Modeling Techniques
Models of Corporation Taxation

James R. Hines Jr.[1] and Nathan Seegert[2]

May 24, 2022

[1]Department of Economics and School of Law, University of Michigan
[2]Department of Finance, University of Utah

The theory of the firm is built on giants of economics

1. Incomplete list of giants
    - Irving Fisher
    - Paul Samuelson
    - Robert Solow
    - Franco Modigliani
    - Merton Miller
    - Ronald Coase

2. Much of the discussion here follows
    - James R. Hines Jr.'s graduate course notes.
    - *Capital Income Taxation and Resource Allocation* by Hans-Werner Sinn.

3. I am going to poorly summarize some of their insights as it furthers our goals here:

How do we write down a model of firm behavior?

Goal: enhance modeling skills and economic intuition

1. You have great institutional knowledge.

2. My understanding is you want to know some models.

3. I am going to provide some economic and model insights
   - Some language differences—PLEASE stop and ask.
   - Some stylistic differences—PLEASE stop and ask.

Models are simplifications of the world

"What a useful thing a pocket-map is!" I remarked.

"That's another thing we've learned from your Nation," said Mein Herr, "map-making. But we've carried it much further than you. What do *you* consider the *largest* map that would be really useful?"

"About six inches to the mile."

"Only *six* inches!" exclaimed Mein Herr. "We very soon got to six yards to the mile. Then we tried a *hundred* yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of a *mile to the mile*!"

"Have you used it much?" I enquired.

"It has never been spread out, yet," said Mein Herr: "the farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well."

from Lewis Carroll, Sylvie and Bruno Concluded, Chapter XI, London 1895

The goal of a model

Several years ago I gave a seminar about some of my research. I started out with a very simple example. One of the faculty in the audience interrupted me to say that he had worked on something like this several years ago, but his model was "much more complex." I replied "My model was complex when I started, too, but I just kept working on it till it got simple!"

Hal Varian "How to Build an Economic Model in Your Spare Time."

Preliminary building blocks

Models need three things

1. Players—who is making a decision (e.g., firm, shareholder, CEO).

2. Strategies—what can the players do (e.g., choose investment levels).

3. Payoffs—what do the players receive (e.g., firm value or utility).

In my writing, I like to spell these out right away and in this order.

Preliminary building blocks

Models are used to highlight trade offs

1. Is your model about a new trade off? (e.g., dividends versus mergers).
2. Is your model about a new feature that affects the tradeoff (e.g., information revelation).
3. Make sure everything supports the novel aspect of your model.

Models start out simple and progress as we add features

1. We will start with the very basic models.
2. These models will be missing a lot of important details.
3. The hope is that these models can be the jumping off point for you to use in your own work
4. and the tools we learn can help build hypotheses from these models.

Outline

1. Separation theorem.
    - How do we shift value across time?
    - Robinson Crusoe model.
2. Two period model.
    - How much debt should a business have?
    - Modligani-Miller.
3. Expected utility with CARA utility.
    - Do firms always maximize firm value?
    - An agency model of corporate behavior.
4. Adding taxes to our two period model.
    - Do corporate taxes distort investment?
5. User cost of capital and effective tax rates ETR.
    - How do tax depreciation methods distort investment?
6. Add personal taxes to our two period model.
    - Do dividend taxes distort investment?
    - New view vs old view.
7. Comparative statics and total differentiation.
    - How does inflation distort investment?
8. The envelope theorem.
9. Sufficient statistics.
    - How do corporate tax rates affect total value in the economy?
10. Structural parameter estimation
    - How elastic are firms?

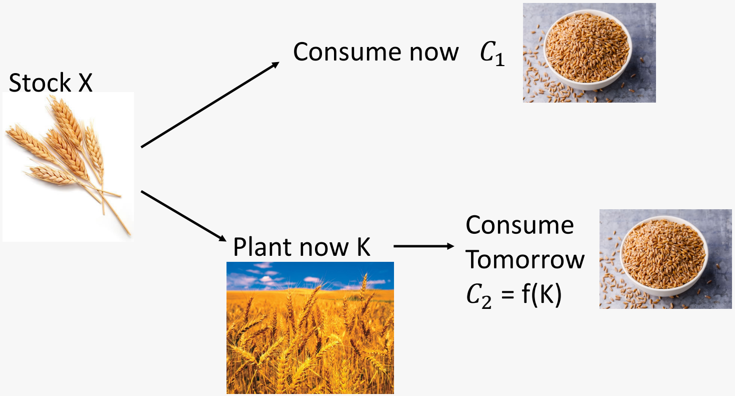
Separation Theorem

How do we shift value across time?

Robinson Crusoe

X amount of wheat

Strategies: Consume now or invest



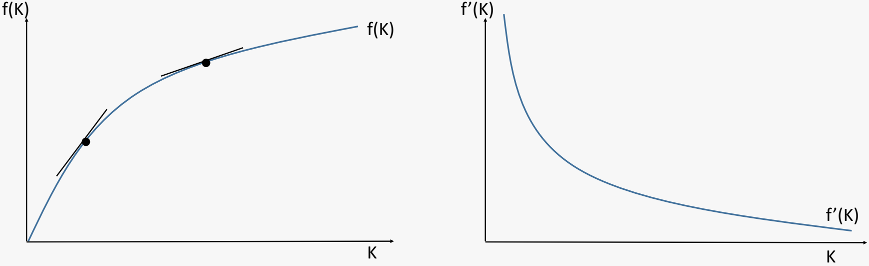
Wheat can be consumed or planted

$$X = C_1 + K \tag{1}$$

Consumption tomorrow is a function of the wheat planted now

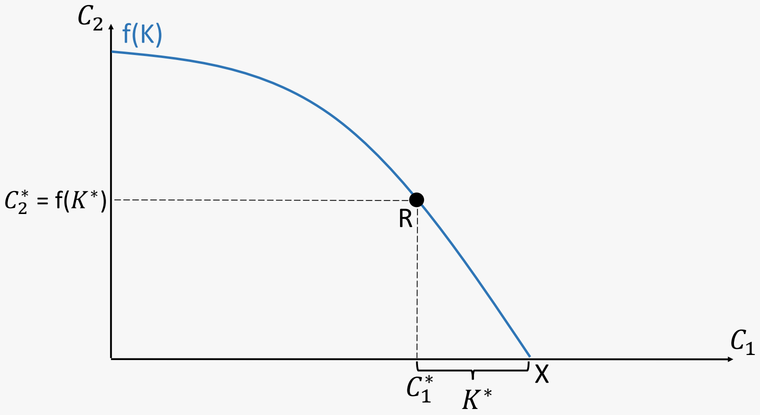
$$C_2 = f(K) \tag{2}$$

Assumptions on the production function



1. $f(0) = 0$, no production without some planting.
2. $f'() > 0$, the more you plant the more yield.
3. $f''() < 0$, the more you plant the lower the marginal yield.
   - **Diminishing returns** only so much room on the island, as you plant more use worse land or over crowd the wheat such that doubling the seed will not double the yield.

Transformation from C_1 to C_2



- Diminishing returns, get less C_2 for each unit of K as K increases.

Payoffs: utility over consumption

$$max_{C_1, C_2, K} \quad U(C_1, C_2) \tag{3}$$

Constraints

1. Cannot consume more today than you have $0 \leq C_1 \leq X$.
2. The sum of consumption today and investment cannot be more than you have $K + C_1 \leq X$.
3. What you consume tomorrow is the yield from production $C_2 = f(K)$.

Assumptions

1. $\partial U(C_1, C_2)/\partial C_1 \equiv U_1 > 0$.
2. $\partial U(C_1, C_2)/\partial C_2 \equiv U_2 > 0$.

Indifference curves are combinations of C_1 and C_2. with the same utility

- Strict quasi-concavity, $U_1 > 0$ and $U_2 > 0$.
- Slope of the indifference curve is the marginal rate of substitution (MRS) U_1/U_2.
- Diminishing returns in consumption.

Household maximization

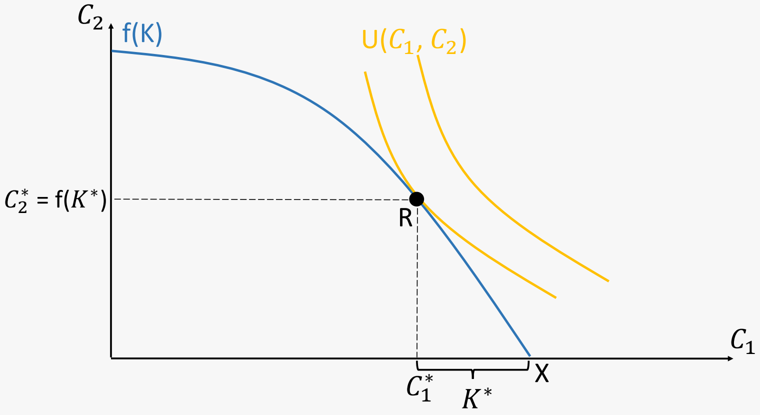
$$max_{C_1, C_2, K} \;\; U(C_1, C_2) \qquad s.t. \;\; 0 \leq C_1 \leq X \;\; \& \;\; X = K + C_1 \;\; \& \;\; C_2 = f(K) \quad (4)$$

$$\begin{aligned} \mathcal{L} = & U(C_1, C_2) + \lambda(X - C_1 - K) + \gamma(f(K) - C_2) \\ & \frac{\partial \mathcal{L}}{\partial C_1} : U_1 = \lambda \\ & \frac{\partial \mathcal{L}}{\partial C_2} : U_2 = \gamma \\ & \frac{\partial \mathcal{L}}{\partial K} : \lambda = \gamma f'(K) \end{aligned} \qquad (5)$$

$$\rightarrow U_1 = \gamma f'(K) = U_2 f'(K)$$

- First-order condition: $U_1 / U_2 = f'(K)$.

