focus on state tax changes. To focus on contemporaneous price changes, we identify the tax as of the first quarter in each year and match this to pricing data in the quarter. Following, Chetty et al. (2009) we obtain state level prices by averaging the prices of all reporting metro areas in the state.[21] In addition, we include income, population, unemployment, and the drinking age as controls. We estimate

$$q^j_{st} = \alpha + \beta\tau^j_{st} + X_{st}\rho + \zeta_s + \zeta_t + \epsilon_{st}, \tag{26}$$

where q_{jt} is the excise-tax-inclusive prices of product $j = B,\ W,\ S$ (for beer, wine, and spirits) in state s and period t. Then, τ is the excise tax in dollars on alcohol, X_{st} are state demand and cost shifters, including the controls listed above. Given alcohol is subject to a sales tax rate, we include the sales tax rate on alcohol in the regression. Finally, ζ_s and ζ_t are state and year fixed effects. We estimate this equation separately for each of the three product classifications and cluster standard errors at the state level. This equation, as in Besley and Rosen (1999), implicitly assumes that taxes on wine and spirits do not influence beer demand. Thus, we modify this equation to account for multiple products, by again separately estimating for each j:

$$q^j_{st} = \alpha + \sum_j \beta^j\tau^j_{st} + X_{st}\rho + \zeta_s + \zeta_t + \epsilon_{st}. \tag{27}$$

This equation accounts for the indirect effects through other taxes due to multiple taxes changing and non-zero cross-price elasticities. Then, for example, when considering beer prices, the interpretation of the coefficient on beer taxes in (27) will measure the direct effect of beer taxes along with any feedback effects, while the coefficient on wine and spirit taxes will identify the effect due to additional taxes changing. On the other

[21] Results are robust to using an unweighted average and a weighted average where we use the population of the city reporting the price as weights.

hand, the coefficient on (26) will include all of these effects in the beer tax coefficient. Empirically, terms (A) and (B) cannot be separately identified in the approach.

Table 6 presents the baseline results. We present results for the average price in a state (columns without a prime) and the population-weighted average price (columns with a prime). To make coefficients within a column comparable, we calculate all taxes as if they were on an equivalent volume of alcohol for all products.[22] Notice that a one dollar increase in the excise tax on beer increases its price by 0.96 dollars. This result is similar to the prior literature that finds evidence consistent with (at least) full pass through. However, when accounting for multiple taxes, a one dollar increase in the beer tax increases its price by 0.34 dollars (term A and B). Increases in the wine and spirit tax have significant effects on beer prices (term C). These effects are economically meaningful and statistically significant. Given the effects are positive, this suggests that wine and spirits are substitutes for beer. Although (27) identifies term A and B jointly and term C separately, term C can only be non-zero if the cross-price elasticities are non-zero. Results are similar when looking at wine prices: the own-price effect falls after accounting for other taxes and other taxes have meaningful effects.[23]

Finally, table A.3 presents an interaction specification that allows us to interpret a marginal effect. To run this specification, we enter only the own-product tax rate, but also interact it with dummy variables indicating if the taxes on the other products change or if multiple products' taxes change. We then present the marginal effects of the own-tax price change on prices. This provides a direct test of when overshifting will occur in our theory and also addresses concerns in the prior regression related to the possible colinearity of tax rates. While not controlling for the magnitudes of the other tax changes, as noted in our theory, the own-tax rate will capture the indirect effects in such a reduced form analysis. For beer, when only the beer tax changes, prices rise by 0.92 dollars. But, when beer and wine taxes rise, prices rise by a \$1.44. As predicted by theory, overshifting only arises when multiple taxes change simultaneously. When both

[22]In the beer price regression, all taxes are for buying the equivalent of a 0.5625 gallons of alcohol; in the wine regression, all taxes are for purchasing the equivalent of 1.5 liters of alcohol. While this standardization allows for easy comparison within a column, interpreting coefficients across different products is difficult. For this reason, in table (A.2) we present results where the taxes are in different units corresponding to the same volumes for the priced items in the ACCRA.

[23]With wine, even after accounting for other taxes, the own-price effect indicates overshifting. While this may not be possible in our model, this could be a result of unobservable cost determinants. So long as these unobservables affect both the first and second specification in the same manner, we are not concerned about this issue.

wine and spirit taxes change, the results are closer to when only beer taxes change.[24]

To summarize, we find that both channels of our model – related products and simultaneous tax changes – are important.

5 Implications for Empirical Analysis

5.1 Empirical Models Used in the Literature

The traditional empirical model of commodity price incidence for retail sales taxes (Besley and Rosen 1999)[25] is a reduced form equation to explain the log (tax-exclusive) price for commodity i in jurisdiction j in year t as a function of the tax rate τ, observable characteristics of the city or commodity in X, and jurisdiction and time fixed effects

$$ln(p_{ijt}) = \beta\tau_{ijt} + \gamma X_{ijt} + \zeta_j + \zeta_t + \varepsilon_{ijt}. \tag{28}$$

Besley and Rosen (1999) show that β can be translated to the change in the consumer price. If the tax rate is small, then β can be directly interpreted as the amount of *over-shifting* of the tax to the consumer. Thus, if $\beta = 0$ the price is fully born by the consumer, if $\beta < 0$ the price incidence is split between the consumer and the producer, and if $\beta > 0$ the tax is over-shifted to the consumer. This reduced-form estimation strategy has now become common in the literature. Studies of the VAT use the tax-inclusive price.

Besley and Rosen (1999) use city-level prices for specific products to estimate the pass through rate. They find large variation of the pass through rate across different types of products. Besley and Rosen (1999) estimate $\beta = 0$ for some products like eggs and tissue paper, but find $\beta > 0$ for many products like Big Macs, bananas or Monopoly games. Recall that β is the extent of overshifting so that a $1 of revenue increase in the tax, increases the tax inclusive price by approximately $1 + \beta$.[26]

[24]The prior analysis indicates that both wine and spirits are substitutes with beer, but this analysis suggests the spirits may be complements. However, the coefficient on spirits here is insignificant. This could be due to the fact that in many states spirits are not subject to taxation because they are sold from state controlled stores (see, for example, Miravete et al. 2018) and our tax data do not have state markups in these stores.

[25]We refer to the paper of Besley and Rosen (1999), but our comments apply to almost all papers in this tax incidence literature using this method. We simply use this paper as an example because of the clear presentation of results and the interesting heterogeneity.

[26]In the U.S. setting, other studies that find overshifting include Kenkel (2005) and Poterba (1996). However, most studies finding overshifting, cite imperfect competition as an example of overshifting,

Studies have formalized the model by comparing treated commodities with untreated commodities in the context of natural experiments. This is done in a formal difference-in-difference framework by estimating

$$ln(p_{it}) = \alpha post_t + \beta post_t \times treat_i + \gamma X_{it} + \zeta_i + \zeta_t + \varepsilon_{ijt} \tag{29}$$

for firm or product i in year t. The variable "treat" takes on the value of one for products or firms experiencing a tax reform and the variable "post" is a value that takes on one for the post-reform period. The design usually involves comparing the treated product or firm with another product that closely resembles it. For example, Kosonen (2015) studies a tax reform on hairdressers and uses "other labor intensive services carried out in small businesses that closely resembles hairdressers, but are not affected by the reform."[27]

5.2 Implications for Empirical Methodology

First, our study influences the optimal research design for empirical studies. When employing a difference-in-difference design using taxed commodities as a treatment group and untaxed commodities (or firms) as a control group, assuming no pre-period differences, researchers will estimate $\beta = \hat{p}_{treat} - \hat{p}_{control}$ for producer prices. Thus, if the treatment products are substitutes for the control products, $\hat{p}_{treat} \geq -1$ and $\hat{p}_{control} \geq 0$. As such, estimates of β will be biased and will overestimate the true incidence by producers and underestimate the true commodity tax incidence born by consumers. If the treatment products are complements to the control products, then $\hat{p}_{treat} \geq -1$ and $\hat{p}_{control} \leq 0$. Again, estimates of β will be biased but perhaps more concerning is that these estimates will overestimate the true commodity tax incidence born by consumers.[28] Of course, the literature is well-aware that using control products in the research design requires any tax changes in the treated sector to not affect the control sector. For this reason, the literature has focused on ruling out "control" products

but none of these studies cite complementarities or substitutabilities.

[27] Kosonen (2015) argues that it is unlikely these other sectors are affected because even if you substitute to other services, haircuts are still necessary. The paper does not rule out complementarities. Benzarti et al. (2018) and Benzarti and Carloni (2019) also rely on difference-in-difference estimation with the latter studying the restaurant industry and selecting a control group of "services that are comparable to the restaurant industry because of their similar nature, but not directly substitutable with restaurants." Indeed, most are also not likely complements.

[28] This ignores multi-market incidence, which the empirical literature has yet to address.

that are substitutes, but more focus should be given to eliminating complementary products. Given product inter-dependencies can never be completely ruled out, our theoretical framework could be used to help bound estimates of incidence. For example, if product 1 is treated (tax change) and product 2 is a control (no tax change), we have

$$\beta = \hat{p}_1 - \hat{p}_2 = \left[\rho_1 + (1+\rho_1)\frac{\eta_{12}\eta_{21}}{\left|\widetilde{H}\right|} - \frac{\eta_{21}\mu_2}{\left|\widetilde{H}\right|} \right] \hat{\tau}_1 = \left\{ \rho_1 + \frac{\eta_{21}}{\left|\widetilde{H}\right|} \left[(1+\rho_1)\eta_{12} - \mu_2 \right] \right\} \hat{\tau}_1. \tag{30}$$

As an alternative to using taxed and untaxed products in the same jurisdiction, researchers might consider comparing the same products in different jurisdictions (assuming these jurisdictions are sufficiently far away such that cross-border shopping is not relevant). In this manner, the researcher can compare prices of commodities subject to the sales tax ignoring demand-side interdependencies. Such an avenue for research may be especially promising in decentralized countries.

5.3 Implications for Interpretation

5.3.1 Elasticities and Incidence

Given the standard partial equilibrium formula, it is tempting to use tax incidence estimates as a tool to infer something about the *relative* supply and demand elasticities. The standard partial equilibrium incidence formula, (4a), suggests that if the incidence is split equally between consumers and producers, that the supply and demand elasticity are equal. If the empirical incidence estimates suggest the consumer pays more of the tax, we generally infer that demand is relatively more inelastic. Our model suggests that this is not the case. As the incidence formula is now a function of more than two elasticities, the relative own-price elasticities cannot be inferred. For example, suppose that a researcher estimated the incidence of -0.47 in the third row of table 2. The demand and supply elasticities in market 1 are not approximately equal. In fact, the demand elasticity is 50% larger than the supply elasticity. Instead of using pass-through to infer relative elasticities, researchers wishing to infer something about the supply and demand elasticities might exploit shocks to supply and demand resulting from, for example, factors that constrain supply chains and alter the elasticity of supply (Marion and Muehlegger 2011).

5.3.2 Overshifting

Second, our study affects the interpretation of the estimated coefficient β. Traditionally, this coefficient has been interpreted in the context of the standard partial equilibrium model where the consumer incidence is clearly bounded between zero and one. Empirical studies often justify overshifting ($\beta > 0$) of the tax to the consumer as evidence consistent with imperfect competition.[29] While imperfect competition is certainly an explanation of $\beta > 0$, our analysis suggests that interpreting β as evidence of market structure is misleading. In particular, we show that when the tax rate on a multiple products increases, as in the case of broad-based taxes on multiple commodities like retail sales or VATs studied in the literature, the estimate of β may be greater than one even though the markets may be competitive. This arises if the product may be related to other taxed products that are also a small share of expenditures. Thus, although it is often argued that overshifting implies imperfect competition similar to that modeled in Delipalla and Keen (1992), we argue that imperfect competition is not necessary for overshifting to occur.

In cases where the tax being considered is a broad based consumption tax (value added tax or even a general sales tax), our model suggests that a sales tax rate increase will also directly impact many untaxed commodities in the consumers consumption basket. Thus, overshifting may arise if the product being studied is a large share of the market. However, for those commodities with all cross-price elasticities close to zero, the partial equilibrium analysis may still apply. In turn, this might be one explanation for why Besley and Rosen (1999) find full pass-through rates for products like tissue paper and eggs, but find overshifting for products like bananas, soda, or Big Macs. Heterogeneous pass-through rates need not be a result of different elasticities of demand in the presence of imperfect competition, but rather, may simply result from different sets of products having different cross-price elasticities.

Our model suggests a further complication for measuring incidence.[30] Even if a tax on a given product is not overshifted to the consumer side of the market, it still

[29] Besley and Rosen (1999) have a section titled "making sense of the results" where they offer two explanations: (1) imperfect competition or (2) unobserved common effects.

[30] Zoutman et al. (2018) show that with two restrictions, a single tax can be used to identify both a demand and supply elasticity. The appendix to their paper extends their model to multiple products. With multiple goods, their model applies if the variation in the tax rate for each good is independent and the variation in prices caused by each tax is linearly independent. In our model, these conditions – especially the first one – may not hold because, by definition, a single change in a broad based tax will not be independent.

remains possible that consumers may face a higher share of the burden of the tax than estimated. Our model makes clear that the incidence is the weighted aggregation of the price changes across all relevant commodities. In the case of a tax on alcohol, this means that the total consumer incidence must consider not only the price change in the alcohol market, but also the price change of all related substitutes and complements. A partial equilibrium analysis of a single commodity market will not represent the incidence borne by consumers resulting from a tax increase; a complete analysis requires studying price changes in the directly affected market but also in the indirectly affected market. Future research might attempt to study multi-market incidence.

5.3.3 Cascading

If an empirical study assumes that business-to-business transactions escape taxation, the study will determine the incidence based off of the statutory sales tax rate on final sales. When doing this, the study might estimate more than full pass-through of the tax to the consumer. However, such a result could be explained by the possibility of tax cascading where the effective tax increase on a product is likely to be much greater than the statutory tax rate. For products that do not have many tangible inputs, the statutory sales tax rate is likely to be a good approximation. However, for products that use taxable tangible inputs in the intermediate stage, the effective tax rate is likely much higher. The use of statutory rather than effective tax rates may explain what empirical studies observe as overshifting. However, if studying the change in price relative to the effective tax rate, overshifting cannot arise if all cross-price elasticities are zero. To address this, researchers might use state-level input-output tables to at least partially account for the possibility of cascading. Using these input-output tables, and assuming the supply curve in the input market is approximately horizontal, the researcher can determine the effective cumulative tax rate that would be born by the consumer across different industries and then benchmark the empirical changes in prices observed in the data against these effective tax rate changes rather than the statutory changes. If the input supply curve is not horizontal, assuming taxes on both inputs and outputs are equal, the fraction of the tax borne by consumers should be benchmarked against the effective change in $\hat{\tau} + \alpha\hat{\tau} + \alpha\hat{p}_2$ rather than $\hat{\tau}$. Note that $\hat{\tau}$ and α are directly observed to the researcher from statutory tax changes and input-output tables. However, $\hat{p}_2$ is unknown and would need to be estimated; but if it is reasonable to assume the elasticity of supply in the input market was close to perfectly elastic, in which case, the effective tax change becomes approximately $\hat{\tau} + \alpha\hat{\tau}$.

Tax cascading can also arise in the VAT setting, if for example, some commodity such as financial services are *exempt* from VAT and refunds are not given for the taxes on the inputs used in their production as is the case under the credit-invoice system (Keen 2013).[31] In the case of exempt goods, the VAT can be passed through to consumers because the final stage lacks a credit against these exempt stages because the exempt dealer need not register for VAT. As in the sales tax, the presence of exempt goods may result in higher effective tax rates, which may result in the empirical study inaccurately estimating the true fraction of incidence borne by the consumer when benchmarking the price change to the statutory rate rather than the effective rate.

6 Conclusion

We present a simple model of tax incidence with multiple products with varying degrees of interdependency, multiple products subject to taxation, and inputs possibly subject to taxation. We show that the standard partial equilibrium formula for tax incidence with a single product is not necessarily a good approximation to the general equilibrium incidence. Just because a product has a small market does not lead to equivalence of the partial and general equilibrium incidence. In a multiproduct setting, product interdependency implies that overshifting of the tax to the consumer and Edgeworth's Paradox is possible even with perfect competition. Tax cascading may also result in the empiricist perceiving "overshifting" if benchmarking price changes relative to the statutory rather than effective tax rate. We present a cautionary, but important, tale for the interpretation of empirical estimates.

