Rushing to Opportunity: Gentrification, Entrepreneurship, and City Growth*

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Abstract

The growth of many industries, neighborhoods, and cities differ with some growing slowly and others experiencing rapid change—i.e., rushes. To explain these differences and illuminate the mechanisms of growth, we develop a model centered on a new trade-off between time-varying fundamentals and time-invariant—but rank-dependent—opportunities. Early growth depends on the opportunities new entities provide, whether from accumulating entrepreneurship human capital in firms, real estate in neighborhoods, or land in cities. Our model can explain the existence of rushes and their size. We provide suggestive empirical evidence on industry growth, neighborhood change, and city growth consistent with the model predictions.

Keywords: Opportunities; rapid change; entrepreneurship; neighborhood change; city growth.

JEL Classification: J24, R12, R14, L16.

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1 Introduction

The growth of some neighborhoods, industries, and cities is slow and steady, while others experience periods of rapid change. Neighborhoods rise, fall, segregate, or gentrify rapidly. Industries experience booms or busts. Cities may see large population swings when there is a sudden influx of new residents. To rationalize these patterns, we propose a novel model where *agents trade off time-varying benefits and time-invariant—but rank-dependent—opportunities. Benefits* depend on the stock of other agents in an industry, neighborhood, or city at any given time; whereas *opportunities* are locked-in by the time an agent arrives there. To fix ideas, consider the choice between two industries, an established, large one and a new, small one. Each pays wages that increase as its size grows over time, and thus the established industry initially pays more while the new one pays less. If this was the end of the story, agents would not move to the new industry and forfeit higher wages in the established one. Yet, moving early may confer extra gains that agents seek to exploit. The new industry may, for example, offer rank-dependent opportunities that compensate for the lower initial wages: workers in new industries generally have a higher rank in the younger firms and are given a broader portfolio of tasks, which allows them to accumulate more human capital.

The addition of rank-dependent opportunities to standard models of cities, human capital, and gentrification provides novel insights on their growth and solves a persistent coordination problem in those models. Without rank-dependent opportunities, individuals are unwilling to move to new entities—a general term that can refer to a firm, a city, or a neighborhood, among others—before the time-varying benefits in these new entities equal those in established ones. In standard models of urban growth, for example, this coordination problem causes models to fail to explain the slow growth of new cities before old cities become grossly oversized (Anas, 1992).[1] With rank-dependent opportunities, individuals are generally willing to forgo some time-varying benefits in order to secure higher opportunities. The creation and growth of cities, industries, and neighborhoods, therefore, depend critically on the shape of these opportunities.[2]

[1]Henderson and Venables (2009) provide a new urban model to solve the coordination problem using fixed assets such as land.

[2]Our proposed mechanism builds on insights from timing games beginning with Smith (1974), Fudenberg, Gilbert, Stiglitz and Tirole (1983), and Fudenberg and Tirole (1985). Our model follows others that have used timing games to investigate games of investment with option value (Boyer, Gravel and Lasserre, 2004; Oprea, Friedman and Anderson, 2009; Anderson, Friedman and Oprea, 2010) and games of preemption and wars of attrition (Park and Smith, 2008; Anderson, Park and Smith, 2017).

We show that when opportunities exist, are initially increasing, and eventually decrease with respect to agents' arrival rank, the new entity must grow suddenly as agents rush to opportunity: many agents rationally move simultaneously from the established entity to the new one, causing sudden growth in the latter. The intuition for this result is that there are benefits to being an early arriver—but not being first. When opportunities first increase with arrival rank, agents rationally wait for others to move as they have no incentive to preempt the rush. The unique rush size must then occur where the marginal opportunity equals the average opportunity and agents have no incentives to outlast the rush. Through a series of comparative statics, we show how the model can explain several counterintuitive empirical growth phenomena. For example, quite surprisingly, a rush occurs earlier and is larger when the opportunities are spread across more agents. The model also predicts that growth will be slower when the difference in opportunities between ranks increases.

When opportunities are monotonically decreasing in agents' arrival rank, the new entities grow slowly, without rushes. The intuition is similar to that of a game of war of attrition: in equilibrium, the larger benefit of being the kth entrant compared to being the $k+1th$ entrant must be offset by the cost of receiving lower benefits in the new entity for a longer duration, thus leading to slow growth. Said differently, if there is a large benefit to being first relative to second, for example, then entrants are willing to move earlier to secure that larger benefit. Yet, growth will be slow and there cannot be a rush because agents would have an incentive to preempt the rush: being the $k-1th$ mover just before the rush allows for discontinuously greater opportunities forever, but for lower time-varying income for an infinitesimal period, which cannot be an equilibrium.