- First-order condition: $U_1/U_2 = f'(K)$.



Optimization can be interpreted in two ways:

1. Marginal change in utility between C_1 and C_2 must equal the marginal change in production. (Marginal rate of substitution equals the marginal rate of transformation MRS = MRT)
    - $U_1/U_2 = f'(K)$.
2. The rate of time preferences $\gamma(C_1, C_2) \equiv -U_1/U_2 - 1$ equals the net marginal product of capital
    - $\gamma(C_1, C_2) = f'(K) - 1$.

- This model told us about the tradeoff between consumption today and tomorrow.

The role of the capital market

- Now, lets see how capital markets change this tradeoff.

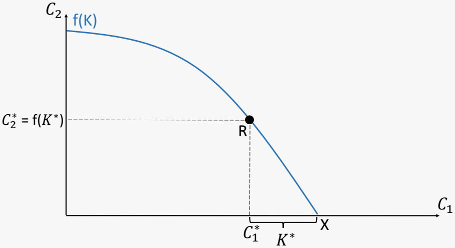
1. Player: a single household with endowment X.
2. Strategies:
   - Consumption now C_1,
   - Borrowing or saving B at interest rate r,
   - Investment K,
3. Payoffs: utility over consumption today and tomorrow $U(C_1, C_2)$

$$C_1 = X - K + B \tag{6}$$

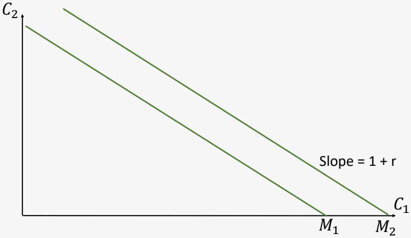
$$C_2 = f(K) - (1 + r)B. \tag{7}$$

Two ways of transforming C_1 and C_2

Production/Investment:
$C_2 = f(K) = f(X - C_1)$.

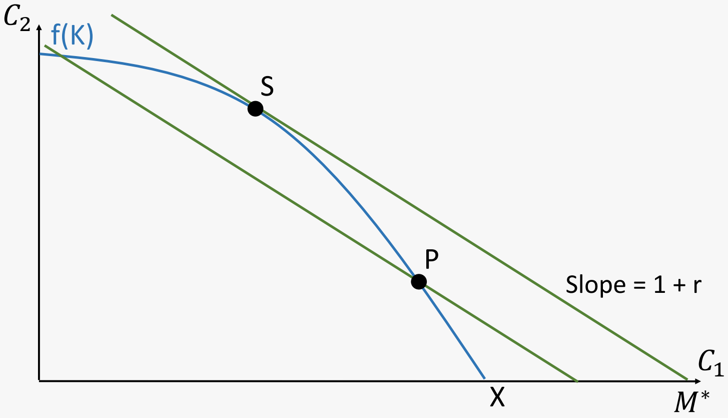


Capital markets:
$C_2 = -(1+r)C_1 + (1+r)X$.



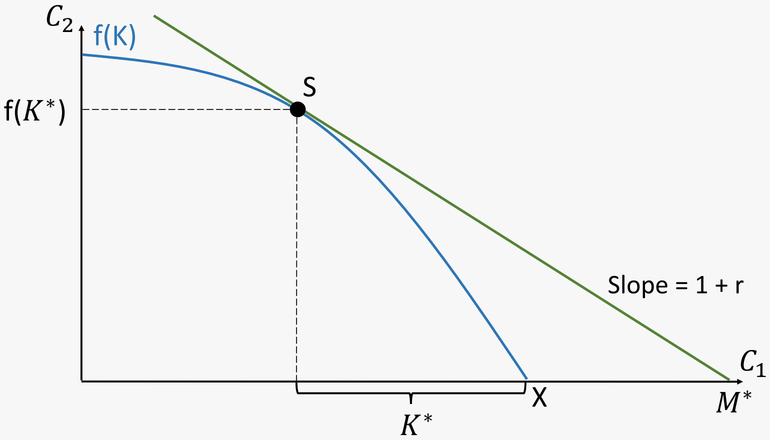
- Wealth is greater at M_1 than M_2.

First, find how much to produce (S or P)



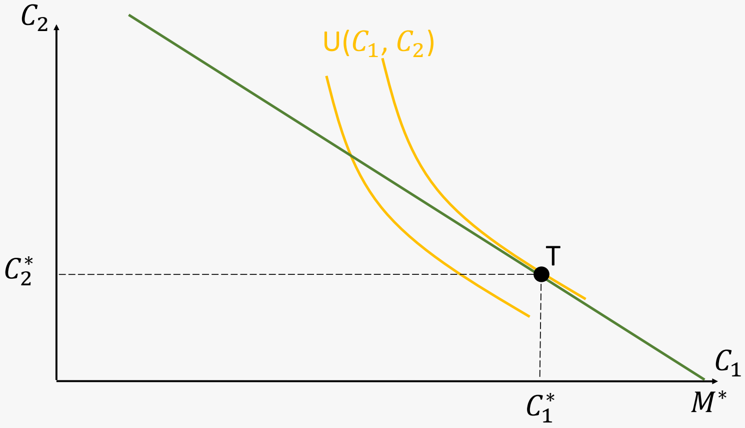
- Maximize wealth M^*, where $f'(K) = 1 + r$

First, find how much to produce



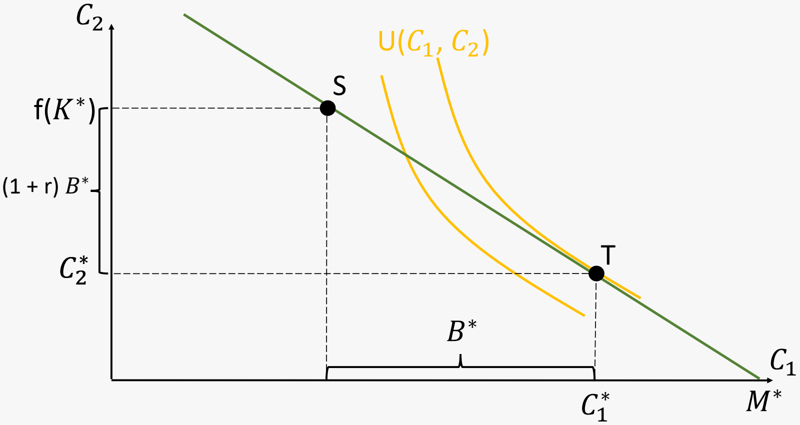
- K^* determines how much to produce $f(K^*)$.
- M^* determines wealth.

Second, find how much to consume



- Maximize utility where $U_1(C_1, C_2)/U_2(C_1, C_2) = 1 + r$, where M^* is given.

Second, find how much to consume



- Start at point S.
- Borrow B^* and repay $B^*(1+r)$ to get to T.

Optimization with capital markets

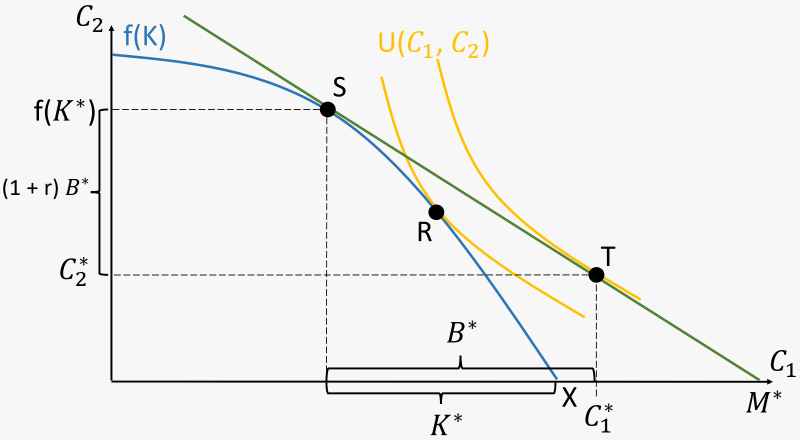
$$max_{C_1} \quad U(C_1, C_2) \qquad s.t. \;\; 0 \leq C_1 \leq X \;\; \& \;\; C_1 = X - K + B \;\; \& \;\; C_2 = f(K) - (1+r)B \tag{8}$$

Optimality conditions for an interior solution

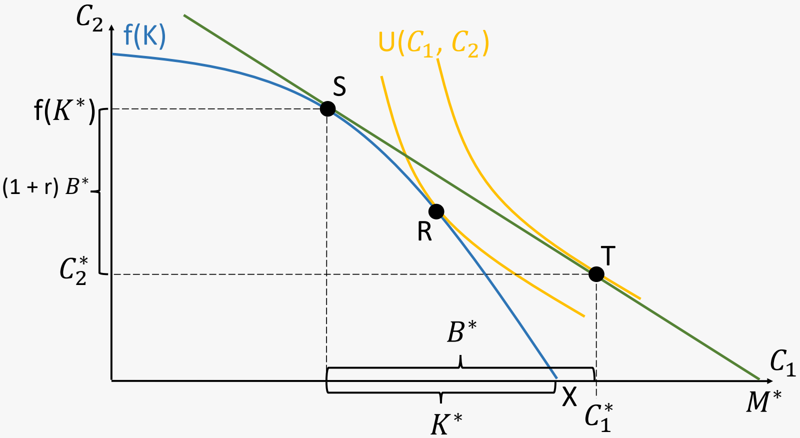
1. $U_1/U_2 = 1 + r$
2. $f'(K) = 1 + r$

Marginal rate of substitution and marginal product of capital has to equal $1 + r$.