More generally, our analysis is appropriate for the study of price effects due to cross-border shopping. Following a characteristics-based approach (Lancaster 1966), products sold in different states could be viewed as highly substitutable given they differ

[31]The discussion of exempt goods is different from that of *zero-rated* (or reduced-rate) goods, where the government does not tax its sale but allows credits for VAT on the inputs. Under the credit-invoice system, the effective VAT rate on a commodity involving a zero-rated good at some stage in the production process will always be the tax rate of the final stage of production. However, under a subtraction method VAT, the effective tax rate will depend on the structure of the production process and which stages are zero-rated. For example, if the final stage is zero-rated, the effective tax rate will be higher than the tax rate on the final stage of production because the zero-rating only applies to the value added in the final stage. Under the credit-invoice system, the statutory rate would equal the effective rate in the example. Researchers studying VAT incidence using multiple countries should carefully determine the structure of credits and account for this when estimating incidence.

only based on only the characteristic of the place of sale. Varying tax rates across states and localities combined with the changing legal status of various products (cigarettes, food, etc.) across states, localities, and Native American Reservations (NAR) creates substantial policy variation (DeCicca et al. 2013). Beyond sales tax differentials at state borders, cigarette tax differentials can be especially large even within a state. In our model, the low-tax rate of other states may have pricing effects on identical products in high-tax state due to substitutability of these two products. The availability of tax-free NAR cigarettes may influence the prices charged for cigarettes by off-reservation retailers. Our model suggests, for example, that a decrease in the tax rate in one jurisdiction could decrease prices in other jurisdictions. We expect this effect will decay with distance as products that are further away from the state border are likely less substitutable. Such characteristics-based approaches have applications to cost shocks in industrial organization and labor economics.

Although our model formalizes pass-through in the form of a taxes, the model generalizes to any shock that causes an increase in marginal cost. Our model provides a cautionary tale for using cost pass through estimates, as has become common, to infer something about market structure. When using pass-through estimates to rule out perfect competition (Pless and van Benthem 2019), researchers must also rule out the presence of demand and supply-side inter-dependencies and multiple cost shocks.

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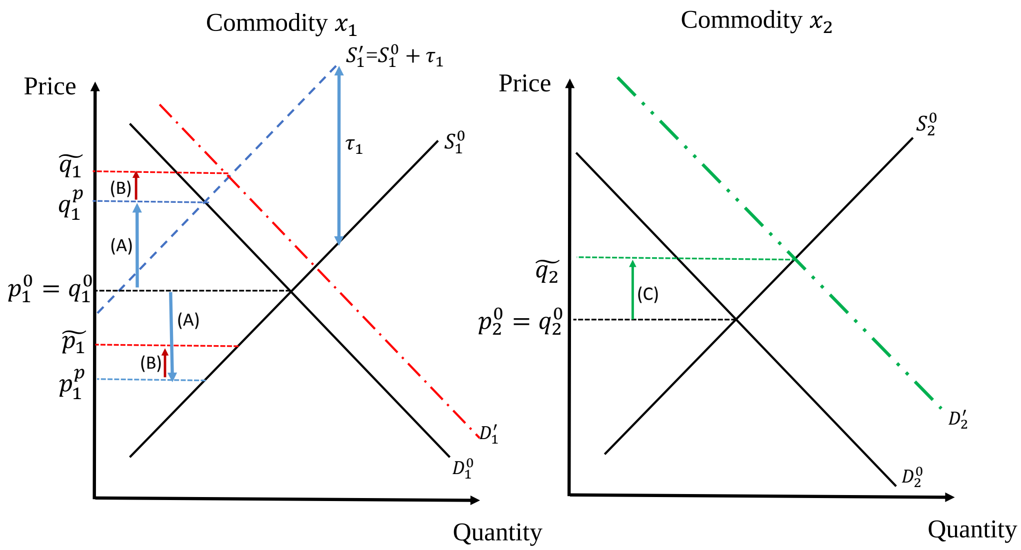
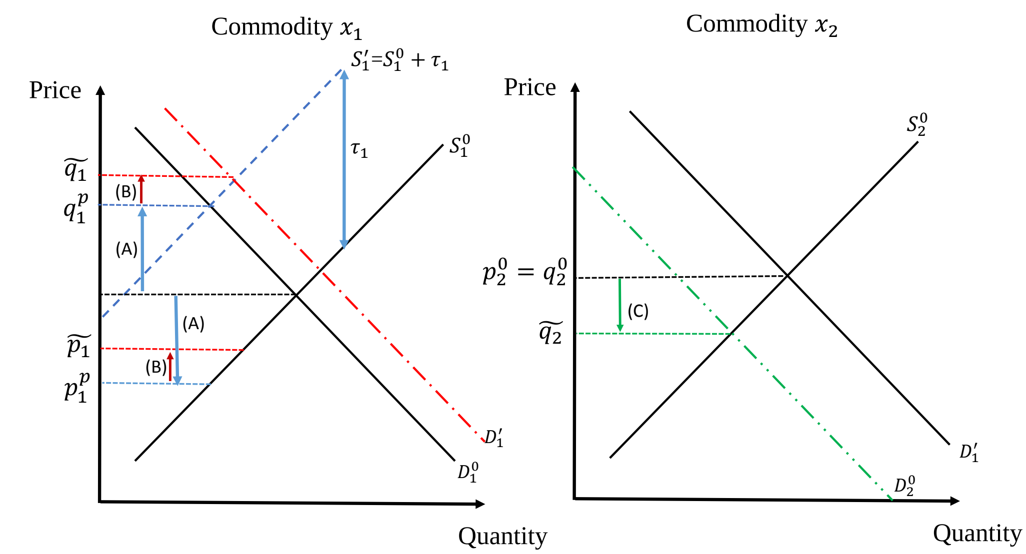
Figure 1: Tax Incidence with Related Commodities

(a) Substitutes

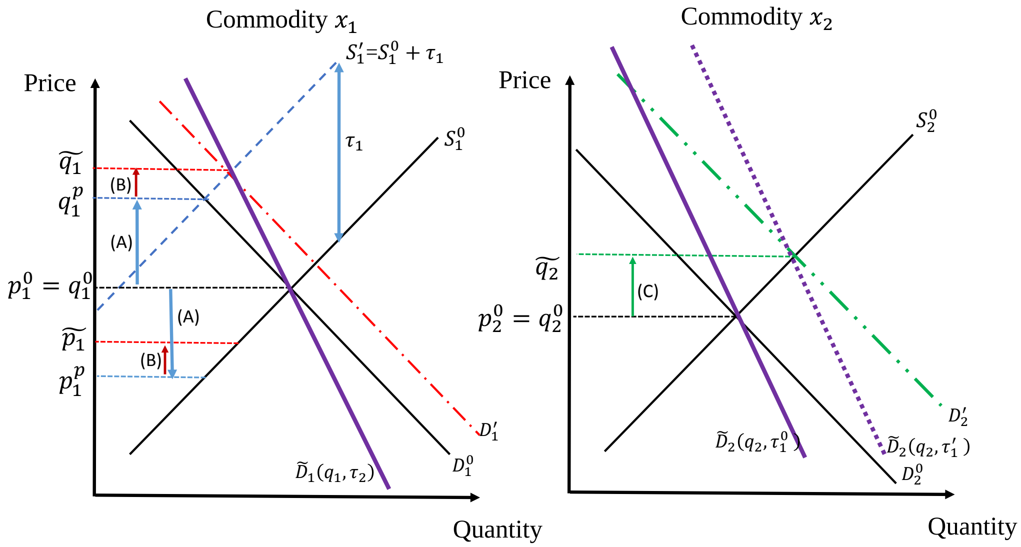




(b) Complements

This figure shows the effects of a tax increase, starting from $\tau_1 = 0$, in the market for x_1. The two commodities are substitutes in panel a and complements in panel b. The initial equilibrium price is given by q_i^0. The partial equilibrium price for consumers is given by q_i^p and by p_i^p for producers. General equilibrium prices are given by $\widetilde{q_i}$ and $\widetilde{p_i}$. The imposition of a tax on x_1 decreases supply as the blue (–) lines indicates. Because of the tax, demand for x_2 changes to the green lines (-..-). The change in q_2 results in a shift in demand for x_1, changing the incidence of the tax to the red line (-.-). Terms (A), (B), and (C) are the shifts defined in (7a) and (8).

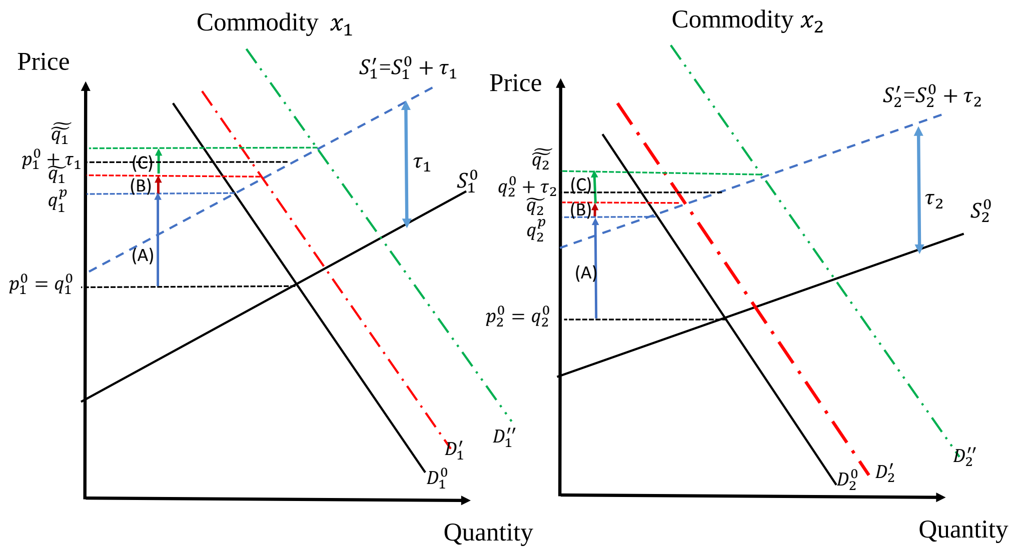
Figure 2: Tax Incidence with Related Commodities: General Equilibrium Demand Curves

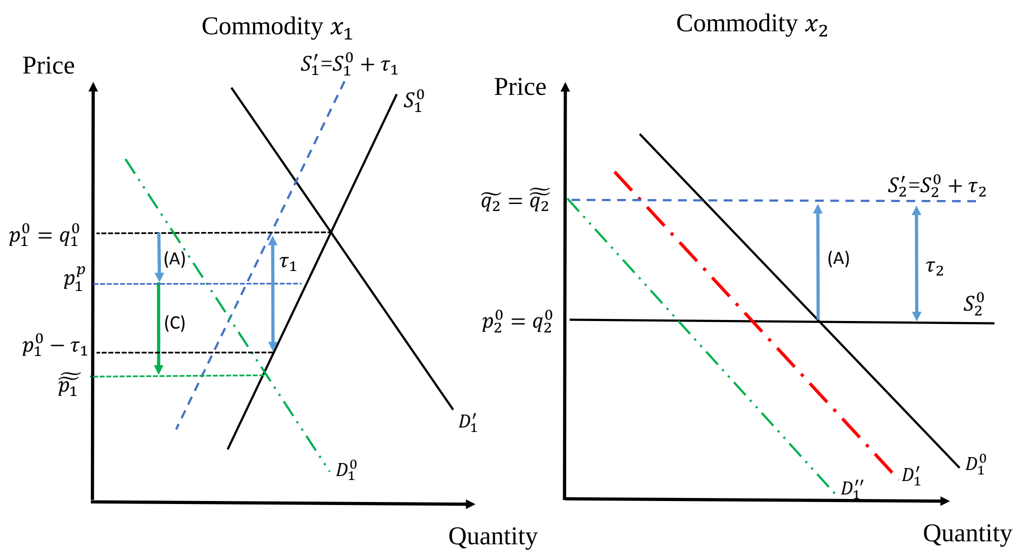


This figure shows the effects of a tax increase, starting from $\tau_1 = 0$, in the market for x_1. The two commodities are substitutes in panel a; the graph for complements is omitted. The initial equilibrium price is given by q_i^0. The partial equilibrium price for consumers is given by q_i^p and by p_i^p for producers. General equilibrium prices are given by $\widetilde{q}_i$ and $\widetilde{p}_i$. As before, consider a graphical representation using the partial equilibrium demand curves D_i. The imposition of a tax on x_1 decreases supply as the blue (–) lines indicates. Because of the tax, demand for x_2 changes to the green lines (-..-). The change in q_2 results in a shift in demand for x_1, changing the incidence of the tax to the red line (-.-). Terms (A), (B), and (C) are the shifts defined in (7a) and (8). Then consider the equilibrium when using a general equilibrium demand curve $\widetilde{D}_i$. This GE-demand curve is shown in bold purple and is steeper than the partial equilibrium demand curve. A tax in market one shifts up supply in that market, but shifts out the GE-demand curve in market 2 to the dotted purple line (.....). The price increase in market two dampens the effect on quantity in market one.

Figure 3: Tax Incidence with Taxes on Two Related Commodities

(a) An Example of Overshifting (Substitutes)

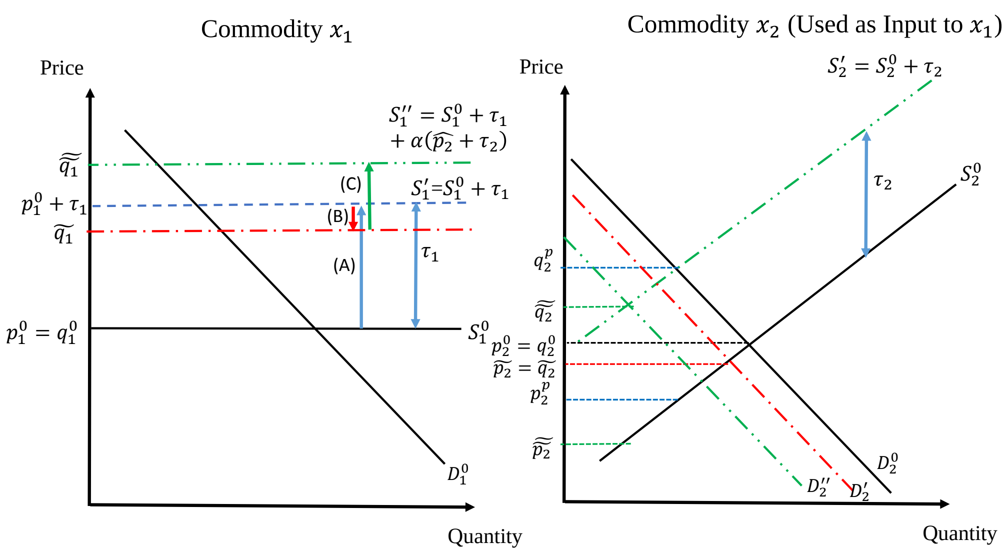




(b) An Example of Undershifting (Complements)

These figure shows the effects of a tax increases in two related markets starting from $\tau_1 = \tau_2 = 0$. In panel b, we assume a case where undershifting is most likely to arise where supply in the second market is perfectly elastic. In panel a, we focus on consumer prices; in panel b, we focus on producer prices. The initial equilibrium price is given by q_i^0. The partial equilibrium prices after the tax increase are q_i^p for the consumer and p_i^p for the producer. The prices accounting only for the feedback from one market are given by $\widetilde{q_i}$ and $\widetilde{p_i}$, and the general equilibrium prices are given by $\widetilde{\widetilde{q_i}}$ and $\widetilde{\widetilde{p_i}}$. The standard partial equilibrium impact of a tax is indicated by the blue lines (–). Because of the taxes on related products, demand for the other commodity changes to the red lines (-.-). The tax in the other market causes an additional feedback denoted by the green lines (-..-). Terms (A), (B), and (C) are the shifts defined in (7a) and (8).

Figure 4: Tax Incidence with Cascading



This figure shows the effects of tax increases starting from $\tau_1 = \tau_2 = 0$, in the markets for commodities x_1 and x_2 where x_2 is also used as an input to produce x_1 and where α is the number of units of x_2 used in production of a unit of x_1. The two commodities have zero cross-price elasticities. The partial equilibrium prices are given by q_i^p for the consumers and p_i^p for the producers. Prices resulting from the reduction of demand for x_2 as an input are given by $\widetilde{q}_i$ and $\widetilde{p}_i$. Full general equilibrium prices accounting for taxes in both markets are given by $\widetilde{\widetilde{q}}_i$ and $\widetilde{\widetilde{p}}_i$. Terms (A), (B), and (C) are the shifts defined in (18a) and (18b). The effective tax change in market 1, in general equilibrium, is given by $\tau_1 + \alpha(\tau_2 + \widehat{p}_2)$. For simplicity, we assume market 1 has a perfectly elastic supply.