We illustrate the general idea by applying our framework to settings where rank-dependent opportunities matter: cities, industries, and neighborhoods. We add rank-dependent opportunities to standard models and show that in the updated models, the opportunity function characterizes growth and can produce rushes.

First, starting with cities, we show how the question of city formation may be viewed as the outcome of a rush.[3] Being early to a new place allows one to secure the best location, which may compensate for moving from a larger place that provides higher income to a smaller one that provides lower income. Using the creation of Lexington and Louisville, Kentucky, as an example, we illustrate the comparative statics prediction that

[3] Glaeser (2013) provides historic examples of booms and busts in the US land and housing markets.

a rush occurs earlier and is larger when the opportunities are spread across more agents. The land surrounding Louisville is heterogeneous in suitability to build, due to distance to the river and rapids and the presence of excessive swampland. By constrast, Lexington is located in the center of the inner Bluegrass Region, which provides vast amounts of fertile and homogeneous land. Put differently, Louisville's opportunity function was relatively steep while Lexington's was relatively flat. This implies that the latter should have an earlier and larger rush, a prediction borne out in the data.

Second, we highlight the existence of rushes in neighborhoods by analyzing gentrification. Being an early mover to a less affluent neighborhood provides opportunities—especially in terms of real estate appreciation—yet is risky if the neighborhood does not improve. Our model predicts that *rapid* change occurs more systematically in initially more risky places where the opportunity function is flatter. Using block-level information for New York, Boston, and Philadelphia, we show these predictions are borne out in the data: conditional on gentrification, rapid change occurs more frequently in more risky places that are closer to the CDB (Rosenthal and Maloney, 2022), have an older housing stock, lower housing prices, and lower average income.[4] Opportunities provide a novel explanation for faster change in more risky areas.

Last, we posit that industry and entrepreneurship growth can partly be explained by the tradeoff workers face between higher incomes and opportunities to gain entrepreneurship human capital. First, consider finance and the technology industry ('tech') in the 1980s. Finance is well established, while tech was just beginning. Wages were higher in finance, but individuals could be a higher rank in tech firms. As a result of these differences in pay and opportunities, our model predicts a rush in the tech industry—consistent with the tech boom in the 1990s. Second, consider differences in entrepreneurship across cities. Liang, Wang and Lazear (2018) suggest that it is easier to gain human capital in areas that are relatively young, where it is more likely for a young person to be of higher rank within their firm. Consistent with our model, we find that younger counties have more entrepreneurship. For example, Burlington, VT, with a median age of 36.5, had 9 new business application per 1,000 people between 2005 and 2020, while Provo, UT, with a median age of 25, had 16.

The remainder of the paper is organized as follows. Section 2 lays out the general

[4] Our results complement the literature on long-run housing cycles of neighborhood change (Rosenthal, 2008; Rosenthal and Ross, 2015) by looking at a mechanism that explains rapid short-run changes.

model. It characterizes its solution and the existence and size of rushes. Sections 3 to 5 present our three examples and the suggestive empirical evidence. Section 6 concludes. Formal proofs and details on our data are relegated to the appendix.

2 Model

We want to understand how opportunities that arise in new entities shape their creation, rapid transformation, and growth. To this end, we first construct a general model in Section 2.1 and provide its solution in Section 2.2. We then apply it to three examples in Sections 3 to 5.

2.1 General setup

There are two entities, denoted 1 and 2.[5] Time is continuous and indexed by t. There is a potential total population $\overline{N} \equiv \int_0^\infty \widetilde{N}(t)\mathrm{d}t$ of homogenous individuals, where $\widetilde{N}(t)$ denotes the mass at time t. Let $N(t) \equiv \widetilde{N}(t)/\overline{N}$ be the normalized population. We assume its growth $\mathrm{d}N(t)/\mathrm{d}t \equiv \dot{N}(t) \equiv \eta(t) > 0$ is exogenous and known by all individuals. It can be nonmonotonic, i.e., there can be periods of faster or of slower growth.