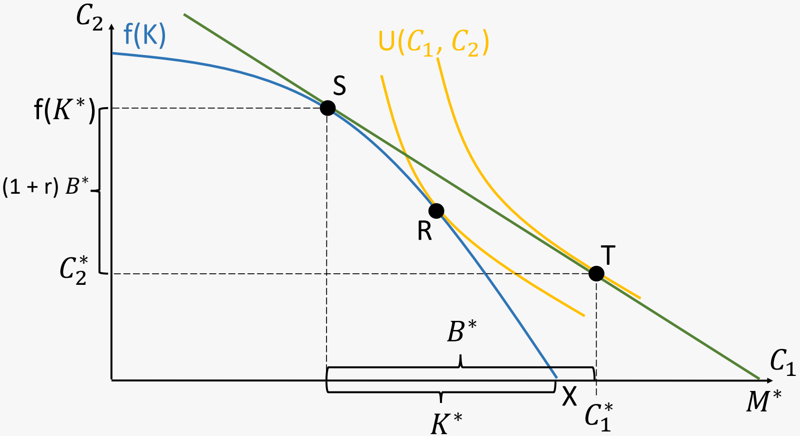
1. **Separation Theorem:** Point S defines the production decision and is independent of household preferences and initial capital endowment.



2. The optimal production decision maximizes wealth M^*.

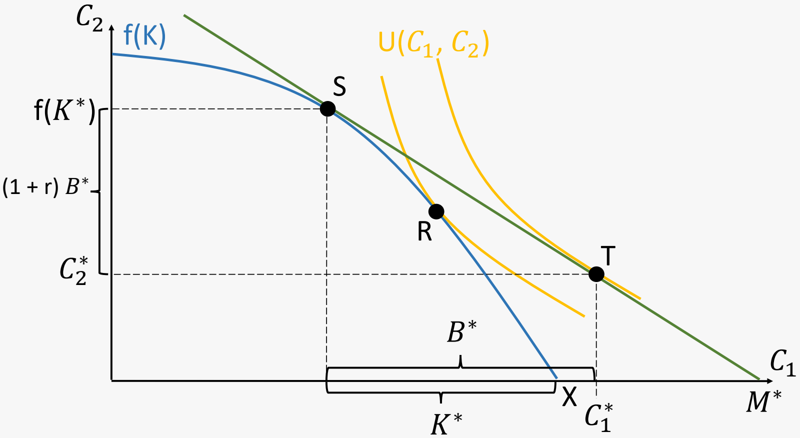


3. Point T defines the consumption decision and is independent of production, once we know M^*.



Capital markets expand the feasible points

4. Utility at point T is greater than at point R, and is a Pareto optimum.



Fisher's model implications

1. **Separation theorem** The production decision is independent of household preferences and initial capital endowment.
    - $f'(K) - 1 = r$.
2. The optimal production decision maximizes wealth and net present value.
    - Wealth $M^* = \frac{f(K)}{1+r} + C_1 - B$.
    - Net present value $= \frac{f(K)}{1+r} - K$.
3. The optimal consumption decision depends on wealth.
    - Production and interest rate only matter as it impacts wealth.
4. This equilibrium is a Pareto optimum
    - No two households could make a mutually beneficial trade.
    - Aggregate production is maximized.
    - No one's utility could be increased without decreasing someone elses.

What else might be important in this model?

1. How could/should this model be extended?

2. What are the limitations of this model?

Extensions of the model

All four results hold even if there are

1. More than two periods.
2. Different capital and consumption goods.
3. Joint ownership of production across households.

This analysis is partial equilibrium

1. It holds fixed r.
2. It is poorly suited to study intertemporal allocations.
3. Solow (1956) model can be incorporated to study capital accumulation.
4. Overlapping generation models Carmichael (1982), Barro (1974), Burbidge (1963), some inconsistency of laissez-faire allocation and social planner.

Basic two period model

How much debt should a business have?

We want to investigate optimal debt issuances

1. How do we build a model to investigate debt issuances?

2. What is the minimum structure needed to gain insights into this problem?

3. What is the key **tradeoff**?
    - Benefit: debt can increase capital.
    - Benefit: debt can increase dividends.
    - Cost: pay back with interest next period.

Cost and benefit of debt

B is debt (bonds, borrowing).

1. Benefit: debt can increase capital or dividends.

$$B = K + D - X \tag{9}$$

- K is capital (investment) used to produce $f(K)$.
- D is dividends (what we consume now).
- X is initial cash on hand (exogenously given).

2. Cost: pay back with interest next period.

$$(1 + r)B/(1 + r) \tag{10}$$

- Pay back $(1 + r)B$, but do so next period.
- r is the interest rate.

Basic model moving forward relabeled dividends and debt

A firm chooses its dividend and debt policies to maximize the value of the firm, which is consumption today plus discounted consumption tomorrow:

$$max_{B,D} \qquad D + \frac{f(K) - (1+r)B}{1+r} = D + \frac{f(X+B-D) - (1+r)B}{1+r} \qquad (11)$$

1. B is debt.
2. Capital is $K = X + B - D$.
3. D is dividends (what we consume now).
4. X is initial cash on hand (exogenously given).
5. r is the interest rate.

Marginal benefit equals marginal cost

A firm chooses its dividend and debt policies to maximize the value of the firm

$$max_{B,D} V = D + \frac{f(X + B - D) - (1 + r)B}{1 + r} \tag{12}$$

First order condition with respect to debt B

$$\partial B : \quad \frac{f'(X + B - D)}{1 + r} - \frac{1 + r}{1 + r} = 0 \tag{13}$$

$$\underbrace{\frac{f'(X + B - D)}{1 + r}}_{\text{marginal benefit}} = \underbrace{\frac{1 + r}{1 + r}}_{\text{marginal cost}}$$

$$\rightarrow f'(K) = 1 + r$$

Marginal benefit equals marginal cost

Firm chooses its dividend and debt policies to maximize the value of the firm

$$max_{B,D} V = D + \frac{f(X+B-D)-(1+r)B}{1+r} \tag{14}$$

First order condition with respect to dividends D

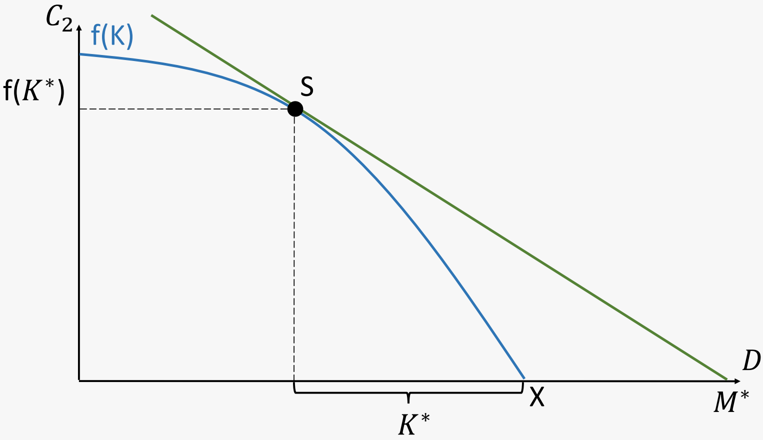
$$\partial D: \quad 1 - \frac{f'(X+B-D)}{1+r} = 0 \tag{15}$$

$$\underbrace{1}_{\text{marginal benefit}} = \underbrace{\frac{f'(X+B-D)}{1+r}}_{\text{marginal cost}}$$

$$\rightarrow f'(K) = 1 + r$$

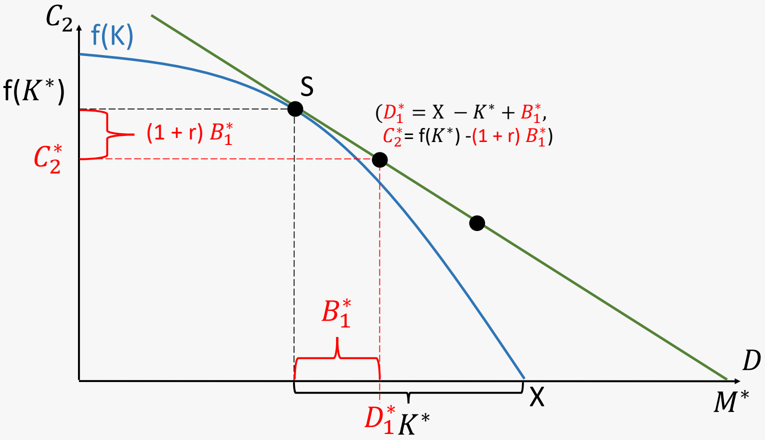
Capital is determined but not dividends or debt

1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.