Table 1: Simulations of Producer Price Changes When One Tax Changes ($\hat{\tau}_1 = 1$)

Own-Price Elasticities				Cross-Price Elasticities			PE	Single Market GE		Multi-market GE Incidence	
$\lvert\eta_{11}\rvert$	$\lvert\eta_{22}\rvert$	μ_1	μ_2	η_{12}	η_{21}	$\frac{q_1 x_1}{q_2 x_2}$	$\hat{p}_1^{PE}$	$\hat{p}_1^{G}$	%△	$I^p_{\tau_1}$	%△
1	1	1	1	0.5	0.5	1	-0.50	-0.47	-7%	-0.33	-33%
1	1	1	1	-0.5	-0.5	1	-0.50	-0.47	-7%	-0.60	+20%
1	1	1	1	1.5	0.5	$\frac{1}{3}$	-0.50	-0.38	-23%	0.08	-115%
1	1	1	1	-1.5	-0.5	$\frac{1}{3}$	-0.50	-0.38	-23%	-0.85	+69%
1	1	1	1	0.5	1.5	3	-0.50	-0.38	-23%	-0.23	-54%
1	1	1	1	-0.5	-1.5	3	-0.50	-0.38	-23%	-0.54	+8%
1	1	10	10	0.5	0.5	1	-0.091	-0.089	-2%	-0.048	-48%
1	1	10	10	-0.5	-0.5	1	-0.091	-0.089	-2%	-0.130	+43%
1	1	10	10	1.5	0.5	$\frac{1}{3}$	-0.091	-0.085	-6%	0.040	-143%
1	1	10	10	-1.5	-0.5	$\frac{1}{3}$	-0.091	-0.085	-6%	-0.210	+131%
1	1	10	10	0.5	1.5	3	-0.091	-0.085	-6%	-0.044	-52%
1	1	10	10	-0.5	-1.5	3	-0.091	-0.085	-6%	-0.127	+40%

This table simulates the producer price incidence for the given elasticities when only the tax in one market changes. Percent changes for the general equilibrium price in the single market is given using the formulas in the text. A positive [negative] percent change means the incidence on the producer increases [decreases] in our general equilibrium formula relative to the partial equilibrium formula. When calculating percent changes, we use the precise numerical incidence value rather than the rounded values in the table. The multimarket incidence is given by the budget-share weighted average of price changes for commodity 1 and 2 and the multimarket percent change is positive [negative] if the incidence on producers across both markets increases [decreases] relative to the partial equilibrium formula. N/A means a constraint on the problem does not hold.

Table 2: Simulations of Producer Price Changes with Two Taxes Change ($\hat{\tau}_1 = \hat{\tau}_2 = 1$)

Elasticities				PE	$\eta_{12} = \eta_{21} = 0.5$		$\eta_{12} = \eta_{21} = -0.5$		$\eta_{12} = 1.5,\ \eta_{21} = 0.5$		$\eta_{12} = -1.5,\ \eta_{21} = -0.5$	
$\lvert\eta_{11}\rvert$	$\lvert\eta_{22}\rvert$	μ_1	μ_2	$\hat{p}_1^{PE}$	$\hat{p}_1^{G}$	%△	$\hat{p}_1^{G}$	%△	$\hat{p}_1^{G}$	%△	$\hat{p}_1^{G}$	%△
1	1	1	1	-0.50	-0.33	-33%	-0.60	+20%	0.08*	-115%*	-0.85	+69%
0.5	1	1	1	-0.33	-0.09	-73%	-0.45	+45%	n/a	n/a	n/a	n/a
1.5	1	1	1	-0.60	-0.47	-21%	-0.68	+14%	-0.18	-71%	-0.88	+47%
1	0.5	1	1	-0.60	-0.27	-45%	-0.64	+27%	n/a	n/a	n/a	n/a
1	1.5	1	1	-0.50	-0.37	-26%	-0.58	+16%	-0.06	-88%	-0.76	+52%
1	1	0.5	1	-0.67	-0.45	-31%	-0.82	+23%	0.11*	-117%*	-1.22**	+83%**
1	1	1.5	1	-0.40	-0.26	-34%	-0.47	+18%	0.06*	115%*	-0.65	+62%
1	1	1	0.5	-0.50	-0.36	-27%	-0.55	+9%	0	-100%	-0.67	+33%
1	1	1	1.5	-0.50	-0.32	-37%	-0.63	+26%	0.12*	-124%*	-0.94	+88%

This table simulates the producer price incidence for the given elasticities. * Indicates overshifting and ** indicated Edgeworth's Paradox. Percent changes are given using the formulas in the text. A positive [negative] percent change means the incidence on the producer increases [decreases] in our general equilibrium formula relative to the partial equilibrium formula. When calculating percent changes, we use the precise numerical incidence value rather than the rounded values in the table. N/A means a constraint on the problem does not hold.

Table 3: Simulations of Producer Price Changes When Supply Elasticities Are Large and Two Taxes Change ($\hat{\tau}_1 = \hat{\tau}_2 = 1$)

Elasticities				PE	$\eta_{12} = \eta_{21} = 0.5$		$\eta_{12} = \eta_{21} = -0.5$		$\eta_{12} = 1.5,\ \eta_{21} = 0.5$		$\eta_{12} = -1.5,\ \eta_{21} = -0.5$	
$\lvert\eta_{11}\rvert$	$\lvert\eta_{22}\rvert$	μ_1	μ_2	$\hat{p}_1^{PE}$	$\hat{p}_1^G$	%△	$\hat{p}_1^G$	%△	$\hat{p}_1^G$	%△	$\hat{p}_1^G$	%△
1	1	10	10	-0.09	-0.05	-48%	-0.13	+43%	0.03*	-143%*	-0.21	+131%
0.5	1	10	10	-0.05	-0.002	-95%	-0.09	+87%	n/a	n/a	n/a	n/a
1.5	1	10	10	-0.13	-0.09	-32%	-0.17	+29%	-0.01	-95%	-0.24	+87%
1	0.5	10	10	-0.09	-0.05	-49%	-0.13	+46%	n/a	n/a	n/a	n/a
1	1.5	10	10	-0.09	-0.05	-46%	-0.13	+42%	0.03*	-137%*	-0.20	+125%
1	1	9.5	10	-0.10	-0.05	-48%	-0.14	+43%	0.04*	-143%*	-0.22	+131%
1	1	10.5	10	-0.09	-0.05	-37%	-0.10	+14%	0.01*	113%*	-0.12	+42%
1	1	10	9.5	-0.09	-0.05	-47%	-0.13	+43%	0.03*	-143%	-0.21	+130%
1	1	10	10.5	-0.09	-0.05	-48%	-0.13	+43%	0.04*	-143%*	-0.21	+132%

This table simulates the producer price incidence for the given elasticities. * Indicates overshifting and ** indicated Edgeworth's Paradox. Percent changes are given using the formulas in the text. A positive [negative] percent change means the incidence on the producer increases [decreases] in our general equilibrium formula relative to the partial equilibrium formula. When calculating percent changes, we use the precise numerical incidence value rather than the rounded values in the table. N/A means a constraint on the problem does not hold.

Table 4: Overshifting and Undershifting with Leontieff-Cobb Douglas Utility Function

A. Overshifting with $U = (min\,[16x_1, x_3])^{.95}\, x_2^{.05}$

μ_2	1			2			10		
μ_1	.25	.5	1	.25	.5	1	.25	.5	1
Partial Equilibrium $\hat{p}$	-0.012	-.006	-0.003	-0.012	-0.006	-0.003	-0.012	-0.006	-0.003
Feedback Effect (term (b) (7a))	0.006	0.003	0.001	0.004	0.002	0.001	0.001	0.001	0.000
Impact of τ_2 (term (c) (7a))	0.102	0.051	0.026	0.135	0.068	0.034	0.181	0.091	0.046
General Equilibrium $\hat{p}$	0.096	0.048	0.024	0.127	0.064	0.032	0.170	0.085	0.043

B. Undershifting with $U = (min\,[x_1, x_2])^{.1}\, x_3^{.9}$

μ_2	1			2			10		
μ_1	**.25**	.5	1	**.25**	.5	1	**.25**	.5	1
Partial Equilibrium $\hat{p}$	**-0.642**	-0.473	-0.310	**-0.642**	-0.473	-0.310	**-0.642**	-0.473	-0.310
Feedback Effect (term (b) (7a))	**0.089**	0.091	0.074	**0.048**	0.050	0.042	**0.010**	0.011	0.009
Impact of τ_2 (term (c) (7a))	**-0.554**	-0.383	-0.237	**-0.595**	-0.424	-0.269	**-0.633**	-0.463	-0.301
General Equilibrium $\hat{p}$	**-1.108**	-0.766	-0.474	**-1.190**	-0.847	-0.537	**-1.265**	-0.925	-0.602

This table presents simulations for the specific utility functions above.

Table 5: Comovement of Alcohol Excise Tax Rates

	# Changes	Comovement with Beer	Comovement with Wine	Comovement with Spirit	No Co-movement	All Co-movement
		same direction / opposite direction				
Beer Tax	80	-	47 / 0	41 / 0	29	
Wine Tax	63	47 / 0	-	43 / 0	10	37
Spirit Tax	50	41 / 0	43 / 0	-	3	

This table shows the number of tax changes on beer, wine and spirit. Then, we list the number of simultaneous tax changes, noting if they move in the same or opposite directions. The second to last column notes the number of tax changes where none of the other taxes change. The final column lists the number of tax changes where all three taxes comove in the same direction.

Table 6: Effect of Excise Taxes on Beer and Wine Prices

	(1)	(2)	(3)	(4)	(1')	(2')	(3')	(4')
	Beer Prices		Wine Prices		Beer Prices		Wine Prices	
Beer Tax	0.968	0.340		0.915	0.973	-0.212		0.252
	(0.737)	(0.417)		(1.280)	(0.747)	(0.261)		(1.454)
Wine Tax		0.263*	1.154*	1.090**		0.484**	2.076**	1.336**
		(0.156)	(0.630)	(0.532)		(0.192)	(0.852)	(0.630)
Spirit Tax		0.076***		-0.0657		0.146***		0.286***
		(0.023)		(0.0745)		(0.047)		(0.092)
Controls	Y	Y	Y	Y	Y	Y	Y	Y
State FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Observations	1,859	1,859	1,859	1,859	1,859	1,859	1,859	1,859
States	51	51	51	51	51	51	51	51

This table presents the results from equations (26) and (27) for beer and wine prices. In columns without a prime, the price is the average (unweighted) price in the state. In columns with a prime, the price is the average (weighted by population across all cities) price in the state. Columns (1)-(2) are for beer and columns (3)-(4) are for wine. All regressions include time fixed effects, state fixed effects, and a full vector of controls including the state sales tax rate on alcohol. Taxes are in dollars for but in this table, we convert the tax to the same volume unit as the dependent variable; thus, for beer prices, we use the tax on a six pack of beer and convert the liquor and wine tax to be for the same amount of alcohol volume as a six pack. These units corresponds to the prices. Standard errors are clustered at the state level. *** 99%, ** 95%, * 90%.

A Appendix (online only)

A.1 Impacts of Taxes on Income

Define the income of consumers of commodity i to be

$$y_i = wL_i + \sum_{j=1}^{3} \phi_j^i \left(p_j x_j - c_j(x_j)\right) + \nu^i \sum_{j=1}^{3} \tau_j x_j \tag{A.1}$$

where L_i is the labor supply of consumers of commodity i, $\phi_j^i \in [0,1]$ and $\nu^i \in [0,1]$. Then totally differentiating (A.1) yields

$$\begin{array}{c} dy_i = \sum_{j=1}^{3} \phi_j^i \left(p_j - c_j^{'}(x_j)\right) \left(\frac{\partial x_j}{\partial q_1}\left(dp_1 + d\tau_1\right) + \frac{\partial x_j}{\partial q_2}\left(dp_2 + d\tau_2\right) + \frac{\partial x_j}{\partial q_3} dp_3\right) \\ +\nu^i \sum_{j=1}^{2} \tau_j \left(\frac{\partial x_j}{\partial q_1}\left(dp_1 + d\tau_1\right) + \frac{\partial x_j}{\partial q_2}\left(dp_2 + d\tau_2\right) + \frac{\partial x_j}{\partial q_3} dp_3\right) \\ + \sum_{j=1}^{3} \phi_j^i x_j dp_j + \nu^i \sum_{j=1}^{2} x_j d\tau_j. \end{array} \tag{A.2}$$

where we assume $d\tau_3 = 0$. Using the fact that profit maximization requires that $p_j - c_j^{'}(x_j) = 0,\ j = 1, 2, 3$ we can simplify (A.2) to

$$dy_i = \quad \nu^i \sum_{j=1}^{2} \tau_j x_j \left(\varepsilon_{j1}\left(\hat{p}_1 + \hat{\tau}_1\right) + \varepsilon_{j2}\left(\hat{p}_2 + \hat{\tau}_2\right) + \varepsilon_{j3}\hat{p}_3\right) + \sum_{j=1}^{3} \phi_j^i q_j x_j \hat{p}_j + \nu^i \sum_{j=1}^{2} q_j x_j \hat{\tau}_j \tag{A.3}$$

where $\hat{p}_j = \frac{dp_j}{q_j}$, $\hat{\tau}_j = \frac{d\tau_j}{q_j}$, and $\varepsilon_{ij} = \frac{\partial x_i}{\partial q_j}\frac{q_j}{x_i}$. Finally, we can express (A.3) as

$$\hat{y}_i = \frac{dy_i}{y_i} = \quad \nu^i \sum_{j=1}^{3} \widetilde{\tau}_j B_j^i \left(\varepsilon_{j1}\left(\hat{p}_1 + \hat{\tau}_1\right) + \varepsilon_{j2}\left(\hat{p}_2 + \hat{\tau}_2\right) + \varepsilon_{j3}\hat{p}_3\right) + \sum_{j=1}^{3} \phi_j^i B_j^i \hat{p}_j + \nu^i \sum_{j=1}^{2} q_j B_j^i \hat{\tau}_j \tag{A.4}$$

where $B_j^i = \frac{q_j x_j}{y_i}$ and $\widetilde{\tau}_j = \frac{\tau_j}{q_j}$.

In the one consumer/producer case with $\nu^i = \phi_j^i = 1$ we have

$$dy = \sum_{j=1}^{2} q_j x_j \left(\hat{p}_j + \hat{\tau}_j\right) + p_3 x_3 \hat{p}_3 + \sum_{j=1}^{2} \tau_j x_j \left(\varepsilon_{j1}\left(\hat{p}_1 + \hat{\tau}_1\right) + \varepsilon_{j2}\left(\hat{p}_2 + \hat{\tau}_2\right) + \varepsilon_{j3}\hat{p}_3\right) \tag{A.5}$$

and

$$\hat{y} = \frac{dy}{y} = \sum_{j=1}^{2} B_j \left(\hat{p}_j + \hat{\tau}_j\right) + B_3 \hat{p}_3 + \sum_{j=1}^{3} \widetilde{\tau}_j B_j \left(\varepsilon_{j1}\left(\hat{p}_1 + \hat{\tau}_1\right) + \varepsilon_{j2}\left(\hat{p}_2 + \hat{\tau}_2\right) + \varepsilon_{j3}\hat{p}_3\right) \tag{A.6}$$

where $B_j = \frac{q_j x_j}{y}$. Impacts on Demand with Income Effects

Totally differentiating the demand for commodity i gives

$$\varepsilon_{i1}\left(\hat{p}_1+\hat{\tau}_1\right)+\varepsilon_{i2}\left(\hat{p}_2+\hat{\tau}_2\right)+\varepsilon_{i3}\hat{p}_3+\delta_i\hat{y}_i-\mu_i\hat{p}_i=0,\ i=1,2,3. \tag{A.7}$$

Then using the Slutsky decomposition, $\varepsilon_{ij}=\eta_{ij}-B_j\delta_i$ and substituting $\hat{y}$ for $\hat{y}_i$ using (A.6) gives

$$\begin{gathered}\eta_{i1}\left(\hat{p}_1+\hat{\tau}_1\right)+\eta_{i2}\left(\hat{p}_2+\hat{\tau}_2\right)+\eta_{i3}\hat{p}_3-\mu_i\hat{p}_i\\ +\delta_i\textstyle\sum_{j=1}^{3}\widetilde{\tau}_jB_j\left(\varepsilon_{j1}\left(\hat{p}_1+\hat{\tau}_1\right)+\varepsilon_{j2}\left(\hat{p}_2+\hat{\tau}_2\right)+\varepsilon_{j3}\hat{p}_3\right)=0,\ i=1,2,3.\end{gathered} \tag{A.8}$$

or

$$\begin{gathered}\left(\eta_{i1}+\delta_i\sum_{k=1}^{3}\widetilde{\tau}_kB_k\varepsilon_{k1}\right)\left(\hat{p}_1+\hat{\tau}_1\right)+\left(\eta_{i2}+\delta_i\sum_{k=1}^{3}\widetilde{\tau}_kB_k\varepsilon_{k2}\right)\left(\hat{p}_2+\hat{\tau}_2\right)\\ +\left(\eta_{i3}+\delta_i\sum_{k=1}^{3}\widetilde{\tau}_kB_k\varepsilon_{k3}\right)\hat{p}_3-\mu_i\hat{p}_i=0,\ i=1,2,3.\end{gathered} \tag{A.9}$$