Let $N_1(t)$ and $N_2(t)$ denote the populations in entities 1 and 2 at time t, respectively. All individuals begin in entity 1, defined as the *established entity*, and choose a time $\tau \in [0, \infty)$ to move to entity 2, defined as the *new entity*. Let $\dot{N}_2 \equiv m(t) \geq 0$ denote the population growth in entity 2, which is entirely due to individuals that move from 1 to 2. Since total population growth is $\eta(t)$, it must be that $\dot{N}_1(t) = \eta(t) - m(t)$.

The mass of individuals who have moved to entity 2 before time τ defines the population there at τ: $N_2(\tau) \equiv M(\tau) = \int_0^\tau m(t)\mathrm{d}t$, and $\dot{M}(t) = m(t)$. Individuals who move to entity 2 at time τ are given a rank equal to the mass of individuals $M(\tau)$ who moved before them. We explain later under which assumptions we can index individuals unambiguously by their rank. For simplicity, individuals who move to entity 2 are assumed to subsequently stay there forever. This implies that $m(t) \geq 0$. We derive conditions below to ensure this assumption is satisfied.[6]

[5] This assumption is imposed for convenience and can be easily relaxed to the more general case with numerous entities. See Seegert (2015) for the context of cities.

[6] This assumption is unnecessary if all individuals who move at time τ receive the average opportunity. In that case, individuals have no incentives to move back to location 1. It is also unnecessary if there exist large fixed costs of moving, i.e., if the cost of moving back to location 1 is sufficiently large.

Individuals move between entities based on utility differences. Each entity $i = 1, 2$ provides utility from two sources: (i) rank-independent but time-varying benefits $Y_i(t)$, hereafter *income*; and (ii) rank-dependent but time-invariant opportunities, hereafter *opportunities*. We assume income in i depends on i's population but not directly on time: $Y_i(t) = Y_i(N_i(t))$. We further assume it is continuously differentiable and subject to first economies and then diseconomies of scale as follows:

$$Y_i(0) = 0, \quad Y_i'(N_i) \gtreqless 0 \quad \text{for} \quad N_i \lesseqgtr \widehat{N}_i, \tag{1}$$

where $\widehat{N}_i$ defines the (unique) income-maximizing population of entity i. Assumptions (1) ensure that, as the population grows, there are benefits for individuals to concentrate in one entity but that it is not efficient for all individuals to indefinitely concentrate in the same entity. For ease of exposition, we assume that the total population at time zero is greater than entity 1's income-maximizing population: $N(0) > \widehat{N}_1$ so that $Y_1'(0) < 0$. Put differently, there are decreasing returns to population in the established entity 1, which provides incentives to leave it and move to the new entity. Yet, since $Y_2(0) = 0$, nobody would make the first move based on income alone, i.e., if there were no opportunities in the new entity.

Without loss of generality, we normalize opportunities in entity 1 to zero. Opportunities in entity 2 depend on individuals' ranks, i.e., on when they moved to that entity. We model them by an *opportunity function*, $R(M(\tau)) > 0$ for all $M(\tau)$. We assume that R is continuously differentiable and single peaked. The average opportunity between ranks M and $M + \Delta M$ is defined as $R_M(\Delta M) = (1/\Delta M) \int_M^{M+\Delta M} R(m)\mathrm{d}m$. Opportunities are eventually decreasing in rank but can initially be increasing:

$$R'(M) \gtreqless 0 \quad \text{for} \quad M \lesseqgtr \widehat{M}, \quad \text{and} \quad R(M) < R_0(M) \quad \text{for some} \quad M > \widehat{M}, \tag{2}$$

where $\widehat{M} \geq 0$ defines the (unique) opportunity-maximizing population of entity 2. Regularity assumptions (2) exclude opportunity functions that are always increasing, initially decreasing and then increasing, and initially increasing and then decreasing at an insufficient rate (we assume opportunities must eventually fall below those of the first movers to the new entity). Observe that the opportunity an individual secures by moving to entity 2 solely depends on his rank $M(\tau)$, which itself depends on the time τ of the move. After that, the opportunity is locked-in and stays constant over time.