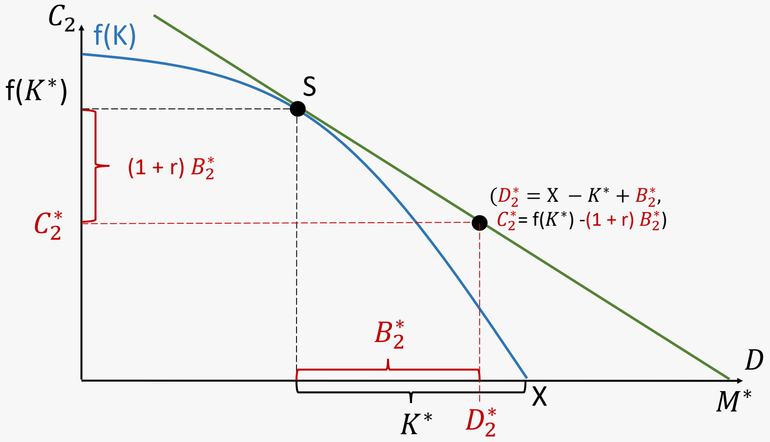
Capital is determined but not dividends or debt

1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.



Capital is determined but not dividends or debt

1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.



Modligani-Miller in our basic model

1. The optimal debt and dividend policies are indeterminate!

2. Value remains constant with an increase in debt and higher dividend payments (or the reverse).

3. Of course, this is not the end of story because there are taxes.

What else might be important in this model?

1. What interest rate matters for investment? Long-run or short-run?

2. How would depreciation be included in the model?

- Example: Operation twist.

Expected Utility with CARA utility

Do firms always maximize firm value?

Do firms always maximize profits?

Most of the economics literature focuses on firm value maximization, but the reality is more complicated (Jensen and Meckling, 1976; Smith and Stulz, 1985).

1. We want to investigate agency problems between managers and stock holders (who want firm value).

2. Consider two potential agency problems
    - Different incentives (e.g., empire building) for the manager.
    - Different risk preferences for the manager (e.g., risk averse).

Do firms always maximize profits?

Managers choose investment to maximize their utility, which consists of their wealth, firm value, and their benefits from empire building.

$$u = w_0 + \alpha \mu_V(K) - \frac{1}{2}\rho\sigma^2(K) + g(K) \tag{16}$$

- w_0 external wealth.
- $\mu_V(K)$ expected value of the firm depends on investment K.
- α weight that firm value enters manager's utility.
- ρ risk aversion parameter.
- $\sigma^2(K)$ variance of firm value, which depends on investment K.
- $g(K)$ benefit from empire building, $g'(K) > 0, g''(K) < 0$.

This simple formula can be derived from CARA utility and a normal distribution of firm value or CRRA utility and a log normal distribution of firm value.

- e.g., CARA utility, $U = -e^{-\rho(w_0 + \alpha V(K) + g(K))}$.

Modeling compensation packages of managers

Now, allow shareholders to compensate managers to align incentives.

1. Effective ownership δ through accumulation of stock and options net of dispositions.
    - To account for managers having other incentives (e.g., empire building).
2. Compensation convexity through vega, ν—such as option grants.
    - To account for managers being more risk averse than shareholders.
3. Together, these features update manager's utility

$$u = w_0 + (\alpha + \delta)\mu_V - \frac{1}{2}(\rho - \nu)\sigma^2 + g(K) \tag{17}$$

What else might be important in this model?

1. How would personal taxes such as dividend taxes affect this model?

2. How would mergers and acquisitions be considered in this model?

Agency model—objective function with dividend taxes

Let τ_d be the dividend tax rate.

$$w_0 + (1 - \tau_d)\delta\mu - \frac{1}{2}(\rho - \nu)\delta_0^2(1 - \tau_d)^2\sigma^2 \tag{18}$$

With dividend taxation, how might compensation committees might want to adjust their recommendations?

1. Hypothesis 1: Higher dividend taxes may require compensation committees to increase δ to get the same incentive alignment.
2. Hypothesis 2: Higher dividend taxes may allow compensation committees to decrease ν to get the same risk preference alignment.

Empirical evidence of personal taxes and CEO compensation

Using the previous model, or something similar, the following research investigates the role of taxes on firm behavior/compensation.

1. Arnemann, Buhlmann, Ruf, and Voget (2022) find higher income taxes on CEOs lowers firm performance.
2. Bennett, Coles, and Wang (2020) find income taxes are *not* paid by the CEO.
3. Coles, Sandvik, and Seegert (2020) find that personal taxes and different compensation incentives provide different incentives for M&A activity and ultimately performance.

- Though very important, now going to go back to ignoring these concerns.

Adding corporate taxes to our two period model

Do corporate taxes distort investment decisions?

Adding corporate taxes to our basic model

We want to investigate whether/how corporate income taxes distort investment.

1. Consider investment from equity issuances E and the tradeoff between today and tomorrow:

   - Cost: $-E$ today.
   - Benefit: higher profits tomorrow $f(X+E)$, where $K = X + E$.

Does the corporate income tax τ_c distort this tradeoff for firms?

Adding corporate taxes to our basic model with equity financing

Shareholders choose equity E to maximize value V, by trading off less income now with higher profits tomorrow.

$$max_E \qquad V = -E + \frac{(1-\tau_c)f(X+E)}{1+r} \tag{19}$$

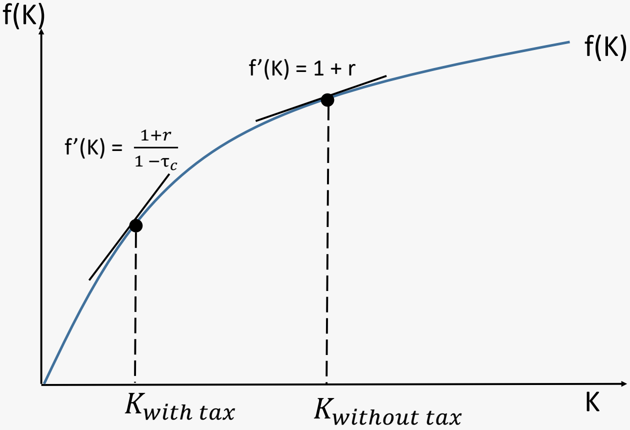
Take the first-order condition

$$\partial E: \qquad -1 + \frac{(1-\tau_c)f'(K)}{1+r} = 0 \tag{20}$$

$$\rightarrow f'(K) = \frac{1+r}{1-\tau_c}$$

- Corporate taxes have a large distortion!

Distortions to investment from corporate taxes and equity financing



Adding corporate taxes to our basic model with debt financing

2. Investment could come from debt B that creates a tradeoff between more production tomorrow and payment with interest tomorrow:

   - Cost: $(1+r)B$ tomorrow.
   - Benefit: higher profits tomorrow $f(X+B)$.

Does the corporate income tax τ_c distort this tradeoff for firms?

- Let $\gamma \in [0,1]$ be the percent of debt costs that are tax deductible.

Corporate taxes in the model with debt investment

Shareholders choose B to maximize firm value trading off higher profits and more debt

$$max_B \qquad V = \frac{(1-\tau_c)\left[f(X+B) - \gamma(1+r)B\right] - (1-\gamma)(1+r)B}{1+r} \qquad (21)$$

- Let $\gamma \in [0, 1]$ be the percent of debt costs that are tax deductible.

Corporate taxes in the model with debt investment

Take the first-order condition

$$\partial B : \quad \frac{(1-\tau_c)\left[f'(K) - \gamma(1+r)\right] - (1-\gamma)(1+r)}{1+r} = 0 \tag{22}$$

$$\rightarrow f'(K) = \gamma(1+r) + (1-\gamma)\frac{1+r}{1-\tau_c}$$

- If $\gamma = 1$, then there is no distortion from corporate taxes if debt is the marginal source of investment.
- If $\gamma = 0$, then there is a large distortion of corporate taxes (Hall and Jorgenson 1967).

What else might be important in this model?

1. Depreciation schedules for tax purposes relative to economic depreciation.

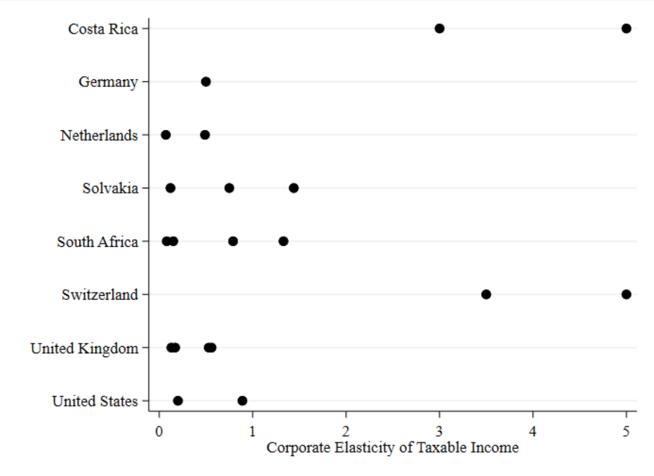
2. How would we empirically test whether corporate taxes distort investment or taxable income?