Then letting

$$\widetilde{\eta}_{ij}\equiv\eta_{ij}+\delta_i\sum_{k=1}^{3}\widetilde{\tau}_kB_k\varepsilon_{kj} \tag{A.10}$$

where $\hat{\tau}_3=0$, we can express (A.9) as

$$\widetilde{\eta}_{i1}\left(\hat{p}_1+\hat{\tau}_1\right)+\widetilde{\eta}_{i2}\left(\hat{p}_2+\hat{\tau}_2\right)+\widetilde{\eta}_{i3}\hat{p}_3-\mu_i\hat{p}_i=0,\ i=1,2,3. \tag{A.11}$$

Finally, when we have x_1 as a small share of the budget and the only taxed commodity ($B_1\approx 0;\ \widehat{\tau}_2=0$) or when both x_1 and x_2 have changes in taxes and both are small shares of the budget ($B_1\approx 0; B_2\approx 0$) the term $\widetilde{\eta}_{ij}$ simplifies to η_{ij} and (A.11) becomes[32]

$$\eta_{i1}\left(\hat{p}_1+\hat{\tau}_1\right)+\eta_{i2}\left(\hat{p}_2+\hat{\tau}_2\right)+\eta_{i3}\hat{p}_3-\mu_i\hat{p}_i=0,\ i=1,2,3. \tag{A.12}$$

A.2 Tax Incidence with 1 Small, Taxed Market

As discussed in Section 2.1 we substitute η_{i1} for $\widetilde{\eta}_{i1}$ and then make the substitution $\frac{q_1x_1}{q_jx_j}\eta_{1j}$ for η_{j1}, j = *2, 3* to obtain

[32] If, for example, x_2 is taxed and has a large market share then $\widetilde{\eta}_{ij}$ will not simplify to η_{ij}. However, as $\widehat{p}_2\approx 0$, $\widehat{p}_3\approx 0$, $\varepsilon_{21}\approx 0$, and $\varepsilon_{31}\approx 0$ when $B_1\approx 0$ and $B_2>0$ and $B_3>0$, using (A.12) to solve for the impacts of changes in τ_1 on equilibria prices still applies.

$$\underbrace{\begin{bmatrix} \eta_{11}-\mu_1 & \widetilde{\eta}_{12} & \widetilde{\eta}_{13} \\ \frac{q_1x_1}{q_2x_2}\eta_{12} & \widetilde{\eta}_{22}-\mu_2 & \widetilde{\eta}_{23} \\ \frac{q_1x_1}{q_3x_3}\eta_{13} & \widetilde{\eta}_{32} & \widetilde{\eta}_{33}-\mu_3 \end{bmatrix}}_{H} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} = \begin{bmatrix} -\eta_{11}\hat{\tau}_1 \\ -\frac{q_1x_1}{q_2x_2}\eta_{12}\hat{\tau}_1 \\ -\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1 \end{bmatrix} \tag{A.13}$$

Solving gives

$$\hat{p}_1 = \begin{bmatrix} -\eta_{11}\hat{\tau}_1\left[\left(\widetilde{\eta}_{22}-\mu_2\right)\left(\widetilde{\eta}_{33}-\mu_3\right)-\widetilde{\eta}_{23}\widetilde{\eta}_{32}\right] \\ +\frac{q_1x_1}{q_2x_2}\eta_{12}\hat{\tau}_1\left[\widetilde{\eta}_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\widetilde{\eta}_{31}\widetilde{\eta}_{32}\right] \\ -\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1\left[\widetilde{\eta}_{12}\widetilde{\eta}_{23}-\widetilde{\eta}_{13}\left(\widetilde{\eta}_{22}-\mu_2\right)\right] \end{bmatrix}|H|^{-1} \tag{A.14a}$$

$$\hat{p}_2 = \begin{bmatrix} \eta_{11}\hat{\tau}_1\left[\frac{q_1x_1}{q_2x_2}\eta_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\eta_{13}\widetilde{\eta}_{23}\right] \\ -\frac{q_1x_1}{q_2x_2}\eta_{12}\hat{\tau}_1\left[\left(\eta_{11}-\mu_1\right)\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\widetilde{\eta}_{31}^2\right] \\ +\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1\left[\left(\eta_{11}-\mu_1\right)\widetilde{\eta}_{23}-\frac{q_1x_1}{q_2x_2}\eta_{12}\widetilde{\eta}_{13}\right] \end{bmatrix}|H|^{-1}, \tag{A.14b}$$

$$\hat{p}_3 = \begin{bmatrix} -\eta_{11}\hat{\tau}_1\left[\frac{q_1x_1}{q_2x_2}\eta_{12}\widetilde{\eta}_{32}-\frac{q_1x_1}{q_3x_3}\eta_{13}\left(\widetilde{\eta}_{22}-\mu_2\right)\right] \\ +\frac{q_1x_1}{q_2x_2}\eta_{12}\hat{\tau}_1\left[\left(\eta_{11}-\mu_1\right)\widetilde{\eta}_{32}-\frac{q_1x_1}{q_3x_3}\widetilde{\eta}_{31}\widetilde{\eta}_{12}\right] \\ -\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1\left[\left(\eta_{11}-\mu_1\right)\left(\widetilde{\eta}_{22}-\mu_2\right)-\frac{q_1x_1}{q_2x_2}\eta_{12}^2\right] \end{bmatrix}|H|^{-1}, \tag{A.14c}$$

and

$$|H| = \begin{array}{c} \left(\eta_{11}-\mu_1\right)\left[\left(\widetilde{\eta}_{22}-\mu_2\right)\left(\widetilde{\eta}_{33}-\mu_3\right)-\widetilde{\eta}_{23}\widetilde{\eta}_{32}\right] \\ -\frac{q_1x_1}{q_2x_2}\eta_{12}\left[\widetilde{\eta}_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\widetilde{\eta}_{31}\widetilde{\eta}_{32}\right] \\ +\frac{q_1x_1}{q_3x_3}\eta_{13}\left[\widetilde{\eta}_{12}\widetilde{\eta}_{23}-\widetilde{\eta}_{13}\left(\widetilde{\eta}_{22}-\mu_2\right)\right], \end{array} \tag{A.15}$$

which simplify to (4a) and (4b) when applying $\frac{q_1x_1}{q_2x_2}=\frac{q_1x_1}{q_3x_3}=0$.

A.3 Tax Incidence with 2 Small, Taxed Markets $\left(\frac{q_1x_1}{q_3x_3}\approx 0,\ \frac{q_2x_2}{q_3x_3}\approx 0\right)$

In the case of two taxed, small markets $\left(\frac{q_1x_1}{q_3x_3}\approx 0,\ \frac{q_2x_2}{q_3x_3}\approx 0\right)$, letting $\eta_{31}=\frac{q_1x_1}{q_3x_3}\eta_{13}$ and $\eta_{32}=\frac{q_2x_2}{q_3x_3}\eta_{23}$ we have

$$\underbrace{\begin{bmatrix} \eta_{11}-\mu_1 & \eta_{12} & \widetilde{\eta}_{13} \\ \eta_{21} & \eta_{22}-\mu_2 & \widetilde{\eta}_{23} \\ \frac{q_1x_1}{q_3x_3}\eta_{13} & \frac{q_2x_2}{q_3x_3}\eta_{23} & \widetilde{\eta}_{33}-\mu_3 \end{bmatrix}}_{H} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} = \begin{bmatrix} -\left(\eta_{11}\hat{\tau}_1+\eta_{12}\hat{\tau}_2\right) \\ -\left(\eta_{21}\hat{\tau}_1+\eta_{22}\hat{\tau}_2\right) \\ -\left(\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1+\frac{q_2x_2}{q_3x_3}\eta_{23}\hat{\tau}_2\right) \end{bmatrix} \tag{A.16}$$

Then solving (A.16) for the prices yields

$$\hat{p}_1 = \begin{bmatrix} -\left(\eta_{11}\hat{\tau}_1+\eta_{12}\hat{\tau}_2\right)\left[\left(\widetilde{\eta}_{33}-\mu_3\right)\left(\eta_{22}-\mu_2\right)-\frac{q_2x_2}{q_3x_3}\eta_{23}\widetilde{\eta}_{23}\right] \\ +\left(\eta_{21}\hat{\tau}_1+\eta_{22}\hat{\tau}_2\right)\left[\eta_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_2x_2}{q_3x_3}\eta_{23}\widetilde{\eta}_{13}\right] \\ -\left(\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1+\frac{q_2x_2}{q_3x_3}\eta_{23}\hat{\tau}_2\right)\left[\eta_{12}\widetilde{\eta}_{23}-\left(\eta_{22}-\mu_2\right)\widetilde{\eta}_{13}\right] \end{bmatrix}|H|^{-1}\ , \tag{A.17a}$$

$$\hat{p}_2 = \begin{bmatrix} \left(\eta_{11}\hat{\tau}_1+\eta_{12}\hat{\tau}_2\right)\left[\eta_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\eta_{13}\widetilde{\eta}_{23}\right] \\ -\left(\eta_{21}\hat{\tau}_1+\eta_{22}\hat{\tau}_2\right)\left[\left(\eta_{11}-\mu_1\right)\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_1x_1}{q_3x_3}\eta_{13}\widetilde{\eta}_{13}\right] \\ +\left(\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1+\frac{q_2x_2}{q_3x_3}\eta_{23}\hat{\tau}_2\right)\left[\left(\eta_{11}-\mu_1\right)\widetilde{\eta}_{23}-\eta_{12}\widetilde{\eta}_{13}\right] \end{bmatrix}|H|^{-1}\ , \tag{A.17b}$$

$$\hat{p}_3 = \begin{bmatrix} -\left(\eta_{11}\hat{\tau}_1+\eta_{12}\hat{\tau}_2\right)\left[\eta_{12}\frac{q_2x_2}{q_3x_3}\eta_{23}-\frac{q_1x_1}{q_3x_3}\eta_{13}\left(\eta_{22}-\mu_2\right)\right] \\ +\left(\eta_{21}\hat{\tau}_1+\eta_{22}\hat{\tau}_2\right)\left[\left(\eta_{11}-\mu_1\right)\frac{q_2x_2}{q_3x_3}\eta_{23}-\frac{q_1x_1}{q_3x_3}\eta_{13}\eta_{12}\right] \\ -\left(\frac{q_1x_1}{q_3x_3}\eta_{13}\hat{\tau}_1+\frac{q_2x_2}{q_3x_3}\eta_{23}\hat{\tau}_2\right)\left[\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{12}\eta_{21}\right] \end{bmatrix}|H|^{-1}\ , \tag{A.17c}$$

and

$$|H| = \begin{bmatrix} \left(\eta_{11}-\mu_1\right)\left[\left(\widetilde{\eta}_{33}-\mu_3\right)\left(\eta_{22}-\mu_2\right)-\frac{q_2x_2}{q_3x_3}\eta_{23}\widetilde{\eta}_{23}\right] \\ -\eta_{21}\left[\eta_{12}\left(\widetilde{\eta}_{33}-\mu_3\right)-\frac{q_2x_2}{q_3x_3}\eta_{23}\widetilde{\eta}_{13}\right] \\ +\frac{q_1x_1}{q_3x_3}\eta_{13}\left[\eta_{12}\widetilde{\eta}_{23}-\left(\eta_{22}-\mu_2\right)\widetilde{\eta}_{13}\right] \end{bmatrix}\ , \tag{A.18}$$

Then letting $\frac{q_1x_1}{q_3x_3}=\frac{q_2x_2}{q_3x_3}=0$, from (A.18), we obtain

$$|H| = \left(\widetilde{\eta}_{33}-\mu_3\right)\left[\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}\right]\ . \tag{A.19}$$

Using (A.19) in (A.17a) with $\frac{q_1x_1}{q_3x_3}=\frac{q_2x_2}{q_3x_3}=0$ gives

$$\hat{p}_1 = \frac{-\eta_{11}(\eta_{22}-\mu_2)+\eta_{21}\eta_{12}}{(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}}\hat{\tau}_1 + \frac{\eta_{12}\mu_2}{(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}}\hat{\tau}_2 \tag{A.20}$$

Simplifying (A.17b) gives an analogous expression for $\hat{p}_2$ and simplifying (A.17c) gives $\hat{p}_3 = 0$. Then an alternative expression for $\hat{p}_1$ can be obtained by letting,

$$\left[\frac{-\eta_{11}}{(\eta_{11}-\mu_1)} + C\right] = \frac{-\eta_{11}(\eta_{22}-\mu_2)+\eta_{21}\eta_{12}}{(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}} \tag{A.21}$$

and solving for C gives:

$$\begin{aligned} C &= \frac{-\eta_{11}(\eta_{22}-\mu_2)+\eta_{12}\eta_{21}}{(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}} + \frac{\eta_{11}}{(\eta_{11}-\mu_1)} \\ &= \frac{(-\eta_{11}(\eta_{22}-\mu_2)+\eta_{12}\eta_{21})(\eta_{11}-\mu_1)+\eta_{11}[(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}]}{[(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}]\ (\eta_{11}-\mu_1)} \\ &= \frac{-\mu_1}{(\eta_{11}-\mu_1)}\frac{\eta_{12}\eta_{21}}{[(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}]} \end{aligned} \tag{A.22}$$

where $\frac{\mu_1}{(\mu_1-\eta_{11})} = 1+\rho_1$ and $\rho_1 = \frac{\eta_{11}}{\mu_1-\eta_{11}}$. Then we obtain $\hat{p}_1$ and others analogously:

$$\hat{p}_1 = \left[\rho_1 + (1+\rho_1)\frac{\eta_{21}\eta_{12}}{\left|\widetilde{H}\right|}\right]\hat{\tau}_1 + \frac{\eta_{12}\mu_2}{\left|\widetilde{H}\right|}\hat{\tau}_2 \tag{A.23a}$$

$$\hat{p}_2 = \left[\rho_2 + (1+\rho_2)\frac{\eta_{21}\eta_{12}}{\left|\widetilde{H}\right|}\right]\hat{\tau}_2 + \frac{\eta_{21}\mu_1}{\left|\widetilde{H}\right|}\hat{\tau}_1 \tag{A.23b}$$

$$\hat{p}_3 = 0 \tag{A.23c}$$

where $\left|\widetilde{H}\right| = (\eta_{22}-\mu_2)(\eta_{11}-\mu_1)-\eta_{12}\eta_{21}$.