An individual's life-time utility depends on his income and—conditional on whether and when he moves from 1 to 2 at time τ—his rank-dependent opportunity profile:

$$\begin{aligned} U\big(\tau, M(\tau)\big) &= \int_0^\tau \mathrm{e}^{-rt} Y_1\big(N(t) - M(t)\big)\mathrm{d}t \\ &\quad + \int_\tau^\infty \mathrm{e}^{-rt} Y_2\big(M(t)\big)\mathrm{d}t + \int_\tau^\infty \mathrm{e}^{-rt} R\big(M(\tau)\big)\mathrm{d}t \\ &= \int_0^\tau \mathrm{e}^{-rt} Y_1\big(N(t) - M(t)\big)\mathrm{d}t + \int_\tau^\infty \mathrm{e}^{-rt} Y_2\big(M(t)\big)\mathrm{d}t + \frac{R\big(M(\tau)\big)}{r}\mathrm{e}^{-r\tau}, \end{aligned} \tag{3}$$

where r denotes the discount rate. Observe that this utility function is differentiable in the time of the move, τ, for any differentiable profile $M(t)$. We show later that this is the case when opportunities are decreasing in time t, in which case there is slow growth of the new entity. Yet, in the general case there may be atoms in the distribution $M(t)$, which precisely occurs when there are periods of rushes.

We define rushes as the case where nobody moves until time τ (i.e., $m(t) = 0$ for $t < \tau$), whereas a mass ΔM of agents move simultaneously at time τ. We show later that there will be at most one rush in equilibrium, so that agents are either part of that rush at period τ or indifferent in moving sometimes after the rush. Assuming that all agents who move during a rush receive the same average opportunity, life-time utility is:

$$U\big(\tau, M(\tau)\big) = \int_0^\tau \mathrm{e}^{-rt} Y_1\big(N(t)\big)\mathrm{d}t + \int_\tau^\infty \mathrm{e}^{-rt} Y_2\big(M(t)\big)\mathrm{d}t + \frac{R_M\big(\Delta M\big)}{r}\mathrm{e}^{-r\tau}. \tag{4}$$

On top of assumptions (1) and (2), we impose two additional restrictions to focus on equilibria where population growth in entity 2 begins at an interior time ($t > 0$) and is nonnegative ($m(t) \geq 0$). First, we assume that the initial benefit of staying in entity 1 is greater than that of moving to entity 2, irrespective of the mass ΔM of agents that leave entity 1 at $\tau_0 = 0$.[7] Formally, $\lim_{\tau \to \tau_0} \frac{\mathrm{d}U(\tau, M(\tau))}{\mathrm{d}\tau} \big|_{\tau_0 = 0, M(\tau_0) = \Delta M} > 0$ for all $\Delta M \leq N(0)$. Using (3), this requires that:

$$Y_1\big(N(0) - \Delta M\big) - Y_2(\Delta M) > R(\Delta M) + \frac{\dot{R}(\Delta M)}{r}, \quad \text{for all} \quad 0 \leq \Delta M \leq N(0). \tag{5}$$

[7] Since agents are homogeneous, this implies that $m(0) = 0$. To derive conditions for this to hold, we consider what would happen if there was a single atom at $t = 0$. The latter implies that $M(0) > 0$, but the function M is smooth afterwards (there is no second atom). We can hence write a (right) derivative for life-time utility in τ and look at the limit as τ goes to zero.

We can view the left-hand side of (5) as the difference in the flow payments (the benefits) in entities 1 and 2, whereas the right-hand side is the continuation benefit from higher opportunities received by moving now versus moving later. Condition (5) states that the initial benefit of staying in entity 1 at $\tau_0 = 0$ is greater than that of moving to entity 2, irrespective of the mass ΔM of moving agents.

Second, we impose a condition that allows us to focus on the fundamental trade-off in the model: waiting in entity 1, which initially offers higher income, or moving to entity 2, thereby forfeiting income in the short run but benefiting from better rank-dependent opportunities in the long run. To this end, we impose two conditions on the utility function—a primitive of the model. We assume utility increases with the timing of the move, not taking into account changes in $M(t)$, whereas utility decreases with the mass of people that have moved before some point in time:

$$\frac{\partial U(\tau, M)}{\partial \tau} > 0, \qquad \text{and} \qquad \frac{\partial U(\tau, M)}{\partial M} < 0. \tag{6}$$

The first condition means that opportunities in entity 2 become more valuable over time, provided nobody moves to exploit them. This entices individuals ceteris paribus to remain in the established entity and not move too early. The second condition means that opportunities in entity 2 become less valuable at each point in time if more agents have moved there before that point and grabbed those opportunities. This entices agents ceteris paribus to not wait too long before moving to the new entity.[8]

2.2 Equilibrium

New entities provide opportunities. The price individuals pay for those opportunities is the difference between the higher income they would have received in the original entity and the lower income they accept in the new entity when moving there. In equilibrium, for agents to move the price of the opportunity must equal its benefit.