Empirical estimates—How elastic are firms?

Use changes in tax rates from tax schedules (bunching).

- Gruber and Rauh (2007); Coles, Patel, Seegert, and Smith (2021); Dwenger and Steiner (2012); Lediga, Riedel, and Strohmaier (2019); Krapf and Staubli (2020); Bukovina, Lichard, Palguta, and Zudel (2021); Bachas and Soto (2021); Massenz and Bosch (2022).

Empirical estimates of the distortions of corporate taxes



- Bunching estimates of distortion of corporate taxes.

User cost of capital and effective tax rate ETR

How do tax depreciation methods distort investment?

Tax rules on investment

We want to understand how tax rules impact investment.

1. Firms have depreciation allowance a_t at time t on a dollar of investment.
   - Accelerated depreciation or any other schedule.
   - $\int a_t dt = 1$, and $z \equiv \int e^{-\rho t} a_t dt$.
   - Capital depreciates exponentially at rate δ; $K_t = Ee^{-\delta t}$.
   - Firms may receive a contemporaneous investment tax credit of κ per dollar invested.
2. We could do this in continuous time (and most of the literature does), but we can get a lot from just a two period model.
3. Modeling goals: explore how to use the user cost of capital and effective tax rate ETR to investigate tax distortions.

Investment with depreciation and discount rate

To follow the continuous time literature, we can update the model as below:

$$max_E \qquad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E \tag{23}$$

1. E is equity.
2. c is after-tax cost of putting a dollar into the firm.
3. $K = X - D + E$ is capital in period 2.
4. δ is the capital depreciation rate.
5. ρ is the rate at which owners discount after-tax flows.
6. z is the depreciation allowance.
7. κ is the investment tax credit.

Tax adjusted user cost of capital

First order condition

$$\partial E : -c + \frac{(1-\tau_c)f'(K)}{\delta+\rho} + \tau_c z + \kappa = 0 \tag{24}$$

$$f'(K) = \frac{c-\kappa-\tau_c z}{1-\tau_c}(\rho+\delta)$$

- The right side is the user cost of capital.

Tax adjusted user cost of capital

User cost of capital

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \quad (25)$$

1. If $c = 1$, then this is the Hall-Jorgenson tax-adjusted user cost of capital.
2. If $\tau_c = 0$, the rental cost of capital is $c(\rho + \delta)$, which reflects the time value of money and cost of depreciation interacted with the expenditure level.
3. Everything else, is the impact of taxation.

Consider different depreciation methods

1. Let investments be depreciated at economic depreciation, then $z = \delta/(\rho + \delta)$.

2. Let investments be expensed immediately, then $z = 1$.
    - If $\kappa = 0$ and $c = 1$, then we can see that immediate expensing returns us to the cost of capital without taxes.

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \qquad (26)$$
$$= \frac{1 - \tau_c}{1 - \tau_c}(\rho + \delta)$$
$$= \rho + \delta$$

- Obviously, depreciation is more complicated than either of these scenarios in practice.

Effective tax rates (ETR)

Consider the investment level induced by the condition:

$$\rho \equiv [f'(K) - \delta](1 - ETR). \tag{27}$$

that defines the effective tax rate

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta}. \tag{28}$$

The ETR provides the "single" tax rate that produces the same investment level given by a combination of tax parameters.

Combinations of tax parameters

User cost of capital

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{29}$$

Now, we can consider different combinations of tax parameters and find the effective tax rate.

- Economic deprecation $z = \delta/(\rho + \delta)$.
- Immediate expensing $z = 1$.
- Equity financed investment $c = 1$.
- Debt financed investment $c < 1$.
- investment tax credit κ.

Effective tax rates (ETR) scenario 1

General model

$$max_E \qquad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E \tag{30}$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{31}$$

Scenario 1: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Immediate expensing of investment $z = 1$.

Scenario 1 user cost of capital:

$$f'(K) = \rho + \delta \tag{32}$$

Effective tax rates (ETR) scenario 1

Use scenario 1 user cost of capital and the formula for ETR to find the ETR

Scenario 1 user cost of capital $c = 1$, $\kappa = 0$, and $z = 1$:

$$f'(K) = \rho + \delta \tag{33}$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta}. \tag{34}$$

$$= \frac{\rho + \delta - \delta - \rho}{\rho + \delta - \delta}$$

$$= 0.$$

- In this scenario immediate expensing leads to no distortions!

Effective tax rates (ETR) scenario 2

General model

$$max_E \qquad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta+\rho} + \tau_c zE + \kappa E \tag{35}$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1-\tau_c}(\rho+\delta) \tag{36}$$

Scenario 2: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Depreciation allowances equal to economic depreciation $z = \delta/(\rho+\delta)$.

Scenario 2 user cost of capital:

$$f'(K) = \rho/(1-\tau_c) + \delta \tag{37}$$

Effective tax rates (ETR) scenario 2

Use scenario 2 user cost of capital and the formula for ETR to find the ETR

Scenario 2 user cost of capital $c = 1, \kappa = 0, z = \delta/(\rho + \delta)$:

$$f'(K) = \rho/(1 - \tau_c) + \delta \tag{38}$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta}. \tag{39}$$

$$= \frac{\rho/(1 - \tau_c) + \delta - \delta - \rho}{\rho/(1 - \tau_c) + \delta - \delta}$$

$$= \tau_c.$$

- In this scenario economic depreciation leads to a distortion that increases with the corporate tax rate.

Effective tax rates (ETR) scenario 3

General model

$$max_E \qquad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E \tag{40}$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{41}$$

Scenario 3: Consider a firm with

1. Debt-financed investment $c = 1 - \tau_c$.
    - $c = (r(1-\tau_c) + \delta)/(\rho + \delta) = 1 - \tau_c$
    - $c = 1 - \tau_c$ with the simplification, $\delta = 0$, $\rho = r$.
2. No investment tax credit $\kappa = 0$
3. depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Scenario 3 user cost of capital:

$$f'(K) = \rho \tag{42}$$

Effective tax rates (ETR) scenario 3

Use scenario 3 user cost of capital and the formula for ETR to find the ETR

Scenario 3 user cost of capital $c = 1 - \tau_c$, $\delta = 0$, $\kappa = 0$ and $z = \delta/(\rho + \delta)$:

$$f'(K) = \rho \tag{43}$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho - \rho}{\rho} = 0 \tag{44}$$

- If there is debt finance and tax depreciation is economic deprecation there is no distortion.
- If there is debt finance and tax depreciation that is more rapid than economic depreciation, then the ETR is **negative**.

ETR can be used to measure/investigate distortions

ETR $= 0$ implies no distortion from taxation. This occurs when

1. Equity financing of investment and immediate expensing.
2. Debt financing of investment and depreciation is allowed at economic depreciation.
   - In both cases, all investment costs are deductible.

ETR and user cost of capital are helpful to understand when and how taxes distort investment.

What else might be important in this model?

1. What other depreciation schedules might we want to model and how would they change investment behavior?

2. What other behavior may depreciation schedules change?

Empirical evidence

Use changes in depreciation (via bonus depreciation) to look at affect on investment.

1. Early literature found large investment responses (House and Shapiro, 2008; Zwick and Mahon, 2017).
    - Use differences across industries in investment.
    - Manufacturing longer lived capital than software developers and thus have more benefits from bonus depreciation.

2. These estimates might be too large though if competition is not taken into account (Patel and Seegert, 2020).
    - Investment is a strategic variable and responses to tax incentives depend on how competitive or concentrated the market is.
    - Industries with longer lived capital likely also more concentrated due to large fixed costs.

Personal taxes in the two period model

Do dividend taxes distort investment?

Do dividend taxes distort investment behavior?

Firms choose dividends and equity policy D and E, to maximize firm value by trading off dividends or equity today and production tomorrow.

$$V = D - E + \frac{f(X - D + E) + X - D + E}{1 + r} \quad (45)$$

- Today firms can pay D dividends or ask for equity E.
- Tomorrow capital $K = X - D + E$ produces $f(K)$ and the firm liquidates and gives back K.[1]

[1] This is important because of rules on dividend taxes between equity and retained earnings (Chetty and Saez, 2010).

Do dividend taxes distort investment behavior?

Dividend taxes make dividends less valuable today, maybe firms will **over-invest**. But maybe not?

$$V = (1-\tau_d)D - E + \frac{(1-\tau_d)[(1-\tau_c)f(X-D+E) + X - D] + E}{1+r} \tag{46}$$

- Dividend taxes τ_d are paid on dividends today, but not rebated to equity.
- Dividend taxes paid on production and retained earnings tomorrow, but not equity.
- For comparison, model corporate income tax τ_c.