A.3.1 Impacts of Taxes when the supply of x_3 is elastic

In this case, we assume a perfectly elastic supply of x_3, a frequent alternative assumption to having x_1 and x_2 be small markets. Then the cost function for x_3 can be expressed as $c_3(x_3) = \overline{c}_3 x_3$. Then profits in the production of x_3 are given by $\pi_3 = (p_3 - \overline{c}_3)\, x_3 = 0$ as perfect competition requires $p_3 = \overline{c}_3$, price equals marginal cost. This being the case, the income of a single consumer is

$$y = wL + \sum_{j=1}^{2} (p_j x_j - c_j(x_j)) + \sum_{j=1}^{2} \tau_j x_j \tag{A.24}$$

where L_i is the labor supply of consumers of commodity i. Then totally differentiating (A.24) yields

$$dy = \textstyle\sum_{j=1}^{2} \tau_j \left(\frac{\partial x_j}{\partial q_1} (dp_1 + d\tau_1) + \frac{\partial x_j}{\partial q_2} (dp_2 + d\tau_2) \right) + \sum_{j=1}^{2} p_j x_j \hat{p}_j + \sum_{j=1}^{2} x_j d\tau_j. \tag{A.25}$$

Using the fact that profit maximization requires that $p_j - c_j^{'}(x_j) = 0,\ j = 1, 2$ we can simplify (A.25) to

$$dy = \sum_{j=1}^{2} q_j x_j (\hat{p}_j + \hat{\tau}_j) + \sum_{j=1}^{2} \tau_j x_j (\varepsilon_{j1} (\hat{p}_1 + \hat{\tau}_1) + \varepsilon_{j2} (\hat{p}_2 + \hat{\tau}_2)) \tag{A.26}$$

and

$$\hat{y} = \frac{dy}{y} = \sum_{j=1}^{2} B_j (\hat{p}_j + \hat{\tau}_j) + \sum_{j=1}^{3} \widetilde{\tau}_j B_j (\varepsilon_{j1} (\hat{p}_1 + \hat{\tau}_1) + \varepsilon_{j2} (\hat{p}_2 + \hat{\tau}_2)) . \tag{A.27}$$

Totally differentiating the demand for commodity i gives

$$\varepsilon_{i1} (\hat{p}_1 + \hat{\tau}_1) + \varepsilon_{i2} (\hat{p}_2 + \hat{\tau}_2) + \delta_i \hat{y} - \mu_i \hat{p}_i = 0,\ i = 1, 2. \tag{A.28}$$

Then using the Slutsky decomposition, $\varepsilon_{ij} = \eta_{ij} - B_j \delta_i$ and substituting $\hat{y}$ using (A.27)

$$\begin{gathered} (\eta_{i1} - B_1 \delta_i) (\hat{p}_1 + \hat{\tau}_1) + (\eta_{i2} - B_2 \delta_i) (\hat{p}_2 + \hat{\tau}_2) + \\ \delta_i \left[\textstyle\sum_{j=1}^{2} B_j (\hat{p}_j + \hat{\tau}_j) + \sum_{j=1}^{2} \widetilde{\tau}_j B_j (\varepsilon_{j1} (\hat{p}_1 + \hat{\tau}_1) + \varepsilon_{j2} (\hat{p}_2 + \hat{\tau}_2)) \right] - \mu_i \hat{p}_i = 0,\ i = 1, 2. \end{gathered} \tag{A.29}$$

gives

$$\begin{gathered} \eta_{i1} (\hat{p}_1 + \hat{\tau}_1) + \eta_{i2} (\hat{p}_2 + \hat{\tau}_2) - \mu_i \hat{p}_i \\ + \delta_i \textstyle\sum_{j=1}^{3} \widetilde{\tau}_j B_j (\varepsilon_{j1} (\hat{p}_1 + \hat{\tau}_1) + \varepsilon_{j2} (\hat{p}_2 + \hat{\tau}_2)) = 0,\ i = 1, 2. \end{gathered} \tag{A.30}$$

Then letting

$$\widetilde{\eta}_{ij} \equiv \eta_{ij} + \delta_i \sum_{k=1}^{3} \widetilde{\tau}_k B_k \varepsilon_{kj} \tag{A.31}$$

we have

$$\widetilde{\eta}_{i1} (\hat{p}_1 + \hat{\tau}_1) + \widetilde{\eta}_{i2} (\hat{p}_2 + \hat{\tau}_2) - \mu_i \hat{p}_i = 0,\ i = 1, 2. \tag{A.32}$$

A.4 Derivation of $\hat{p}_1 > 0$ and $\hat{p}_1 < -1$

A.4.1 Overshifting

Letting $\hat{\tau}_2 = 0$ in (A.20) gives

$$\hat{p}_1 = \frac{-\eta_{11}\left(\eta_{22} - \mu_2\right) + \eta_{21}\eta_{12}}{\left(\eta_{22} - \mu_2\right)\left(\eta_{11} - \mu_1\right) - \eta_{12}\eta_{21}}\hat{\tau}_1 \tag{A.33}$$

Then, if $\hat{p}_1 > 0$ it must be the case that which implies that

$$\mu_1\left(\mu_2 - \eta_{22}\right) > \left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{12}\eta_{21} \tag{A.34}$$

or, simplifying

$$\begin{gathered} \eta_{11}\mu_2 - \left[\eta_{11}\eta_{22} - \eta_{12}\eta_{21}\right] > 0 \\ \text{s.t. } \left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12} > 0. \end{gathered} \tag{A.35}$$

which cannot be satisfied as $\eta_{11}\mu_2 < 0$ and $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$ by the second order condition for expenditure minimization.
Letting $\hat{\tau}_2 = \hat{\tau}_1$ in (A.20) gives

$$\hat{p}_1 = \left[\frac{-\eta_{11}\left(\eta_{22} - \mu_2\right) + \eta_{21}\eta_{12} + \eta_{12}\mu_2}{\left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12}}\right]\hat{\tau}_1. \tag{A.36}$$

Then $\hat{p}_1 > 0$ implies that

$$\frac{-\left[\left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12}\right] - \mu_1\left(\eta_{22} - \mu_2\right) + \eta_{12}\mu_2}{\left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12}} > 0 \tag{A.37}$$

$$-1 + \frac{-\mu_1\left(\eta_{22} - \mu_2\right) + \eta_{12}\mu_2}{\left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12}} > 0 \tag{A.38}$$

which can be expressed as

$$\mu_1\left(\mu_2 - \eta_{22}\right) + \eta_{12}\mu_2 > \left(\mu_2 - \eta_{22}\right)\left(\mu_1 - \eta_{11}\right) - \eta_{12}\eta_{21} \tag{A.39}$$

$$\eta_{11}\left(\mu_2 - \eta_{22}\right) + \eta_{12}\mu_2 + \eta_{12}\eta_{21} > 0 \tag{A.40}$$

or, equivalently,

$$\begin{gathered} \left(\eta_{11} + \eta_{12}\right)\mu_2 - \left(\eta_{11}\eta_{22} - \eta_{12}\eta_{21}\right) > 0 \\ s.t.\ \left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \eta_{21}\eta_{12} > 0 \end{gathered} \tag{A.41}$$

which can be satisfied for some values of η_{11}, η_{12}, η_{11}, η_{22}, μ_1, and μ_2. If $\eta_{11} = 0$ then this becomes

$$\eta_{12}\left(\mu_2+\eta_{21}\right)>0$$
$$\text{s.t. } \left(\mu_2-\eta_{22}\right)\mu_1-\eta_{12}\eta_{21}>0. \tag{A.42}$$

A.4.2 The Edgeworth Paradox ($\hat{p}_1 < -1$)

Letting $\hat{\tau}_2 = 0$ in (A.20) gives

$$\hat{p}_1 = \frac{-\eta_{11}\left(\eta_{22}-\mu_2\right)+\eta_{21}\eta_{12}}{\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}}\hat{\tau}_1 \tag{A.43}$$

Then if $\hat{p}_1 < -\hat{\tau}_1$ it must be the case that

$$\frac{-\left[\left(\mu_1-\eta_{11}\right)\left(\mu_2-\eta_{22}\right)-\eta_{21}\eta_{12}\right]+\mu_1\left(\mu_2-\eta_{22}\right)}{\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}} = \frac{\mu_1\left(\mu_2-\eta_{22}\right)}{\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}}-1<-1 \tag{A.44}$$

which implies that

$$\mu_1\left(\mu_2-\eta_{22}\right)<0 \tag{A.45}$$

which cannot be satisfied. Now consider two equal tax changes. Letting $\hat{\tau}_2 = \hat{\tau}_1$ in (A.20) gives

$$\hat{p}_1 = \left[\frac{-\eta_{11}\left(\eta_{22}-\mu_2\right)+\eta_{21}\eta_{12}+\eta_{12}\mu_2}{\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}}\right]\hat{\tau}_1. \tag{A.46}$$

Then we can express this as

$$\begin{aligned} \hat{p}_1 &= \left[\frac{-\left[\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}\right]-\mu_1\left(\eta_{22}-\mu_2\right)+\eta_{12}\mu_2}{\left(\eta_{11}-\mu_1\right)\left(\eta_{22}-\mu_2\right)-\eta_{21}\eta_{12}}\right]\hat{\tau}_1 \\ &= \left[\frac{-\mu_1\left(\eta_{22}-\mu_2\right)+\eta_{12}\mu_2}{\left(\mu_2-\eta_{22}\right)\left(\mu_1-\eta_1\right)-\eta_{12}\eta_{21}}-1\right]\hat{\tau}_1. \end{aligned} \tag{A.47}$$

Then for $\hat{p}_1 < -1$ it must be the case that

$$\frac{-\mu_1\left(\eta_{22}-\mu_2\right)+\eta_{12}\mu_2}{\left(\mu_2-\eta_{22}\right)\left(\mu_1-\eta_1\right)-\eta_{12}\eta_{21}}-1<-1 \tag{A.48}$$

or

$$\mu_1\left(\mu_2-\eta_{22}\right)+\eta_{12}\mu_2<0. \tag{A.49}$$

which can be satisfied for some values of η_{11}, η_{12}, η_{11}, η_{22}, μ_1, and μ_2.

A.5 Tax Incidence with 2 Small, Taxed Markets $\left(\frac{q_1 x_1}{q_3 x_3} \approx 0,\ \frac{q_2 x_2}{q_3 x_3} \approx 0\right)$ and Supply as a Function of 2 Prices

We now slightly revise our supply functions for x_1 and x_2 to be functions of both p_1 and p_2 with $x_1^s(p_1, p_2)$ and $x_2^s(p_2, p_1)$. We continue with the case of two taxed, small markets $\left(\frac{q_1 x_1}{q_3 x_3} \approx 0,\ \frac{q_2 x_2}{q_3 x_3} \approx 0\right)$, letting $\eta_{31} = \frac{q_1 x_1}{q_3 x_3}\eta_{13}$ and $\eta_{32} = \frac{q_2 x_2}{q_3 x_3}\eta_{23}$ we now have

$$\underbrace{\begin{bmatrix} \eta_{11} - \mu_{11} & \eta_{12} - \mu_{12} & \widetilde{\eta}_{13} \\ \eta_{21} - \mu_{21} & \eta_{22} - \mu_{22} & \widetilde{\eta}_{23} \\ \frac{q_1 x_1}{q_3 x_3}\eta_{13} & \frac{q_2 x_2}{q_3 x_3}\eta_{23} & \widetilde{\eta}_{33} - \mu_3 \end{bmatrix}}_{H} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} = \begin{bmatrix} -\left(\eta_{11}\hat{\tau}_1 + \eta_{12}\hat{\tau}_2\right) \\ -\left(\eta_{21}\hat{\tau}_1 + \eta_{22}\hat{\tau}_2\right) \\ -\left(\frac{q_1 x_1}{q_3 x_3}\eta_{13}\hat{\tau}_1 + \frac{q_2 x_2}{q_3 x_3}\eta_{23}\hat{\tau}_2\right) \end{bmatrix} \tag{A.50}$$

Then solving (A.50) for the prices with $\frac{q_1 x_1}{q_3 x_3} = \frac{q_2 x_2}{q_3 x_3} = 0$ gives

$$\hat{p}_1 = \frac{-\eta_{11}\left(\eta_{22} - \mu_{22}\right) + \eta_{21}\left(\eta_{12} - \mu_{12}\right)}{\left(\eta_{22} - \mu_2\right)\left(\eta_{11} - \mu_1\right) - \left(\eta_{12} - \mu_{12}\right)\left(\eta_{21} - \mu_{21}\right)}\hat{\tau}_1 + \frac{\eta_{12}\mu_{22} - \eta_{22}\mu_{12}}{\left(\eta_{22} - \mu_2\right)\left(\eta_{11} - \mu_1\right) - \left(\eta_{12} - \mu_{12}\right)\left(\eta_{21} - \mu_{21}\right)}\hat{\tau}_2 \tag{A.51}$$

where $\mu_{ij} = \frac{\partial x_i^s}{\partial p_j}\frac{p_j}{x_j}$. Then an alternative expression for $\hat{p}_1$ can be obtained by letting,

$$\left[\frac{-\eta_{11}}{\left(\eta_{11} - \mu_1\right)} + C\right] = \frac{-\eta_{11}\left(\eta_{22} - \mu_2\right) + \eta_{21}\left(\eta_{12} - \mu_{12}\right)}{\left(\eta_{22} - \mu_2\right)\left(\eta_{11} - \mu_1\right) - \left(\eta_{12} - \mu_{12}\right)\left(\eta_{21} - \mu_{21}\right)} \tag{A.52}$$

and solving for C gives:

$$\begin{aligned} C &= \frac{\left(\eta_{12} - \mu_{12}\right)\left[\eta_{11}\mu_{21} - \eta_{21}\mu_{11}\right]}{\left|\widetilde{\widetilde{H}}\right|\left(\eta_{11} - \mu_{11}\right)} \\ &= \frac{\left(\eta_{12} - \mu_{12}\right)\left[-\mu_{21}\rho_1 + \mu_{11}\left(1 + \rho_1\right)\right]}{\left|\widetilde{\widetilde{H}}\right|} \end{aligned} \tag{A.53}$$

where $\left|\widetilde{\widetilde{H}}\right| = \left(\eta_{22} - \mu_2\right)\left(\eta_{11} - \mu_1\right) - \left(\eta_{12} - \mu_{12}\right)\left(\eta_{21} - \mu_{21}\right)$, $\frac{\mu_{11}}{\left(\mu_{11} - \eta_{11}\right)} = 1 + \rho_1$ and $\rho_{11} = \frac{\eta_{11}}{\mu_1 - \eta_{11}}$. Then $\hat{p}_1$ can be expressed as:

$$\hat{p}_1 = \left[\rho_1 + \frac{\left(\eta_{12} - \mu_{12}\right)\left[-\mu_{21}\rho_1 + \mu_{11}\left(1 + \rho_1\right)\right]}{\left|\widetilde{\widetilde{H}}\right|}\right]\hat{\tau}_1 + \frac{\eta_{12}\mu_{22} - \eta_{22}\mu_{12}}{\left|\widetilde{\widetilde{H}}\right|}\hat{\tau}_2 \tag{A.54}$$

Then from (A.51) if $\hat{p}_1 > 0$ it follows that

$$-\eta_{12}\mu_{12} + [\eta_{11}\mu_{22} - (\eta_{11}\eta_{22} - \eta_{12}\eta_{21})] > 0. \tag{A.55}$$

The bracketed term in (A.55) is from (14a) the gives the sign of $\hat{p}_1$ when supplies are only a function of own price and is negative. However, the term $-\eta_{12}\mu_{12}$ is of indeterminate sign and could be positive. By the second order condition $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$ and the term $\eta_{11}\mu_{22} < 0$. While the sign of the term $-\eta_{12}\mu_{12}$ is indeterminate, if the product of the "direct effects," $(\eta_{11}\mu_{22})$ is larger in absolute value than the "indirect effects" $(-\eta_{12}\mu_{12})$ then the term $-\eta_{12}\mu_{12} + \eta_{11}\mu_{22} < 0$ and (A.55) cannot be satisfied.

We can briefly address the possibility of $\hat{p}_1 > 0$, "overshifting" when $\hat{\tau}_1 = \hat{\tau}_2$. Then for overshifting to occur it must be the case from (A.51) that

$$-\eta_{11}(\eta_{22} - \mu_{22}) + \eta_{21}(\eta_{12} - \mu_{12}) + \eta_{12}\mu_{22} - \eta_{22}\mu_{12} > 0 \tag{A.56}$$

subject to $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} > 0$. This can be expressed as

$$[(\eta_{11} + \eta_{12})\mu_{22} - \eta_{11}\eta_{22} - \eta_{12}\eta_{21}] - (\eta_{21} + \eta_{22})\mu_{12} > 0. \tag{A.57}$$

The bracketed term is the condition for overshifting in the case in which supply only depends on own-price, (14a). Then if $\eta_{11} + \eta_{12} > 0$ for the second order condition to be satisfied it must be the case that $\eta_{21} + \eta_{22} < 0$. Then the term $-(\eta_{21} + \eta_{22})\mu_{12}$ will be positive if $\mu_{12} > 0$ ensuring that conditions that give overshifting in the case with supply depending only on own-price also gives overshifting when supply depends on both prices. Alternatively, if $\eta_{11} + \eta_{12} < 0$ it is possible for $\eta_{21} + \eta_{22} > 0$ and for the second order condition to still be satisfied. In this case it is possible for (A.57) to be satisfied if $\mu_{12} < 0$.