[Insert Figure 1 about here.]

To provide the intuition for the trade-offs, we start with a graphical example. The

[8]If conditions (6) do not hold, there may be a time when growth in 2 becomes negative. As stated before, we rule out this case.

simplified Figure 1 depicts the price and benefit of the opportunities in the new entity.[9] First, consider an individual who moves at time $\tau = 6$ for opportunity A. This individual receives an opportunity equal to 8 forever. The price of this opportunity is the income in the original entity (Y_1) net of the income (Y_2) and opportunity (R) in the new entity. Said differently, the area of the red triangle from (6,8), (6,16), and (18,8), which equals 48, quantifies the price paid by the individual for moving to the new entity. As the difference in incomes decreases between the two entities, eventually the income plus opportunity (which was locked in) in the new entity exceeds the income in the original entity (starting at $t = 18$ in our example). The benefit of the opportunity is then given by the area of the blue triangle (18,8), (30,8), (30,0), which equals 48 too. Similarly, an individual who moves at time 12 for opportunity B, pays the price of 27 for a benefit of 27. Observe that, in equilibrium individuals are indifferent between moving at $\tau = 6$ or $\tau = 12$, as required by (7). The growth $m(t)$ of entity 2 is the variable that determines in equilibrium the slopes of the net income and opportunity functions, with respect to time, such that the price equals the benefit.

Let us now more formally analyze the equilibrium. The model defines a game. A player's strategy is the time τ he moves from entity 1 to entity 2 (conditional on being in entity 1). Players receive payoffs according to equation (3), which depend on when the player moves to entity 2 (i.e., τ) and the distribution of when the other players move to entity 2 (i.e., $M(\tau) = \int_0^\tau M(t)\mathrm{d}t$). We now construct an equilibrium toward the goal of proving existence and uniqueness. It is fully characterized by $M(t)$ that implicitly defines a time of creation τ_1 for entity 2 with initial size $\Delta M_1 \geq 0$ at creation.[10] Before creation, we have by definition $m(t) = 0, \forall t < \tau_1$, and at time τ_1 we have $m(t) > 0$ for the first time. After creation, we focus on equilibria without *period of inaction*, defined as equilibria in which entity 2 continually grows once created.[11] Formally:

[9]Figure 1 has three simplifications. First, it gives income and opportunity functions that eventually hit zero, when in reality they have an asymptote there. Second, it disregards discounting. Last, it assumes time and the benefit of the opportunity stop at 30 instead of continuing forever.

[10]Since agents are homogeneous, we restrict the search to symmetric Nash equilibria. With a continuum of agents, looking for pure strategy Nash equilibria—where each individual deterministically picks a time τ to move—is equivalent to looking for mixed strategy equilibria (Sun, 2006)—where individuals mix across times on when to move to entity 2 according to some probability distribution $q(t)$. With a mass $\overline{N} \equiv 1$ of homogeneous agents over time, $q(t) = m(t)$, and $Q(t) = \int_0^t q(s)\mathrm{d}s = \int_0^t m(s)\mathrm{d}s \equiv M(t)$. The equilibrium is characterized by the cumulative distribution $Q(t)$. Following Anderson, Park and Smith (2017), off-equilibrium behavior can be specified such that all Nash equilibria are also subgame perfect.

[11]The equilibrium without periods of inaction is the only equilibrium that survives a trembling-hand refinement defined as a safe equilibrium (see Anderson, Park and Smith, 2017). It is unique, and an

Definition 1. *An equilibrium without periods of inaction is such that* $m(t) > 0$ *for all* $t \geq \tau_1$.

Entity 2 may be created in a 'smooth' way or suddenly via a rush. Assume a mass ΔM move at time τ. To uniquely index individuals by their rank, we assume in that case that: (i) ranks are randomly attributed among the simultaneous movers; and (ii) each individual in the rush receives the same average opportunity, which depends on the mass $M(\tau)$ of individuals who moved before the rush. An equilibrium can be constructed using a *no arbitrage condition* and a *boundary condition* for creation of the new entity. The no arbitrage condition is obtained by ensuring that individuals are indifferent between moving now or in the 'next period'.[12] The boundary condition is obtained by ensuring that no individual wants to preempt the rush.

We can prove the following result (see Appendix A.1 for details).