Consider a model with equity and dividend taxes

Let $D = 0$, and firms choose equity E to maximize firm value.

$$max_E \qquad V = -E + \frac{(1-\tau_d)[(1-\tau_c)f(X+E)+X]+E}{1+r} \tag{47}$$

Take the first-order condition

$$\partial V / \partial E = -1 + \frac{(1-\tau_d)[(1-\tau_c)f'(X+E)]+1}{1+r} = 0 \tag{48}$$

$$f'(X+E) = \frac{r}{(1-\tau_d)(1-\tau_c)}$$

- Dividend tax rate distorts investment similar to corporate taxes.

Consider a model with dividends and dividend taxes

Let $E = 0$, and firms choose dividends D to maximize firm value.

$$max_D \qquad V = (1-\tau_d)D + \frac{(1-\tau_d)[(1-\tau_c)f(X-D) + X - D]}{1+r} \tag{49}$$

$$\partial V/\partial D = (1-\tau_d) - \frac{(1-\tau_d)[(1-\tau_c)f'(X-D)+1]}{1+r} = 0 \tag{50}$$

$$(1-\tau_c)f'(X-D) + 1 = \frac{(1-\tau_d)(1+r)}{(1-\tau_d)}$$

$$f'(X-D) = \frac{1+r}{(1-\tau_c)}$$

- Dividend tax rate drops out—no distortion.

New view vs old view—matter of firm type

Whether dividend taxes distort investment decisions seem to depend on whether the firms are issuing equity or paying dividends.

1. Old view: **distortion**. Cash constrained firms; $D = 0$ and $E > 0$,
2. New view: **no distortion**. Cash rich firms; $D > 0$ and $E = 0$,
3. Cash intermediate firms; $D = 0$ and $E = 0$.
    - Ignore because not that interesting.

New view vs old view—empirical evidence

- Chetty and Saez (2005) document
  1. Dividends increased after the dividend tax cut of 2003.
     - Seems at odds with new view.
  2. The adjustment was rapid.
     - Seems at odds with old view, because supply mechanism would take longer.

- Gordon and Dietz (2008) and Chetty and Saez (2010) propose an agency model based on Jensen and Meckling (1976).

- Yagan (2015) finds that despite increased dividend payments there was no change to corporate investment or employee compensation.
  - Consistent with the new view—but a puzzle, where did the money come from?

- Ohrn and Seegert (2019) include M&A into the model and show it reconciles all of the empirical findings.
  - The model is also consistent with evidence on M&A behavior around 2003.

Comparative statics

How does inflation distort investment?

Remember inflation?

Want to know whether inflation affects real investment.

1. Interest rates r should adjust for inflation π.
   - Irving Fisher 1930 noted nominal interest rates should rise one-for-one with inflation $dr/d\pi = 1$.
2. Interest rates affect real investment.
   - Interest rates are nominal while capital is a real variable (Darby, 1975; Feldstein, 1976).
3. Plausible that inflation, therefore, affects real investment.
4. Modeling tool:
   - Show comparative statics using total differentiation of equilibrium condition.

What do we need in our model?

1. What is the key question?
    - How/does inflation distort the tradeoff and therefore investment?
    - Inflation makes money borrowed today not as costly to payoff tomorrow.
2. What is the key tradeoff we are interested in?
    - Interested in investment.
    - Benefit is more production $f(K)$.
    - Cost is cost of investment (borrowing to look at inflation) rB.

Demand of capital

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \quad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B \tag{51}$$

1. Capital is increasing with borrowing $K = X + B$.
2. After-tax profits net interest payments $(f(K) - rB)(1 - \tau_c)$.
3. Inflationary gains on the stock of nominal borrowing. πB.
4. Capital depreciation δK and value of tax deduction for depreciation $\tau_c \delta K$.
    - Is this last piece necessary for the model?

Partial and general equilibrium analysis

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \qquad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B \tag{52}$$

Take the first-order condition

$$\partial V / \partial B = (1 - \tau_c) f'(K) - (1 - \tau_c) r + \tau \delta - \delta + \pi = 0 \tag{53}$$

$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c}$$

- At this step, you might say, inflation does lead to more investment—BUT, this is ignoring that r changes with π.
- Said differently, we need to think general equilibrium not partial equilibrium.

General equilibrium analysis

To think general equilibrium, we need to allow multiple variables to change at the same time.

First-order condition

$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c} \tag{54}$$

What variables do we think change?

1. Let capital change K.
2. Let interest rates change r.
3. Let inflation change π

Comparative statics: capital wrt inflation

Totally differentiate the first-order condition (allowing K, π, and r to change). Note $f''(K) < 0$.

$$f'(K) - \delta = r - \frac{\pi}{1-\tau_c} \tag{55}$$

$$\text{totally differentiate} \quad f''(K)dK = dr - \frac{d\pi}{1-\tau_c}$$

$$\frac{dK}{d\pi} = -\frac{1}{-f''(K)}\left(\frac{dr}{d\pi} - \frac{1}{1-\tau_c}\right)$$

- How capital responds to inflation depends on how much interest rates respond to inflation.

$$\frac{dK}{d\pi} = \begin{cases} > 0, & \text{if} \quad \frac{dr}{d\pi} < \frac{1}{1-\tau_c} \\ = 0, & \text{if} \quad \frac{dr}{d\pi} = \frac{1}{1-\tau_c} \\ < 0, & \text{if} \quad \frac{dr}{d\pi} > \frac{1}{1-\tau_c} \end{cases}$$

How do interest rates change with inflation?

1. To know whether investment increases or decreases with inflation we need to know how interest rates change with inflation.

2. Remember, Fisher 1930 noted $dr/d\pi = 1$.

3. To solve it in general equilibrium, we need to consider supply of capital (lenders).

4. Lenders receive real after-tax returns (individual tax rate t):

$$\tilde{r} = r(1 - t) - \pi \tag{56}$$

Supply of capital comparative statics

Lenders receive real after-tax returns (individual tax rate t):

$$\tilde{r} = r(1-t) - \pi \tag{57}$$

Totally differentiate

$$d\tilde{r} = (1-t)dr - d\pi \tag{58}$$

$$\frac{d\tilde{r}}{d\pi} = (1-t)\frac{dr}{d\pi} - 1$$

$$\frac{d\tilde{r}}{d\pi} = \begin{cases} < 0, & \text{if} \quad \frac{dr}{d\pi} < \frac{1}{1-t} \\ = 0, & \text{if} \quad \frac{dr}{d\pi} = \frac{1}{1-t} \\ > 0, & \text{if} \quad \frac{dr}{d\pi} > \frac{1}{1-t} \end{cases}$$

For capital markets to clear supply = demand

Let $\tau = t$ Market supply of capital

$$\frac{d\tilde{r}}{d\pi} = \begin{cases} < 0, & \text{if} \quad \frac{dr}{d\pi} < \frac{1}{1-t} \\ = 0, & \text{if} \quad \frac{dr}{d\pi} = \frac{1}{1-t} \\ > 0, & \text{if} \quad \frac{dr}{d\pi} > \frac{1}{1-t} \end{cases}$$

Market demand of capital

$$\frac{dK}{d\pi} = \begin{cases} > 0, & \text{if} \quad \frac{dr}{d\pi} < \frac{1}{1-\tau_c} \\ = 0, & \text{if} \quad \frac{dr}{d\pi} = \frac{1}{1-\tau_c} \\ < 0, & \text{if} \quad \frac{dr}{d\pi} > \frac{1}{1-\tau_c} \end{cases}$$

- For capital markets to clear $\frac{dr}{d\pi} = \frac{1}{1-\tau_c}$.

Implications

1. Interest rate increases more than inflation $\frac{dr}{d\pi} = \frac{1}{1-\tau_c}$.
2. Interest rate adjusts for inflation **AND** tax implications.
3. Capital is unaffected by inflation $\frac{dK}{d\pi} = 0$.

What else might be important in this model?

1. Did modeling Capital depreciation δK and value of tax deduction for depreciation $\tau_c \delta K$ matter?

2. Could you redo the analysis abstracting from depreciation, or setting $\delta = 0$?

3. Could we now test this given current increases in inflation?

The envelope theorem

How do corporate tax rates affect total value in the economy?

Total value (welfare) in the economy

- So far, we have considered firm value solely.
- For tax policy, we may want to consider additional affects of corporate taxes.
- What do we need to include in the model to capture total value in the economy?
- How do corporate taxes distort welfare?

How do corporate taxes distort welfare?

There are several candidates

1. Change firm behavior due to changes in capital K.
2. Change tax reporting ρ of firms.
    - Let fraction μ of firm reporting be a shift in value and $1 - \mu$ be a resource cost.
    - Examples of shifting are transfers to accounting firms or shifting money into a tax preferred vehicle.
    - Examples of resource costs include exerting effort in a law library figuring out credits and deductions.
    - Does it matter if it is a resource cost or shifting?
3. Change taxable income $Y(K, \rho)$ and thus tax revenues.

Follow the analysis in Coles, Patel, Seegert, and Smith (2021) as an application of the envelope theorem.