A.6 Cascading Taxes

As in Section 2.4 totally differentiating (16a) and (16b) gives

$$\underbrace{\begin{bmatrix} \eta_{11} - \mu_1 & \alpha\eta_{11} & \widetilde{\eta}_{13} \\ \alpha\eta_{11} & \eta_{22} - \mu_2 & \widetilde{\eta}_{23} \\ 0 & 0 & \widetilde{\eta}_{33} - \mu_3 \end{bmatrix}}_{H^C} \begin{bmatrix} \hat{p}_1^n \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} = \begin{bmatrix} -\eta_{11}(\hat{\tau}_1 + \alpha\hat{\tau}_2) \\ -(\alpha\eta_{11}\hat{\tau}_1 + \eta_{22}\hat{\tau}_2) \\ 0 \end{bmatrix} \tag{A.58}$$

and solving $\hat{p}_1^n$ for gives

$$\hat{p}_1^n = \frac{-\eta_{11}(\eta_{22}-\mu_2)}{(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2}\hat{\tau}_1 - \frac{\alpha\eta_{11}(\eta_{22}-\mu_2)}{(\eta_{11}-\mu_{11})(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2}\hat{\tau}_2 + \frac{\alpha\eta_{11}(\alpha\eta_{11}\hat{\tau}_1+\eta_{22}\hat{\tau}_2)}{(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2} \tag{A.59}$$

Let

$$\frac{-\eta_{11}(\eta_{22}-\mu_2)+\alpha^2\eta_{11}^2}{(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2} = \frac{-\eta_{11}}{(\eta_{11}-\mu_{11})} + C$$

which implies

$$C = \frac{-\eta_{11}(\eta_{22}-\mu_2)+\alpha^2\eta_{11}^2}{(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2} + \frac{\eta_{11}}{(\eta_{11}-\mu_1)}.$$

or

$$\begin{aligned} C &= \frac{-\eta_{11}(\eta_{22}-\mu_2)(\eta_{11}-\mu_1)+\alpha^2\eta_{11}^2(\eta_{11}-\mu_1)}{\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right](\eta_{11}-\mu_1)} + \frac{\eta_{11}\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right]}{(\eta_{11}-\mu_1)\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right]} \\ &= -\frac{\mu_1\alpha^2\eta_{11}^2}{\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right](\eta_{11}-\mu_{11})} \\ &= \frac{\mu_1}{(\mu_1-\eta_{11})}\frac{\alpha^2\eta_{11}^2}{\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_{22})-\alpha^2\eta_{11}^2\right]} \\ &= (1+\rho_1)\frac{\alpha^2\eta_{11}^2}{\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right]} \end{aligned} \tag{A.60}$$

Then

$$\hat{p}_1^n = \rho_1\hat{\tau}_1 + (1+\rho_1)\frac{\alpha^2\eta_{11}^2}{\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right]}\hat{\tau}_1 + \frac{\alpha\eta_{11}\mu_2}{\left[(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2\right]}\hat{\tau}_2 \tag{A.61}$$

or

$$\hat{p}_1^n = \rho_1\hat{\tau}_1 + (1+\rho_1)\frac{\alpha^2\eta_{11}^2}{|H^C|}\hat{\tau}_1 + \frac{\alpha\eta_{11}\mu_2}{|H^C|}\hat{\tau}_2 \tag{A.62}$$

where $\left|H^C\right| = (\eta_{11}-\mu_1)(\eta_{22}-\mu_2) - \alpha^2\eta_{11}^2 > 0$. Then, solving for $\hat{p}_2$ gives

$$\hat{p}_2 = \frac{-\alpha\eta_{11}(\eta_{11}-\mu_1)+\alpha\eta_{11}^2}{(\eta_{11}-\mu_1)(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2}\hat{\tau}_1 + \frac{-\eta_{22}(\eta_{11}-\mu_1)+\alpha^2\eta_{11}^2}{(\eta_{11}-\mu_{11})(\eta_{22}-\mu_2)-\alpha^2\eta_{11}^2}\hat{\tau}_2 \tag{A.63}$$

which simplifies to

$$\hat{p}_2 = \frac{\alpha\eta_{11}\mu_1}{|H^C|}\hat{\tau}_1 + \frac{-\eta_{22}(\eta_{11}-\mu_1)+\alpha^2\eta_{11}^2}{|H^C|}\hat{\tau}_2 \tag{A.64}$$

Then as $\hat{p}_1 = \hat{p}_1^n + \alpha\hat{p}_2 + \alpha\hat{\tau}_2 = \hat{p}_1^n + \alpha\hat{q}_2$ we solve for $\hat{q}_2 = \hat{p}_2 + \hat{\tau}_2$ using (A.64) gives

$$\begin{aligned} \hat{q}_2 &= \frac{\alpha\eta_{11}\mu_1}{|H^C|}\hat{\tau}_1 + \left[\frac{-\eta_{22}(\eta_{11}-\mu_1)+\alpha^2\eta_{11}^2}{|H^C|} + 1\right]\hat{\tau}_2 \\ &= \frac{\alpha\eta_{11}\mu_1}{|H^C|}\hat{\tau}_1 + \left[-\eta_{22}(\eta_{11}-\mu_1) + \alpha^2\eta_{11}^2 + (\eta_{11}-\mu_1)(\eta_{22}-\mu_2) - \alpha^2\eta_{11}^2\right] \\ &= \frac{\alpha\eta_{11}\mu_1}{|H^C|}\hat{\tau}_1 + \mu_2(\mu_1-\eta_{11})\frac{\hat{\tau}_2}{|H^C|} \end{aligned} \tag{A.65}$$

Then using (A.65) in $\hat{p}_1 = \hat{p}_1^n + \alpha\hat{q}_2$ gives

$$\hat{p}_1 = \rho_1 + \frac{\mu_1}{(\mu_{11}-\eta_{11})}\frac{\alpha^2\eta_{11}^2}{|H^C|}\hat{\tau}_1 + \frac{\alpha\eta_{11}\mu_2}{|H^C|}\hat{\tau}_2 + \alpha\left[\frac{\alpha\eta_{11}\mu_1}{|H^C|}\hat{\tau}_1 + \mu_2\left(\mu_1 - \eta_{11}\right)\frac{\hat{\tau}_2}{|H^C|}\right] \tag{A.66}$$

which can be expressed as

$$\hat{p}_1 = \left\{\rho_1 + \frac{\mu_1\alpha^2\eta_{11}^2}{|H^C|}\left[1 + \frac{1}{(\mu_1-\eta_{11})}\right]\right\}\hat{\tau}_1 + \frac{\alpha\mu_1\mu_2}{|H^C|}\hat{\tau}_2 \tag{A.67}$$

When $\hat{\tau}_1 = \hat{\tau}_2$ this becomes

$$\hat{p}_1 = \left\{\rho_1\left(1 + \frac{\mu_1\alpha^2\eta_{11}}{|H^C|}\right) + \frac{\mu_1\alpha}{|H^C|}\left(\alpha\eta_{11}^2 + \mu_2\right)\right\}\hat{\tau} \tag{A.68}$$

or

$$\hat{p}_1 = \left\{\begin{array}{c} \eta_{11}\left(\left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \alpha^2\eta_{11}^2\right) + \mu_1\alpha^2\eta_{11}^2\left(\mu_1 - \eta_{11} + 1\right) \\ +\alpha\mu_1\mu_2\left(\mu_1 - \eta_{11}\right) \end{array}\right\}\frac{\hat{\tau}}{\left(\mu_1 - \eta_{11}\right)|H^C|} \tag{A.69}$$

and for "overshifting" to occur it must be the case that

$$\begin{gathered}\left\{\eta_{11}\left(\left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \alpha^2\eta_{11}^2\right) + \mu_1\alpha^2\eta_{11}^2\left(\mu_1 - \eta_{11} + 1\right) + \alpha\mu_1\mu_2\left(\mu_1 - \eta_{11}\right)\right\} > 0 \\ \text{s.t. } \left(\eta_{11} - \mu_1\right)\left(\eta_{22} - \mu_2\right) - \alpha^2\eta_{11}^2 > 0\end{gathered} \tag{A.70}$$

A.7 The n-commodity Case

A.7.1 A 4-Commodity Example

We begin by considering tax incidence in a 4-commodity example and then expand it to illustrate incidence in a general n-commodity example. In the 4-commodity case, we assume that commodities $i = 1, ...3$ are "small" markets, that is, $\frac{q_i x_i}{q_4 x_4} \approx 0$, $i = 1, ...3$ with commodity x_4 being the composite commodity. Then allowing for the possibility that taxes on commodities $i = 1, ..., 3$ change (but not on x_4) means that the changes in commodity prices are determined by

$$\begin{bmatrix} \eta_{11}-\mu_1 & \eta_{12} & \eta_{13} & \eta_{14} \\ \eta_{21} & \eta_{22}-\mu_2 & \eta_{23} & \eta_{24} \\ \eta_{31} & \eta_{32} & \eta_{33}-\mu_3 & \eta_{34} \\ 0 & 0 & 0 & \widetilde{\eta}_{44}-\mu_4 \end{bmatrix} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \\ \hat{p}_4 \end{bmatrix} = \begin{bmatrix} -\sum_{i=1}^{3}\eta_{1i}\hat{\tau}_i \\ -\sum_{i=1}^{3}\eta_{2i}\hat{\tau}_i \\ -\sum_{i=1}^{3}\eta_{3i}\hat{\tau}_i \\ 0 \end{bmatrix} \tag{A.71}$$

As assumed in the three commodity case discussed in Section 2, even if changes in the price of the composite commodity, x_4, affect the demand for commodities $x_i,\ i = 1,...3$, $(\eta_{i4} \neq 0,\ i = 1,...,3)$ as the markets for these commodities are small relative to that of x_4, that is, $\frac{q_i x_i}{q_4 x_4} \approx 0,\ i = 1,...3$ and as $\eta_{4i} = \frac{q_i x_i}{q_4 x_4}\eta_{i4}$ it follows that $\eta_{4j} \approx 0,\ j = 1,...,3$. Solving (A.71) for $\hat{p}_1$ gives

$$\hat{p}_1 = \frac{-\sum_{i=1}^{3}\eta_{1i}\hat{\tau}_i\left[(\eta_{22}-\mu_2)(\eta_{33}-\mu_3)-\eta_{32}\eta_{23}\right]+\sum_{i=1}^{3}\eta_{2i}\hat{\tau}_i\left[\eta_{12}(\eta_{33}-\mu_3)-\eta_{32}\eta_{13}\right]+\sum_{i=1}^{3}\eta_{3i}\hat{\tau}_i\left[\eta_{13}(\eta_{22}-\mu_2)-\eta_{12}\eta_{23}\right]}{(\eta_{11}-\mu_1)\left[(\eta_{22}-\mu_2)(\eta_{33}-\mu_3)-\eta_{32}\eta_{23}\right]-\eta_{21}\left[\eta_{12}(\eta_{33}-\mu_3)-\eta_{32}\eta_{13}\right]-\eta_{31}\left[\eta_{13}(\eta_{22}-\mu_2)-\eta_{12}\eta_{23}\right]}. \tag{A.72}$$

It will be convenient to express this in terms of $\mu_i - \eta_{ii}$. Doing so gives

$$\hat{p}_1 = \frac{\sum_{i=1}^{3}\eta_{1i}\hat{\tau}_i\left[(\mu_2-\eta_{22})(\mu_3-\eta_{33})-\eta_{32}\eta_{23}\right]+\sum_{i=1}^{3}\eta_{2i}\hat{\tau}_i\left[\eta_{12}(\mu_3-\eta_{33})+\eta_{32}\eta_{13}\right]+\sum_{i=1}^{3}\eta_{3i}\hat{\tau}_i\left[\eta_{13}(\mu_2-\eta_{22})+\eta_{12}\eta_{23}\right]}{(\mu_1-\eta_{11})\left[(\mu_2-\eta_{22})(\mu_3-\eta_{33})-\eta_{32}\eta_{23}\right]-\eta_{21}\left[\eta_{12}(\mu_3-\eta_{33})+\eta_{32}\eta_{13}\right]-\eta_{31}\left[\eta_{13}(\mu_2-\eta_{22})+\eta_{12}\eta_{23}\right]}. \tag{A.73}$$

Then we can express (A.73) as

$$\hat{p}_1 = \frac{1}{|\widetilde{H}|}\sum_{i=1}^{3}\{\eta_{1i}+\eta_{2i}\beta_2+\eta_{3i}\beta_3\}\hat{\tau}_i \tag{A.74}$$

where $\beta_2 = \frac{\eta_{12}+\frac{\eta_{32}\eta_{13}}{(\mu_3-\eta_{33})}}{(\mu_2-\eta_{22})-\frac{\eta_{32}\eta_{23}}{(\mu_3-\eta_{33})}}$, $\beta_3 = \frac{\eta_{13}+\frac{\eta_{12}\eta_{23}}{(\mu_2-\eta_{22})}}{(\eta_{22}-\mu_2)-\frac{\eta_{32}\eta_{23}}{(\eta_{33}-\mu_3)}}$, and $|\widetilde{H}| = [\mu_1-\eta_{11}-\eta_{21}\beta_2-\eta_{31}\beta_3] > 0$. Inspection of β_2, for example, suggests that the impact of the cross-market effects of taxes on x_1 and x_2 (η_{12}) are tempered by the cross-market effects between both $x_1$and x_3 (η_{13}) and x_2 and x_3 (η_{23}) with different stability condition $(\mu_2-\eta_{22})-\frac{\eta_{32}\eta_{23}}{(\mu_3-\eta_{33})}$. Finally, an alternative expression is

$$\hat{p}_1 = \rho_1\hat{\tau}_1 + (1+\rho_1)\frac{(\eta_{21}\beta_2+\eta_{31}\beta_3)}{|\widetilde{H}|}\hat{\tau}_1 + \sum_{i=2}^{3}\{\eta_{2i}\beta_2+\eta_{3i}\beta_3\}\hat{\tau}_i \tag{A.75}$$

If commodity x_3 is unrelated to commodities x_1 or x_2 (have zero cross-price elasticities with them, $\eta_{13} = \eta_{31} = \eta_{23} = \eta_{32} = 0$) then (A.75) simplifies to (7a), the case with two small markets. Then changes in taxes in unrelated markets (x_3) have no impact on the price change of x_1.

A.7.2 The General N-Commodity Case

We now briefly demonstrate how the prior simple examples generalize to an arbitrary number of distinct markets, Specifically, we let there be a set $\mathbf{K}$ containing K commodities and another set $\mathbf{M}$ of M commodities that have non-zero cross-price elasticities, amongst them and finally a composite commodity (x_c). All markets except for the composition commodity are assumed to be "small", meaning there are no income effects. A general form of the system determining the impacts of tax changes is given by

$$\underbrace{\begin{bmatrix} \eta_{11}-\mu_1 & \cdots & \eta_{1k} & \eta_{1,k+1} & \cdots & \eta_{1,k+m} & \widetilde{\eta}_{1c} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \eta_{k1} & \cdots & \eta_{kk}-\mu_k & \eta_{k,k+m} & \cdots & \eta_{k,k+m} & \widetilde{\eta}_{kc} \\ \eta_{k+1,1} & \cdots & \eta_{k+1,k} & \eta_{k+1,k+1}-\mu_{k+1} & \cdots & \eta_{k+1,k+m} & \widetilde{\eta}_{k+1,c} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \eta_{k+m,1} & \cdots & \eta_{k+m,k} & \eta_{k+1,k+1} & \cdots & \eta_{k+m,k+m}-\mu_{k+m} & \widetilde{\eta}_{k+1,c} \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \widetilde{\eta}_{cc}-\mu_c \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} \hat{p}_1 \\ \vdots \\ \hat{p}_k \\ \hat{p}_{k+1} \\ \vdots \\ \hat{p}_{k+m} \\ \hat{p}_c \end{bmatrix}}_{\underline{\hat{p}}} = \underbrace{\begin{bmatrix} \sum_{i\in K}\eta_{1i}\hat{\tau}_i + \sum_{i\in M}\eta_{1i}\hat{\tau}_i \\ \vdots \\ \sum_{i\in K}\eta_{ki}\hat{\tau}_i + \sum_{i\in M}\eta_{ki}\hat{\tau}_i \\ \sum_{i\in K}\eta_{k+1,i}\hat{\tau}_i + \sum_{i\in M}\eta_{k+1,i}\hat{\tau}_i \\ \vdots \\ \sum_{i\in K}\eta_{k+m,i}\hat{\tau}_i + \sum_{i\in M}\eta_{k+m,i}\hat{\tau}_i \\ 0 \end{bmatrix}}_{\frac{dx}{d\tau}} \tag{A.76}$$

Solving for $\hat{p}_1$ using Cramer's rule gives

$$\hat{p}_1 = \frac{\left(\sum_{i\in\mathbf{K}}\eta_{1i}\hat{\tau}_i + \sum_{i\in\mathbf{M}}\eta_{1i}\hat{\tau}_i\right)|H_{11}^T| + \sum_{j\in\mathbf{K}}\eta_{1j}\,(-1)^{1+i}\,|H_{1j}^T| + \sum_{j\in\mathbf{M}}\eta_{1j}\,(-1)^{1+i}\,|H_{1j}^T| + \eta_{1c}\,(-1)^{1+k+m+1}\,|H_{1c}|}{|H|} \tag{A.77}$$

where H^T is the matrix created from the matrix H by replacing its first column with the column $\frac{dx}{d\tau}$ from (A.76); H_{1j} is the $(K+M)\times(K+M)$ matrix obtained by deleting row 1 and column j and $|H_{1j}|$ is determinant of the minor of H_{ij}. To demonstrate that tax changes of commodities that are unrelated to commodity x_1 and any its complements or substitutes, have no affect on $\hat{p}_1$, we assume $\eta_{ij} \neq 0,\ i,j \in \mathbf{K}$; $\eta_{ij} = 0,\ i \in \mathbf{K},\ j \in \mathbf{M}$; $\eta_{ij} \neq 0,\ i,j \in \mathbf{M}$ — commodities in the set $\mathbf{K}$ and commodities in the set $\mathbf{M}$ have zero cross-price elasticities with each other. As before, we assume that $\widetilde{\eta}_{ic} \neq 0$ and $\eta_{ci} = 0\,\forall\, i \in \mathbf{K},\mathbf{M}$. Then given these assumptions, in the first term of (A.77), $\sum_{i\in M}\eta_{1i}\hat{\tau}_i$ equals zero, the third term equals zero as