Theorem 1 (Existence and uniqueness of an equilibrium)**.** *There exists a unique* ε*-safe mixed strategy Nash equilibrium.*

Proof. The proof of Theorem 1 is shown in five steps in Appendix A.1. First, initially moving to the new entity is worse than staying, given the regularity condition in equation (5). Second, eventually the second entity is formed because population growth deteriorates income in the first entity. Third, there is a unique starting time where the benefits of moving to the second entity exactly equals the cost. Fourth, the implicit function theorem determines a unique growth pattern after the second entity is started. Note the indifference condition in equation (7) creates a differential equation $\partial U/\partial\tau = W_M m(t) + W_\tau = 0$ with a known solution. Fifth, this equilibrium has no period of inaction, satisfies the conditions for an $\varepsilon-$safe equilibrium, and no other equilibrium exists without period of inaction that is not $\varepsilon-$safe (Anderson, Park and Smith, 2017). □

extremal equilibrium such as this has the earliest starting time. Milgrom and Roberts (1994) suggest focusing on such extremal equilibria.

[12]Formally, $\partial U\big(\tau, M(\tau)\big)/\partial\tau = 0$ whenever $m(t) > 0$. Differentiating (3), using Leibnitz's rule, and rearranging, this condition is given by:

$$\mathrm{e}^{-r\tau}\left[Y_1\big(N_1(\tau) - M(\tau)\big) - Y_2(N_2(\tau)) - R(M(\tau))\right] - \frac{\dot{R}(M(\tau))}{r} = 0. \tag{7}$$

Recall that (3) is differentiable if M has no atoms, but that it is not generally differentiable at τ_1 when there is an atom. In that case, (7) must hold for $t > \tau_1$ (using a right derivative at $t = \tau_1$), whereas $U(\tau) \leq U(\tau_1)$ for all $\tau \in [0, \tau_1)$ in equilibrium (no agents wants to preempt the rush).

Having established existence of equilibrium, we can analyze how growth of entity 2 changes with the fundamentals of the opportunity function. Consider first how growth changes as the opportunity function becomes flatter—said differently, as the difference in opportunities individuals receive shrinks—i.e., $|R'(M)|$ becomes smaller.

Proposition 1 (Rate of growth)**.** *A flatter opportunity function causes the growth of entity 2 to be faster when individuals are moving to entity 2.*

Proof. Rearranging equation (7), evaluated at $\tau = 0$ and using $\dot{R}(M(\tau)) = -R'(M(\tau))m(\tau)$, directly yields

$$m(\tau) = \frac{r\left[Y_1\big(N_1(\tau) - M(\tau)\big) - Y_2(N_2(\tau)) - R(M(\tau))\right]}{-R'(M(\tau))}, \tag{8}$$

which is larger for smaller (absolute) values of $R'(M(\tau))$. □

Equation (8) shows that the growth rate of entity 2 increases as the opportunity function becomes 'flatter'. The result that small differences in opportunities encourage growth may at first seem counterintuitive. To understand it, remember that, in equilibrium, differences in opportunities must be offset by differences in incomes an individual receives. Therefore the benefit in terms of opportunities from being mover M rather than mover $M + \varepsilon$ must be offset by the fact that by moving earlier an individual foregoes more net income for a longer period. Consider the extreme case where everyone would receive the same opportunities, regardless of when they move to entity 2, i.e., a flat opportunity function. In this case, there cannot be any period of prolonged slow growth, because individuals that move early will always have an arbitrage opportunity by waiting. In other words, there is a first-mover penalty. The alternative is then that entity 2 'grows infinitely fast' by experiencing one giant rush of agents. This result highlights the importance of the shape of the opportunity function for determining the pattern of equilibrium growth.

2.3 Rushes

The model may deliver (interior) equilibria with slow growth, where $m(t) > 0$ over some interval and where the no arbitrage condition holds (e.g., as in Figure 1). However, the model may also deliver (corner) equilibria with rushes: nobody moves from 1 to 2 before τ_1, while at $t = \tau_1$ there is a sudden movement of a mass ΔM of agents to entity 2 which steadily grows afterwards ($m(t) > 0$ for $t \geq \tau_1$ as we look at equilibria without periods of inaction per Definition 1). Formally, a rush corresponds to an equilibrium that exhibits atoms. It can only occur when the trade-off between income and opportunities is non-monotone. Proposition 2 provides the conditions for entity 2 to be formed by a rush.