Firms maximize firm value

Write firm value in second period value

$$max_{K,\rho} \qquad V = -rK + (1-\tau_c)(f(K)-\rho) + \rho - c(\rho) \qquad (59)$$

- Firms choose capital K and amount of reporting ρ.
- Taxable income $Y = f(K) - \rho$.
- Cost of reporting $c(\rho)$ and benefit of reporting $\tau_c \rho$.
- Profits $f(K)$.

Total value in the economy

Total value in the economy.

$$
\begin{aligned}
TV =& [-rK + (1-\tau_c)(f(K)-\rho) + \rho - c(\rho)] && \text{Firm value} \qquad (60)\\
&+ \tau_c(f(K)-\rho) && \text{Tax revenue}\\
&+ \mu c(\rho) && \text{Cost of reporting}
\end{aligned}
$$

Cost of reporting to the extent that it shifts to accounting and law firms and is not a resource cost.

- Pure shift of value $\mu = 1$.
- Pure resource cost $\mu = 0$.

How does total value change with the corporate tax rate?

1. We want to take the derivative $\frac{\partial TV}{\partial(1-\tau_c)}$.
2. Note, that capital and shifting are functions of the corporate tax rate.
3. Do we have to take $\partial K/\partial(1-\tau_c)$ and $\partial \rho/\partial(1-\tau_c)$ everywhere?
4. No, we can apply the envelope theorem!

Rewrite total value in terms of taxable income $Y(K,\rho)$.

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho) \tag{61}$$

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K,\rho) - Y(K,\rho) \qquad \text{direct effect}$$

$$+\tau_c \frac{\partial Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)} \qquad \text{indirect effect}$$

Envelope theorem application

Rewrite total value in terms of taxable income $Y(K, \rho)$.

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho) \qquad (62)$$

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K,\rho) - Y(K,\rho) \qquad \text{direct effect}$$

$$+\tau_c \frac{\partial Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)} \qquad \text{indirect effect}$$

Why did we not take the derivative of K and ρ inside of the square brackets but did outside?

Showing the envelope theorem

Why did we take the derivative of Y and ρ outside of the square brackets but not inside?

Consider the derivative of K and ρ in firm value

$$V = -rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho) \tag{63}$$

$$\frac{\partial V}{\partial(1-\tau_c)} = Y - r\frac{\partial K}{\partial(1-\tau_c)} + (1-\tau_c)\frac{Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + (1-\tau_c)\frac{Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \frac{\partial \rho}{\partial(1-\tau_c)} - c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)} \tag{64}$$

Showing the envelope theorem

Consider the derivative of K and ρ in firm value

$$\frac{\partial V}{\partial(1-\tau_c)} = Y - r\frac{\partial K}{\partial(1-\tau_c)} + (1-\tau_c)\frac{Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + (1-\tau_c)\frac{Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \frac{\partial \rho}{\partial(1-\tau_c)} - c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)} \tag{65}$$

Rearrange

$$\frac{\partial V}{\partial(1-\tau_c)} = Y + \underbrace{\left(-r + (1-\tau_c)\frac{Y(K,\rho)}{\partial K}\right)}_{=0 \text{ bc FOC}}\frac{\partial K}{\partial(1-\tau_c)} \tag{66}$$

$$+ \underbrace{\left((1-\tau_c)\frac{Y(K,\rho)}{\partial \rho} + 1 - c'(\rho)\right)}_{=0 \text{ bc FOC}}\frac{\partial \rho}{\partial(1-\tau_c)}$$

$$= Y$$

How does total value in the economy change with tax rates?

1. Taking money from firms?
    - No, the direct effect is zero—transfer from firms to the government.
2. Firm value?
    - No, the indirect effect of firm value is zero by the envelope theorem.
3. Tax revenue changes?
    - Yes.
4. Tax reporting?
    - Yes, if reporting is shifting $\mu > 0$.

This motivates understanding the mechanisms of tax reporting.

What are other examples of the envelope theorem?

1. Shepard's lemma: in a cost minimization problem the derivative with respect to the interest rate is capital and the derivative with respect to wages is labor.

2. Le Chatelier's principle: labor is more responsive to a change in the wage in the long run than in the short run because in the long run the firm can adjust its capital.

3. Deadweight loss Harberger (1964) "triangle."

Sufficient statistics

How do corporate tax rates affect total value in the economy?

Is there one parameter that can tell us about distortions in the economy?

1. Feldstein (1999) argued that the elasticity of taxable income with respect to the corporate tax rate captured the welfare gain/cost from taxes.
    - The elasticity of taxable income as a sufficient statistic for welfare analysis.
    - For more on sufficient statistics see Chetty (2009).
2. Many papers have qualified this statement (Doerrenberg, Peich, and Siegloch, 2017; Coles, Patel, Seegert, and Smith, 2021).
3. Follow the analysis in Coles, Patel, Seegert, and Smith (2021) to
    - Demonstrate sufficient statistics.

Total value in the economy

Start again with total value in the economy.

$$\begin{aligned} TV = & [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] && \text{Firm value} \quad (67) \\ & + \tau_c Y(K,\rho) && \text{Tax revenue} \\ & + \mu c(\rho) && \text{Cost of reporting} \end{aligned}$$

Is there one parameter that would be sufficient for understanding $\partial TV / \partial (1-\tau_c)$?

Derive welfare costs of corporate taxes

Take the derivative of total value with respect to the net-of-tax rate.

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K,\rho) - Y(K,\rho) + \tau_c \frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} + \mu c'(\rho) \frac{\partial \rho}{\partial(1-\tau_c)} \tag{68}$$

Rearrange to get terms that we like (note $c'(\rho) = \tau_c$).

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \left(\frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} \frac{1-\tau_c}{Y} + \mu \frac{\partial \rho}{\partial(1-\tau_c)\frac{1-\tau_c}{Y}} \right) \tag{69}$$

Rewrite in terms of elasticities

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \left(e_Y - \mu e_\tau \right) \tag{70}$$

Is the elasticity of taxable income a sufficient statistic?

We know that

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \left(e_Y - \mu e_\tau\right) \tag{71}$$

1. If the cost of tax adjustments is a resource cost ($\mu = 0$), then
   - the elasticity of taxable income is a sufficient statistic for the distortion to total value.
2. If the cost of tax adjustments is partially a transfer ($\mu > 0$), then
   - the elasticity of taxable income is an upper bound on the distortion to total value
   - the distortion to total value decreases with the tax adjustment elasticity e_τ

Structural parameter estimation

How elastic are firms?

Structural estimation

Structural estimation connects the model directly to the empirical estimation.

1. This can be as simple as running an OLS regression.
2. Alternatively, it could require estimation via general method of moments, maximum likelihood, or simulated method of moments.
3. What are the benefits?
   - Identifies exactly what your empirical estimation is telling you.
   - Allows for extrapolation out of sample for policy "experiments."

Let's go through an example following Agostini, Bertanha, Bernier, Bilicka, He, Koumanakos, Lichard, Massenz, Palguta, Patel, Perrault, Riedel, Seegert, and Todtenhaupt (2022).

Standard model of firms with fixed cost

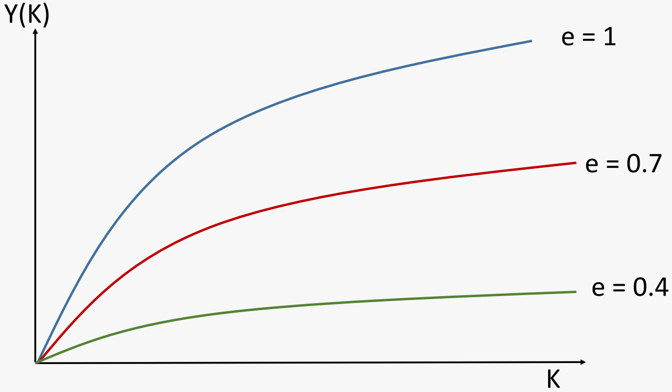
- Firm i chooses how much earnings to distribute as a dividend ($D_i \geq 0$) and how much equity to issue ($E_i \geq 0$).
- Those choices determine period 2 Capital: $K_{2,i} = K_{1,i} + E_i - D_i$.
- Profits net depreciation costs:

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$

- Fixed costs $F_i = exp(X_F' \beta_F + \nu_F)$, normally distributed.
- Productivity $A_i = exp(X_A' \beta_A + \nu_A)$, normally distributed.
- Parameter of interest e tells us how elastic firms are.

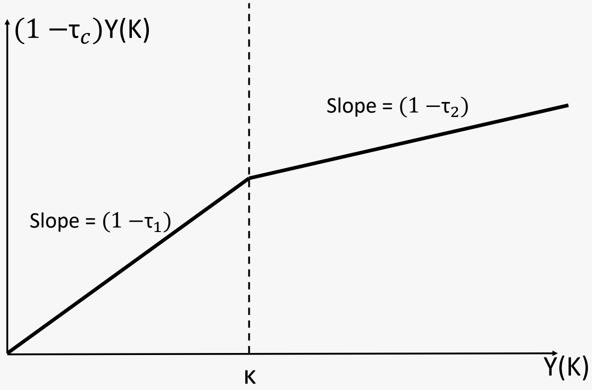
Parameter e tells us how elastic firms are

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$



The tax schedule with a kink in it

Profits below κ taxed at rate τ_1 and profits above κ taxed at rate τ_2, where $\tau_1 < \tau_2$.