$\eta_{1j}=0$ for $j\in\mathbf{M}$, and, finally, the fourth term is zero as well as $|H_{1c}|$ as it has a row of zeros $(n_{c1}=0,\eta_{c2}=0,...,\eta_{c,k+m})$. Then (A.77) simplifies to

$$\hat{p}_1=\frac{\sum_{j\in K}\sum_{i\in K}\eta_{i1}\hat{\tau}_i|H_{j1}^{K_T}|}{|H^K|} \tag{A.78}$$

where H^K is the $K\times K$ matrix, $H^K=\begin{bmatrix}\eta_{11}-\mu_1 & \cdots & \eta_{1K}\\ \vdots & \vdots & \vdots\\ \eta_{K1} & \cdots & \eta_{KK}\end{bmatrix}$ and $H^{K_T}=\begin{bmatrix}\sum_{i\in\mathbf{K}}\eta_{1i}\hat{\tau}_i & \cdots & \eta_{1k}\\ \vdots & \vdots & \vdots\\ \sum_{i\in\mathbf{K}}\eta_{ki}\hat{\tau}_i & \cdots & \eta_{kk}\end{bmatrix}$
where the block diagonal structure of the matrix H gives the determinant $|H|=(\eta_{cc}-\mu)\,|H^K||H^M|$ where $H^K=\begin{bmatrix}\eta_{11}-\mu_1 & \cdots & \eta_{1k}\\ \vdots & \vdots & \vdots\\ \eta_{k1} & \cdots & \eta_{kk}\end{bmatrix}$ and $H^M=\begin{bmatrix}\eta_{k+1,k+1}-\mu_{k+1} & \cdots & \eta_{k+1,k+m}\\ \vdots & \vdots & \vdots\\ \eta_{k+1,k+m} & \cdots & \eta_{k+m,k+m}\end{bmatrix}$. Then $|H_{1j}^T|=|H_{1j}^{K_T}|\,(\eta_{cc}-\mu)\,|H^K||H^M|$ where H^{K_T} is obtained by replacing the element $j1$ of H^K by $\sum_{i\in K}\eta_{j1}\hat{\tau}_i$. Then as the elements of $|H_{j1}^{K_T}|$ and those of $|H^K|$ only include the elements of the submatrix $\mathbf{K}$, the elasticities of the commodities with non-zero cross-price elasticities with x_1 and zero-cross price elasticities with the commodities in set $\mathbf{M}$ it follows from (A.78) that the change in the price of x_1 is only affected by taxes on the set of inter-related commodities $\mathbf{K}$ and not commodities with zero cross-price elasticities with this set.

A.8 Specific Utility Function Derivations

As in the text, for the case of $x_1$and x_2 to be substitutes, let utility be given by

$$U\left(x_1x_2,x_3\right)=\left(min[\beta_1x_1,\beta_3x_3]\right)^{\alpha}x_2^{1-\alpha} \tag{A.79}$$

Let $w_1$be the income devoted to x_1 and x_3. Then utility maximization requires that $\beta_1x_1=\beta_3x_3$and that $q_1x_1+q_2x_2=w_1$. Then solving gives $x_1=\frac{\beta_3}{\beta_3q_1+\beta_1q_3}w_1$ and $x_3=\frac{\beta_1}{\beta_3q_1+\beta_1q_3}w_1$ with $min[\beta_1x_1,\beta_3x_3]=\frac{\beta_1\beta_3}{\beta_3q_1+\beta_1q_3}w_1$ giving

$$U=\left(\frac{\beta_1\beta_3}{\beta_3q_1+\beta_1q_3}w_1\right)^{\alpha}\left(\frac{w-w_1}{q_2}\right)^{1-\alpha} \tag{A.80}$$

Solving A.80 for w_1 gives $w_1=\alpha w$ so the Marshallian demand functions become

$$x_i=\alpha\frac{\beta_j}{\beta_3q_1+\beta_1q_3}w,\; i,j=1,3;\; i\neq j \tag{A.81}$$

and

$$x_2 = (1-\alpha)\,\frac{w}{q_2} \tag{A.82}$$

Then the indirect utility function can be expressed as

$$V(q_1, q_2, q_3, w) = \frac{kW}{(\beta_3 q_1 + \beta_1 q_3)^{\alpha}\, q_2^{1-\alpha}},\ \ k = (\beta_1 \beta_3 \alpha)^{\alpha}\,(1-\alpha)^{1-\alpha} \tag{A.83}$$

Inverting (A.83) yields the expenditure function,

$$e(q_1, q_2, q_3, U) = (\beta_3 q_1 + \beta_1 q_3)^{\alpha}\, q_2^{1-\alpha}\frac{U}{k}. \tag{A.84}$$

Differentiating (A.84) with respect to each of the prices yields the compensated demand functions,

$$h_1(q_1, q_2, q_3, U) = \alpha\beta_3\,(\beta_3 q_1 + \beta_1 q_3)^{\alpha-1}\, q_2^{1-\alpha}\frac{U}{k} = \alpha\beta_3\frac{e(q_1, q_2, q_3, U)}{(\beta_3 q_1 + \beta_1 q_3)}, \tag{A.85}$$

$$h_2(q_1, q_2, q_3, U) = (1-\alpha)\,(\beta_3 q_1 + \beta_1 q_3)^{\alpha}\, q_2^{-\alpha}\frac{U}{k} = (1-\alpha)\,\frac{e(q_1, q_2, q_3, U)}{q_2}, \tag{A.86}$$

and

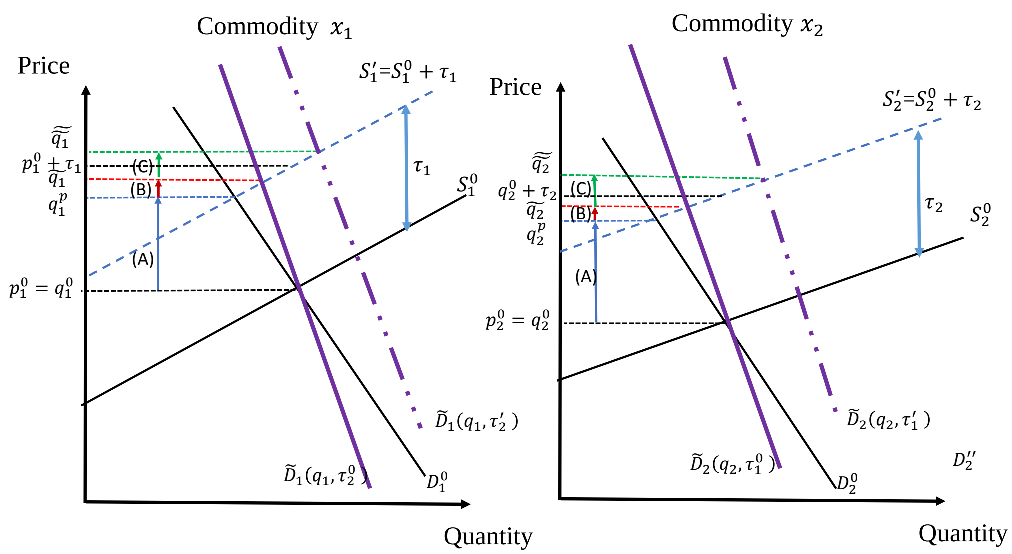
$$h_3(q_1, q_2, q_3, U) = \alpha\beta_1\,(\beta_3 q_1 + \beta_1 q_3)^{\alpha-1}\, q_2^{1-\alpha}\frac{U}{k} = \alpha\beta_1\frac{e(q_1, q_2, q_3, U)}{(\beta_3 q_1 + \beta_1 q_3)}, \tag{A.87}$$

For the case of x_1 and x_2 being complements, the form of the demand equation for $x_2$is now given by (A.87) and the form for x_3 is given by (A.86).

A.9 Additional Figures

The appendix shows all simulation figures as described in the text.

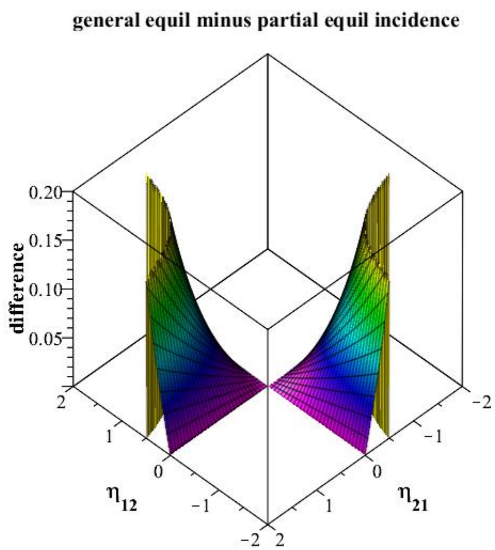
Figure A.1: Tax Incidence with Taxes on Two Related Commodities:General Equilibrium Demand Curves



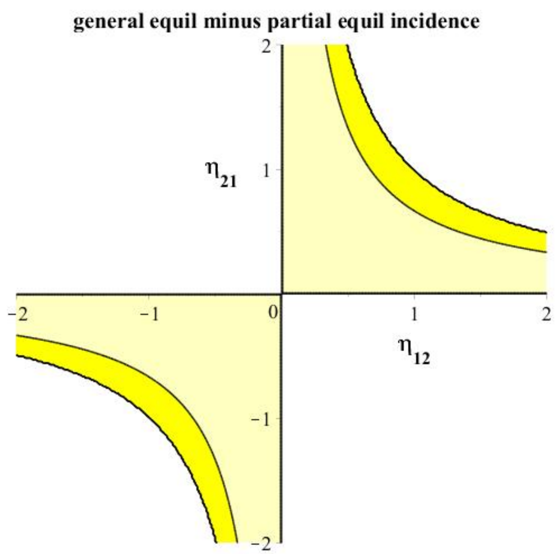
These figure shows the effects of a tax increases in two related markets starting from $\tau_1 = \tau_2 = 0$. The initial equilibrium price is given by q_i^0. The partial equilibrium prices after the tax increase are q_i^p for the consumer. The prices accounting only for the feedback from one market are given by $\widetilde{q}_i$, and the general equilibrium prices are given by $\widetilde{\widetilde{q}}_i$. The analysis using partial equilibrium demand curves is given in figure 3a. Terms (A), (B), and (C) are the shifts defined in (7a) and (8). Then consider the equilibrium when using a general equilibrium demand curve $\widetilde{D}_i$. This GE-demand curve is shown in bold purple and is steeper than the partial equilibrium demand curve. A tax in market one shifts up supply in that market, but shifts out the GE-demand curve in market 2. The price increase in market two dampens the effect on quantity in market one. But then, the tax in market 2 shifts the GE-demand curve in market one.

Figure A.2: Baseline Simulations for Producer Prices in Market 1 with Small Markets, A Tax Only in Market 1 and $\mu_1 = \mu_2 = |\eta_{11}| = |\eta_{11}| = 1$

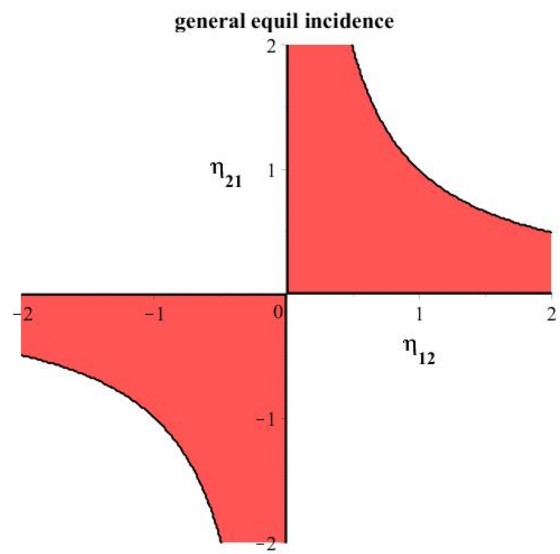
(a) Precise Difference in GE and PE Formula



(b) Contour of Difference in GE and PE Formula



(c) Area Where Incidence Is [0,1] and Stable



Given the elasticities used in the simulations, the partial equilibrium incidence on producers is always -0.50. Percentage deviations can be calculated by dividing the estimates above by this number.

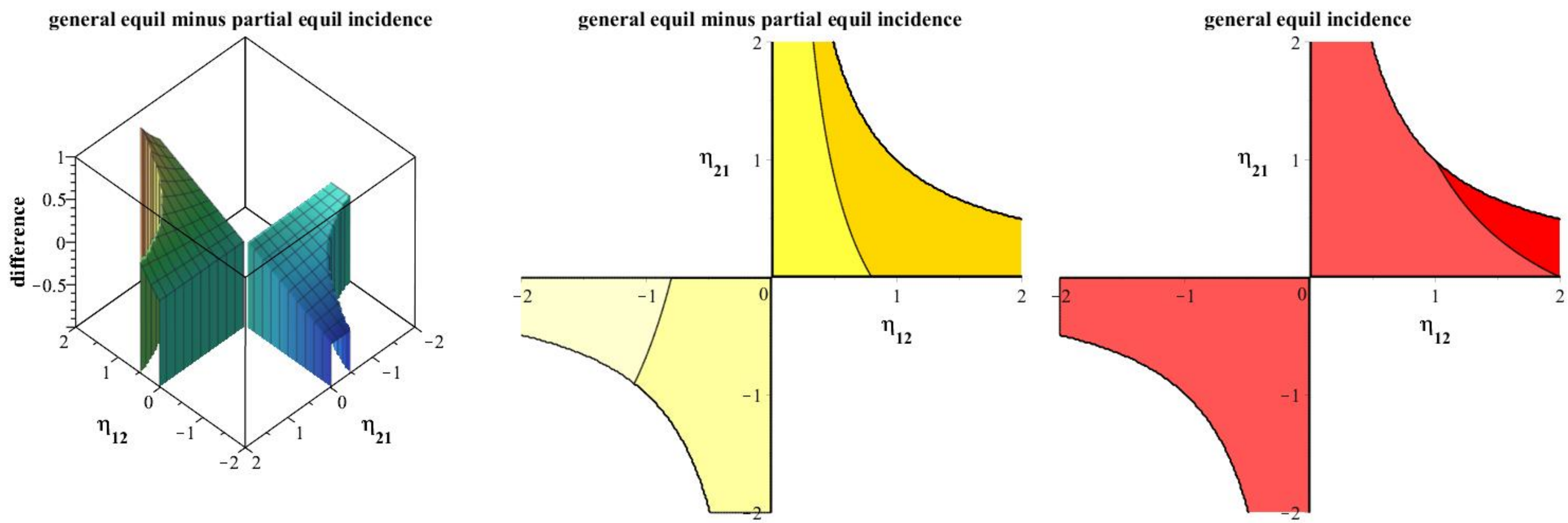
Figure a legend: The shaded areas correspond to cross-price elasticities where the GE model satisfies all of the constraints. White areas correspond to a violation of a constraint on the problem. The shaded height gives the deviation from the partial equilibrium formula $(\hat{p}_i^G|_{\hat{\tau}_2=0} - \rho_i)$. For example, a value of 0.15 means that the change in the producer price is -0.35 instead of the -0.50 PE formula.

Figure b legend: As the 3d graph can be difficult to interpret, this figure shows contour plots corresponding to the level of the deviation. White areas correspond to a violation of a constraint on the problem. Light yellow are areas where the deviation from partial equilibrium is between [0, 0.10] and dark yellow areas are areas where the deviation from partial equilibrium is greater than 0.10 cents per dollar.

Figure c legend: White areas correspond to a violation of a constraint on the problem. Medium red areas are where the general equilibrium incidence on producers is between $\hat{p}_1 \in [-1, 0]$. With only one tax, no areas correspond to overshifting or undershifting.

Figure A.3: Baseline Simulations for Producer Prices in Market 1 with Small Markets, A Tax in Both Market 1 and 2 and $\mu_1 = \mu_2 = |\eta_{11}| = |\eta_{11}| = 1$

(a) Precise Difference in GE and PE Formula (b) Contour of Difference in GE and PE Formula (c) Area of [0,1] Incidence and Overshifting



Given the elasticities used in the simulations, the partial equilibrium incidence on producers is always -0.50. Percentage deviations can be calculated by dividing the estimates above by this number.