Proposition 2 (Existence of rushes)**.** *A necessary and sufficient condition for every equilibrium to involve a rush is that the opportunity function is nonmonotonic and initially increasing.*

Proof. See Appendix A.2. □

The intuition underlying Proposition 2 is depicted in Figure 2. First, consider a monotonically decreasing opportunity function, as in panel (a).[13] A rush does not exist in this case because there is always an incentive to preempt it. The reason is as follows. Individuals in the rush receive the average opportunity $R_0(\Delta M)$ (recall that entity 2 is still empty, hence $R_M(\Delta M) = R_0(\Delta M)$), and, when the opportunity function is decreasing, this is less than the initial opportunity $R(0)$. Thus, an individual who preempts the rush receives a discontinuously larger opportunity forever, but forfeits some income over an infinitesimal period. The former always dominates the latter and it follows that there can be no rush at $\tau_1 > 0$ in that case. It follows that when the opportunity function is monotonically decreasing, any equilibrium involves slow growth where $m(t) > 0$ and where the arbitrage condition holds.

[Insert Figure 2 about here.]

Second, consider a nonmonotonic and initially increasing opportunity function, as in panel (b) of Figure 2. The new entity cannot be created by slow growth in this case because there is an incentive to wait. No agent wants to move first because doing so entails both lower opportunities than that of the next mover *and* lower income. Hence, agents wait. As time goes by, income in entity 1 eventually decreases enough so that moving to entity 2 becomes more attractive. Yet, no agent will move individually since

[13] This is a generalization of the illustration in Figure 1 to the case where the opportunity and income functions are not linear and do not hit zero. Whereas we depict Figure 1 as a function of time t, we plot Figure 2 as a function of the mass $M(t)$ of agents who have moved by time t. As explained before, we can alternatively think about this as the agents' rank.

there cannot be slow growth (no agent has an incentive to be first since opportunities are initially increasing). Thus, there must be a rush.

Proposition 3 provides results on the timing and the size of the rush.

Proposition 3 (Timing and size of rushes)**.** *There is at most one rush, and it occurs at time τ_1, when entity 2 is created. In an equilibrium with a rush, the size ΔM_1 of a rush at time τ_1 is unique and occurs where the marginal opportunity equals the average opportunity, $R(\Delta M_1) = R_0(\Delta M_1)$, i.e., at the maximum average opportunity.*

Proof. We build on the intuition of Proposition 2 to show that there exists a rush at time τ_1. By Proposition 2, we know that, for there to be a rush, in equilibrium the opportunity function must be nonmonotonic and initially increasing. By the reasoning in Proposition 2, entity 2 cannot be created by slow growth if the opportunity function is initially increasing because there will be an incentive for individuals to wait. Therefore entity 2 must be created by a rush.[14]

The size of the equilibrium rush is such that there is no incentive to preempt the rush and there is no incentive to outlast it. Let ΔM_1 denote the mass of individuals who rush at time τ_1 to receive the average opportunity $R_0(\Delta M_1)$, graphed in panel (b) of Figure 2. Define three points: A, B, and C. Point A defines the peak opportunity $\widehat{M}$. Point B defines the point at which the average opportunity equals the marginal opportunity $R(\Delta M_1) = R_0(\Delta M_1)$. Point C defines the point at which the average opportunity equals the opportunity of the first mover: $R_0(M_{\text{max}}) = R(0)$. To ensure there is no incentive to preempt the rush, its size cannot be larger than point C.[15] To ensure there is no incentive to outlast the rush, its size cannot be smaller than point B. In equilibria without periods of inaction (recall Definition 1), individuals must receive the same opportunities being in a rush or moving right after the rush (recall $m(t) > 0$ for $t > \tau_1$; this is only possible if the individual is indifferent between rushing at τ_1 or moving afterwards). Otherwise individuals would be unwilling to move right after the rush. This occurs at the unique point where the marginal opportunity equals the average opportunity, $R(\Delta M_1) = R_0(\Delta M_1)$, point B in Figure 2. The value ΔM_1 is the unique equilibrium rush size for equilibria without periods of inaction.[16]

[14]Thus, in equilibrium $M(t)$ is discontinuous at most once and this occurs at τ_1 when $M(t) = 0$.

[15]Figure 2 is drawn with $M(\tau)$ on the x-axis, i.e., the size of the rush.