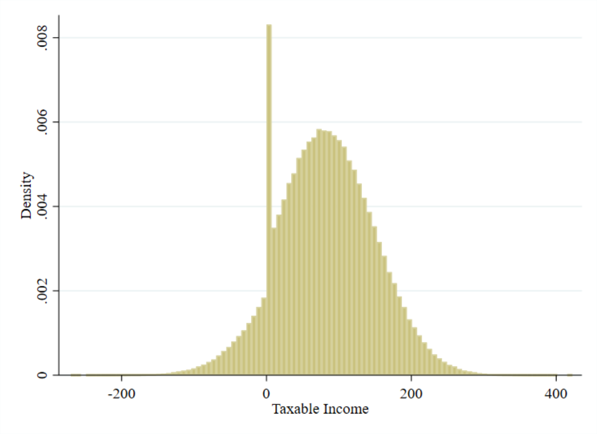
Firms maximize shareholder value subject to the corporate tax schedule

Profits below κ taxed at rate τ_1 and profits above κ taxed at rate τ_2, where $\tau_1 < \tau_2$.

$$
\begin{aligned}
max_{K_{2,i}} \quad V = & D_i - E_i + \frac{K_{2,i}}{1+r} \\
& + \mathbb{1}(Y_i(K_{2,i}) \leq \kappa)\frac{(1-\tau_1)Y_i(K_{2,i})}{1+r} \\
& + \mathbb{1}(Y_i(K_{2,i}) > \kappa)\frac{(1-\tau_1)\kappa + (1-\tau_2)(Y_i(K_{2,i}) - \kappa)}{1+r}
\end{aligned}
$$

Taxable income is given by piecewise function with bunching based on e

$$Y_i^* = \begin{cases} \frac{1+e}{e} r^{-e}(1-\tau_1)^e A_i - F_i, & A_i \leq \underline{A}(e,\kappa,\tau_1) \\ \kappa, & \underline{A}(e,\kappa,\tau_1) < A_i < \overline{A}(e,\kappa,\tau_2) \\ \frac{1+e}{e} r^{-e}(1-\tau_2)^e A_i - F_i, & A_i \geq \overline{A}(e,\kappa,\tau_2) \end{cases}$$



Two step estimation: using variation in productivity and fixed cost

- Fixed costs $F_i = exp(X_F' \beta_F + \nu_F)$, normally distributed.
- Productivity $A_i = exp(X_A' \beta_A + \nu_A)$, normally distributed.

Case 1: $Y < \kappa$

$$
\begin{aligned}
Y &= \underbrace{\frac{1+e}{e} r^{-e} (1-\tau_1)^e}_{\lambda_1} A_i - F_i \\
&= X_A \beta_A \lambda_1 + X_F \beta_F (-1) + \lambda_1 \nu_A - \nu_F
\end{aligned}
$$

Case 2: $Y > \kappa$

$$
\begin{aligned}
Y &= \underbrace{\frac{1+e}{e} r^{-e} (1-\tau_2)^e}_{\lambda_2} A_i - F_i \\
&= X_A \beta_A \lambda_2 + X_F \beta_F (-1) + \lambda_2 \nu_A - \nu_F
\end{aligned}
$$

Two step estimation: using variation in productivity and fixed cost

1. The conditional expectation when $Y < \kappa$ (similarly for $Y > \kappa$)

$$\mathbb{E}[Y|X_A, X_F, Y < \kappa] = X_A(\beta_A\lambda_1) + X_F(-\beta_F) - w_1 \frac{\phi\left(\frac{X_A(-\beta_A\lambda_1)+X_F(\beta_F)}{w_1}\right)}{\Phi\left(\frac{X_A(-\beta_A\lambda_1)+X_F(\beta_F)}{w_1}\right)}$$

2. Ratio of coefficients on productivity:

$$\frac{\beta_A\lambda_1}{\beta_A\lambda_2} = \frac{\frac{1+e}{e} r^{-e}(1-\tau_1)^e}{\frac{1+e}{e} r^{-e}(1-\tau_2)^e} = \frac{(1-\tau_1)^e}{(1-\tau_2)^e}$$

3. Derive the parameter e

$$e = ln\left(\frac{\beta_A\lambda_1}{\beta_A\lambda_2}\right)\frac{1}{ln(1-\tau_1) - ln(1-\tau_2)}$$

Implications, extensions, and limitations

Firms respond to tax rates

$$\varepsilon_i = e\left(1 + \frac{F_i}{Y_i}\right) \tag{72}$$

1. Implication: Firms with higher taxable incomes have lower elasticities.
   - Consistent with empirical evidence in Devereux et al. (2014).
2. Extension: Include profit shifting.
3. Limitation: Assume that e is a structural parameter that captures all firm responsiveness.

Conclusion

Modeling helps focus the reader on the tradeoff in your work

Modeling takeaways:

1. Begin with the tradeoff you are interested in studying.
    - You can start from many models that already exist.
    - Define the players, strategies, and payoffs.
2. Add in features of interest.
    - Depreciation schedules
    - Mergers and acquisitions
    - Tax reporting
    - Inflation
3. Let your model ebb and flow.
    - Add features to test whether conclusions are robust.
    - Delete features that are robust.
4. Have fun and be creative.

References

Agostini, Claudio, Marinho Bertanha, Govindadeva Bernier, Katarzyna Bilicka, Yuxuan He, Evangelos Koumanakos, Tomas Lichard, Gabriella Massenz, Jan Palguta, Elena Patel, Louis Perrault, Nadine Riedel, Nathan Seegert, Maximilian Todtenhaupt. 2022. The elasticity of corporate taxable income across countries. *Working paper* .

Arnemann, Laura, Florian Buhlmann, Martin Ruf, Johannes Voget. 2022. The effect of taxes on ceo performance. *Working paper* .

Bennett, Benjamin, Jeffrey L. Coles, Zexi Wang. 2020. How executive compensation changes in response to personal income tax shocks (who pays the ceos income taxes?). *Working paper* .

Chetty, Raj. 2009. Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annual Review of Economics* **1**(1) 451–488.

Chetty, Raj, Emmanuel Saez. 2005. Dividend taxes and corporate behavior: Evidence from the 2003 dividend tax cut. *Quarterly Journal of Economics* **120(3)** 791–833.

Chetty, Raj, Emmanuel Saez. 2010. An agency theory of dividend taxation. *American Economic Journal: Economic Policy* **2(3)** 1–21.

Coles, Jeffrey L., Elena Patel, Nathan Seegert, Matthew Smith. 2021. How do firms respond to corporate taxes? *Journal of Accounting Research* **n/a**(n/a).

Coles, Jeffrey L., Jason Sandvik, Nathan Seegert. 2020. Taxes, managerial compensation, and the market for corporate control. *Working paper* .

Darby, Michael R. 1975. The financial and tax effects of monetary policy on interest rates. *Economic Inquiry* **13**(2) 266–276.

Devereux, Michael P, Li Liu, Simon Loretz. 2014. The elasticity of corporate taxable income: New evidence from uk tax records. *American Economic Journal: Economic Policy* **6**(2) 19–53.

Doerrenberg, Philipp, Andreas Peich, Sebastian Siegloch. 2017. The elasticity of taxable income in the presence of deduction possibilities. *Journal of Public Economics* **151** 41–55.

Feldstein, Martin. 1999. Tax avoidance and the deadweight loss of the income tax. *Review of Economics and Statistics* **81**(4) 674–680.

Feldstein, Martin S. 1976. Inflation, income taxes, and the rate of interest: A theoretical analysis. *American Economic Review* **66**(5) 809–820.

Gordon, Roger, Martin Dietz. 2008. Dividends and taxes. Alan J. Auerbach, Daniel N. Shaviro, eds., *Institutional Foundations of Public Finance: Economic and Legal Perspectives*. Cambridge, MA: Harvard University Press, 203–224.

Harberger, Arnold C. 1964. The measurement of waste. *The American Economic Review* **54**(3) 58–76.

House, Christopher L., Matthew D. Shapiro. 2008. Temporary investment tax incentives: Theory with evidence from bonus depreciation. *American Economic Review* **98**(3) 737–68.

Jensen, Michael C., William H. Meckling. 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* **3**(4) 305–360.

Ohrn, Eric, Nathan Seegert. 2019. The impact of investor-level taxation on mergers and acquisitions. *Journal of Public Economics* **177** 104038.

Patel, Elena, Nathan Seegert. 2020. Does market power encourage or discourage investment? evidence from the hospital market. *The Journal of Law and Economics* **63**(4) 667–698.

Smith, Clifford W., Rene M. Stulz. 1985. The determinants of firms' hedging policies. *Journal of Financial and Quantitative Analysis* **20**(4) 391–405.

Yagan, Danny. 2015. Capital tax reform and the real economy: The effects of the 2003 dividend tax cut. *American Economic Review* **105**(12) 3531–63.

Zwick, Eric, James Mahon. 2017. Tax policy and heterogeneous investment behavior. *American Economic Review* **107**(1) 217–48.