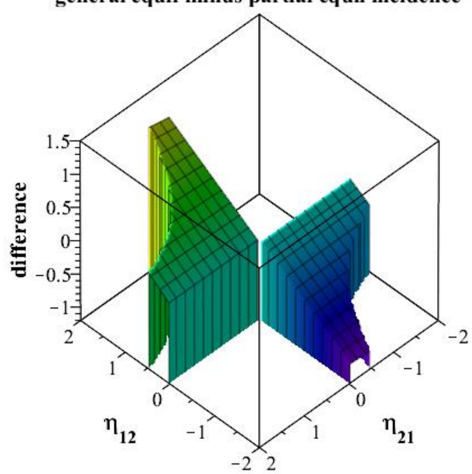
Figure a legend: The shaded areas correspond to cross-price elasticities where the GE model satisfies all of the constraints. White areas correspond to a violation of a constraint on the problem. The shaded height gives the deviation from the partial equilibrium formula $(\hat{p}_i^G|_{\hat{\tau}_1=\hat{\tau}_2} - \rho_i)$. For example, a value of 0.15 means that the change in the producer price is -0.35 instead of the -0.50 PE formula; a value of -0.15 means that the change in the producer price is -0.65 instead of the -0.50 PE formula.

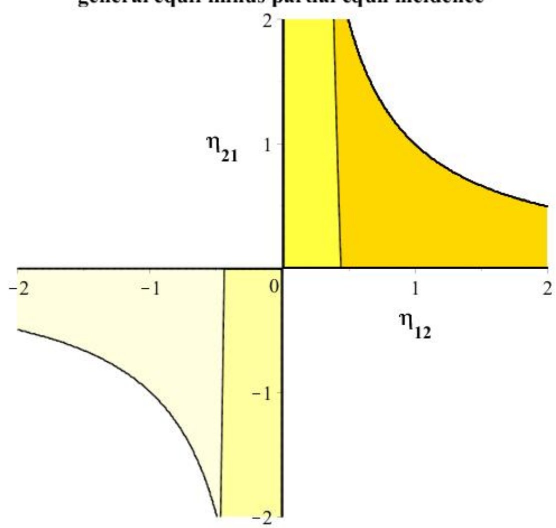
Figure b legend: As the 3d graph can be difficult to interpret, this figure shows contour plots corresponding to the level of the deviation. White areas correspond to a violation of a constraint on the problem. Lightest yellow are areas where the deviation from partial equilibrium is < -0.20, the second lightest area correspond to deviation between $[-0.2, 0]$. The darkest yellow area are areas where the deviation from partial equilibrium is > 0.20 while the second darkest are corresponds to deviations between $[0, 0.20]$ cents per dollar tax change.

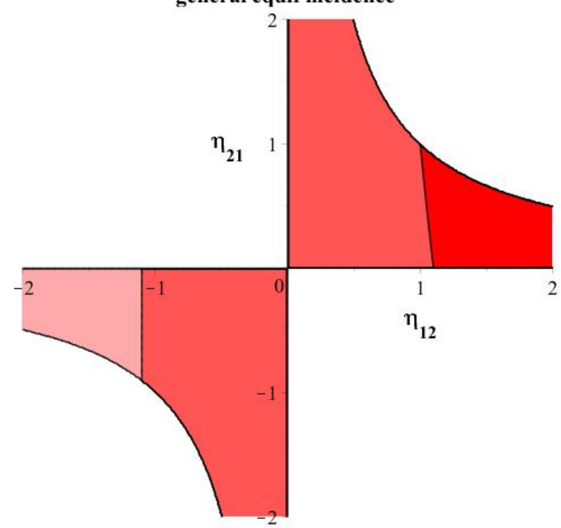
Figure c legend: This figure shows areas in general equilibrium where overshifting and Edgeworth taxation paradox arise. White areas correspond to a violation of a constraint on the problem. Light red areas (none in the parameterization) are where Edgeworth's taxation paradox arises $\hat{p}_1 <= 1$, medium red areas are where the general equilibrium incidence on producers is between $\hat{p}_1 \in [-1, 0]$, dark red areas are where overshifting occurs: $\hat{p}_1 > 0$. In this figure all areas are medium red or dark red, so Edgeworth's paradox never arises for such parameter values.

Figure A.4: Baseline Simulations for Producer Prices in Market 1 with Two Small Markets, A Large Supply Elasticity in Market 2 ($\mu_2 = 10$) and $\mu_1 = |\eta_{11}| = |\eta_{11}| = 1$

(a) Precise Difference in GE and PE Formula (b) Contour of Difference in GE and PE Formula (c) Area of [0,1], Overshifting and Undershifting







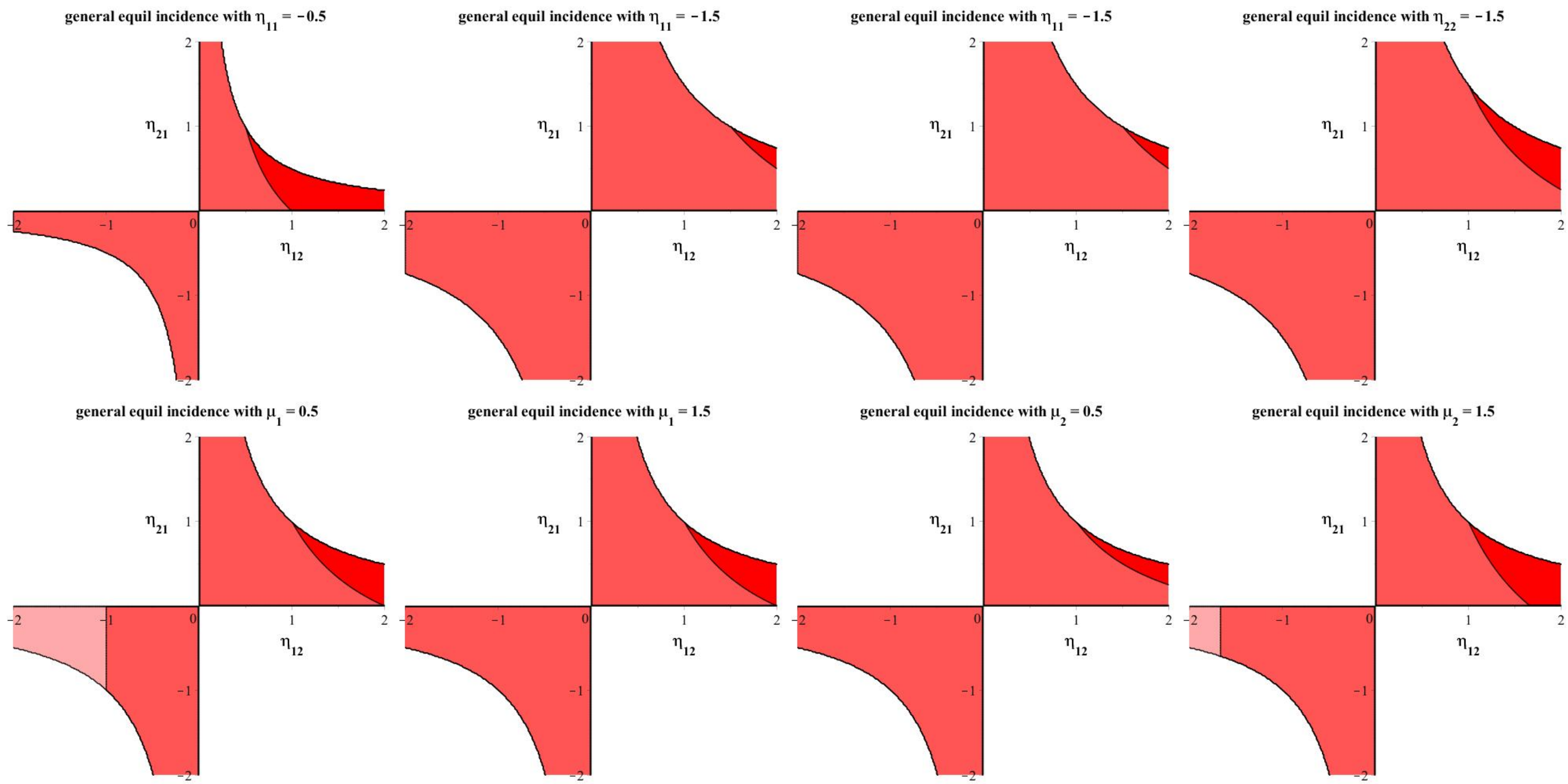
Given the elasticities used in the simulations, the partial equilibrium incidence on producers is always -0.50. Percentage deviations can be calculated by dividing the estimates above by this number.

Figure a legend: The shaded areas correspond to cross-price elasticities where the GE model satisfies all of the constraints. White areas correspond to a violation of a constraint on the problem. The shaded height gives the deviation from the partial equilibrium formula $(\hat{p}_i^G|_{\hat{\tau}_1=2\hat{\tau}_2} - \rho_i)$. For example, a value of 0.15 means that the change in the producer price is -0.35 instead of the -0.50 PE formula; a value of -0.15 means that the change in the producer price is -0.65 instead of the -0.50 PE formula.

Figure b legend: As the 3d graph can be difficult to interpret, this figure shows contour plots corresponding to the level of the deviation. White areas correspond to a violation of a constraint on the problem. Lightest yellow are areas where the deviation from partial equilibrium is < -0.20, the second lightest area correspond to deviation between $[-0.2, 0]$. The darkest yellow area are areas where the deviation from partial equilibrium is > 0.20 while the second darkest are corresponds to deviations between $[0, 0.20]$ cents per dollar tax change.

Figure c legend: This figure shows areas in general equilibrium where overshifting and Edgeworth taxation paradox arise. White areas correspond to a violation of a constraint on the problem. Light red areas are where Edgeworth's taxation paradox arises $\hat{p}_1 <= 1$, medium red areas are where the general equilibrium incidence on producers is between $\hat{p}_1 \in [-1, 0]$, dark red areas are where overshifting occurs: $\hat{p}_1 > 0$.

Figure A.5: Baseline Simulations for Producer Prices in Market 1 with Small Markets, A Tax in Both Market 1 and 2 and All (Except One) Own-price Elasticities = 1



Each figure simulates the producer price change in our general equilibrium model with all of the elasticities μ_1, μ_2, $|\eta_{11}|$, $|\eta_{11}|$ equal to one except for one. The one that does not equal one is given in the title of each figure. We change one elasticity by ± 0.50 as a way of visualizing how incidence changes as one parameter changes. We show only level curves and not the precise incidence values, but the incidence is smoothly increasing or decreasing toward the darker and lighter areas.

Figures legend: This figure shows areas in general equilibrium where overshifting and Edgeworth's taxation paradox arise. White areas correspond to a violation of a constraint on the problem. Light red areas are where Edgeworth's taxation paradox arises $\hat{p}_1 <= 1$, medium red areas are where the general equilibrium incidence on producers is between $\hat{p}_1 \in [-1, 0]$, dark red areas are where overshifting occurs: $\hat{p}_1 > 0$.

Table A.2: Effect of Excise Taxes on Beer and Wine Prices, With Taxes for Specific Products

	(1)	(2)	(3)	(4)	(1')	(2')	(3')	(4')
	Beer Prices		Wine Prices		Beer Prices		Wine Prices	
Beer Tax	0.968	0.340		0.645	0.973	-0.212		0.178
	(0.737)	(0.417)		(0.902)	(0.747)	(0.261)		(1.024)
Wine Tax		0.373*	1.154*	1.090**		0.686**	2.076**	1.336**
		(0.221)	(0.630)	(0.532)		(0.272)	(0.852)	(0.630)
Spirit Tax		0.215***		-0.131		0.416***		0.571***
		(0.064)		(0.149)		(0.134)		(0.185)
Controls	Y	Y	Y	Y	Y	Y	Y	Y
State FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Observations	1,859	1,859	1,859	1,859	1,859	1,859	1,859	1,859
States	51	51	51	51	51	51	51	51

This table presents the results from equations (26) and (27) for beer and wine prices. In columns without a prime, the price is the average (unweighted) price in the state. In columns with a prime, the price is the average (weighted by population) price in the state. Columns (1)-(2) are for beer and columns (3)-(4) are for wine. All regressions include time fixed effects, state fixed effects, and a full vector of controls including the state sales tax rate on alcohol. Taxes are in dollars for a six pack of beer, a 1.5 L bottle of wine, and a 0.75 L bottle of liquor in all regressions. This implies that in the beer price regression, taxes are for each of these three different units. These units corresponds to the prices. Standard errors are clustered at the state level. *** 99%, ** 95%, * 90%.

Table A.1: Simulations of Producer Price Changes When One Product Is Inelastically Supplied and the Other Elastically Supplied and Two Taxes Change ($\hat{\tau}_1 = \hat{\tau}_2 = 1$)

Elasticities				PE	$\eta_{12} = \eta_{21} = 0.5$		$\eta_{12} = \eta_{21} = -0.5$		$\eta_{12} = 1.5,\ \eta_{21} = 0.5$		$\eta_{12} = -1.5,\ \eta_{21} = -0.5$	
$\|\eta_{11}\|$	$\|\eta_{22}\|$	μ_1	μ_2	$\hat{p}_1^{PE}$	$\hat{p}_1^G$	%△	$\hat{p}_1^G$	%△	$\hat{p}_1^G$	%△	$\hat{p}_1^G$	%△
1	1	0.1	10	-0.91	-0.49	-46%	-1.32**	+46%**	0.41*	-146%*	-2.22**	+144%**
0.5	1	0.1	10	-0.83	-0.04	-95%	-1.61**	+94%**	n/a	n/a	n/a	n/a
1.5	1	0.1	10	-0.94	-0.65	-31%	-1.22**	+31%**	-0.04	-95%	-1.82**	+95%**
1	0.5	0.1	10	-0.91	-0.46	-49%	-1.34**	+48%	n/a	n/a	n/a	n/a
1	1.5	0.1	10	-0.91	-0.50	-45%	-1.31**	+44%	0.35*	-139%*	-2.16**	+138%**
1	1	0.05	10	-0.95	-0.68	-29%	-1.27**	+46%	0.44*	-146%*	-2.34**	+145%**
1	1	0.15	10	-0.90	-0.46	-47%	-1.27**	+46%	0.39*	146%*	-2.12**	+144%**
1	1	0.1	9.5	-0.91	-0.49	-46%	-1.32**	+46%	0.41*	-146%*	-2.22**	+144%**
1	1	0.1	10.5	-0.91	-0.48	-47%	-1.33**	+46%	0.42*	-146%*	-2.23**	+145%**

This table simulates the producer price incidence for the given elasticities. * Indicates overshifting and ** indicated Edgeworth's Paradox. Percent changes are given using the formulas in the text. A positive [negative] percent change means the incidence on the producer increases [decreases] in our general equilibrium formula relative to the partial equilibrium formula. When calculating percent changes, we use the precise numerical incidence value rather than the rounded values in the table. N/A means a constraint on the problem does not hold.

Table A.3: Marginal Effects of Own-Tax Tax in Interaction Model

	(1)	(2)	(3)	(4)	(1')	(2')	(3')	(4')
	Beer Prices		Wine Prices		Beer Prices		Wine Prices	
Own Tax Changes	0.968	0.927	1.154*	0.732	0.973	0.772	2.076**	1.651*
	(0.737)	(0.901)	(0.630)	(0.657)	(0.747)	(0.932)	(0.852)	(0.878)
Own Tax and Wine		1.446**				1.388*		
		(0.688)				(0.718)		
Own Tax and Spirit		0.394		0.526		0.226		1.586
		(1.114)		(0.996)		(1.196)		(1.093)
Own Tax and Beer				1.118				1.901*
				(0.768)				(0.981)
				(0.768)				(0.981)
All Taxes		0.913		0.917		0.842		1.837**
		(0.826)		(0.639)		(0.826)		(0.850)
Controls	Y	Y	Y	Y	Y	Y	Y	Y
State FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Observations	1,859	1,859	1,859	1,859	1,859	1,859	1,859	1,859
States	51	51	51	51	51	51	51	51

This table presents the results where the own-product excise tax rate is in the regression and it is interacted with dummies if each of the other product taxes change. We then present the marginal effects (rows in this table) when the own-tax changes alone, when it changes with one of the other taxes, and when all taxes change. In columns without a prime, the price is the average (unweighted) price in the state. In columns with a prime, the price is the average (weighted by population) price in the state. Columns (1)-(2) are for beer and columns (3)-(4) are for wine. All regressions include time fixed effects, state fixed effects, and a full vector of controls including the state sales tax rate on alcohol. Taxes are in dollars corresponding to the units of volume of the good. Standard errors are clustered at the state level and the Delta method is applied for marginal effects. *** 99%, ** 95%, * 90%.