[16]For rush sizes larger than point B, the opportunity in the rush is strictly greater than the following opportunity, $R(\Delta M) < R_0(\Delta M_1)$ for $\Delta M > \Delta M_1$, causing there to be a period of inaction after the rush.

To show that there cannot be a second rush, we proceed by contradiction. Suppose that entity 2 is created at time τ_1 by a rush of size ΔM_1 and that a second rush occurs at time $\tau_2 > \tau_1$. The opportunities for the mass ΔM_2 of individuals in the second rush are given by the running average $R_{M_2}(\Delta M_2)$. These average opportunities during the second rush are strictly less than the ones right after the first rush, i.e., $R_{M_2}(\Delta M_2) < R(M_1) = R(\Delta M_1)$. The reason is that, at the time of the second rush, the average opportunity is decreasing because the equilibrium size of the first rush occurs at the maximum of the average opportunity. Hence, the second rush cannot be an equilibrium because individuals have an incentive to preempt it: by moving just before the rush, they receive discontinuously greater opportunities forever and less income for an infinitesimal period. □

We next consider consider how the size of a rush, ΔM, changes with the opportunity function. Proposition 3 demonstrates that the equilibrium size of the rush is determined by the peak of the average opportunity function. From this condition, several comparative static results follow. They are summarized in the following proposition.

Proposition 4 (Size of a rush). *The size of a rush is unaffected by a proportional or a level change in the opportunity function; i.e., $R_\phi(M) = \phi R(M)$ or $R_\phi(M) = R(M) + \phi$. The size of a rush decreases as the domain is compressed; i.e., $R_\phi(M) = R(\phi M)$, with $\phi > 1$; and it increases as the domain is stretched; i.e., $R_\phi(M) = R(\phi M)$, with $0 < \phi < 1$.*

Proof. See Appendix A.3. □

The first two comparative static results show that policies or events that cause a proportional or level shift of the opportunity function—such as a proportional tax or subsidy—will not affect the size of a rush. The third comparative static result shows that policies that spread the opportunities over fewer people will cause rushes to be smaller.

[Insert Figure 3 about here.]

Figure 3 provides examples of growth in entity 2 with proportional shifts and stretches (or compression) of the opportunity function. The size of the rush is given by the upward

The period of inaction is costly to the individuals that rush because they receive a lower income in entity 2 for a longer period. In equilibrium, the period of inaction is uniquely determined for a given rush size, such that the benefit individuals receive from a larger opportunity equals the cost incurred by receiving lower income for a longer period.

jump on the y axis, which begins at some time given on the x axis. Both panels depict growth with nonmonotone opportunity functions that cause entity 2 to be formed by a rush. Panel (a) of Figure 3 depicts growth with a rush and two comparative growth paths with proportional shifts in the opportunity function. The rush size does not change, but the timing of the rush and the growth after the rush are both affected by a proportional shift. For example, the long dashed line depicts growth with a proportional increase in opportunities, which in comparison to the baseline growth path, begins earlier and has slower equilibrium growth. Panel (b) of Figure 3 depicts growth with a rush and two comparative growth paths with a stretched or a compressed opportunity function, respectively. The stretched opportunity function (long dashed line) has a larger rush that occurs earlier, because the income in entity 2, after the rush, is larger due to the larger rush. After creation, the stretched opportunity function also leads to faster growth in the new entity because it is flatter (see Proposition 1).

We now illustrate the broad applicability of our general framework using three examples. The first example considers the decision by individuals to stay in an established city or move to a new one. The second considers the decision to stay in an established neighborhood or to move to a gentrifying one. The last example considers the decision to stay in an established firm or leave to start a new one or join a startup.

3 City growth

We investigate the growth of cities to highlight the existence of rushes and the predictions of Proposition 2. We consider the opportunity of claiming better land in new cities. The opportunity function differs across cities based on differences in the distribution of land quality. In some cases, this leads to a rush and, in other cases, to slow growth.

3.1 Motivating example: Lexington and Louisville, Kentucky

We compare the creation and growth of Lexington and Louisville, Kentucky, to highlight the potential for rushes given by Proposition 1. Louisville and Lexington, Kentucky, provide an interesting comparison of the creation and growth of cities because they are only seventy-five miles apart and were chartered within two years of each other (1780 and 1782, respectively). Anecdotally, land was an important determinant for the growth of these cities, as it is in many contexts of urban growth (Wade, 1996). Louisville is located next to the falls of the Ohio River, which were the only navigational barrier on the river at the time. The falls created a stopping point—a portage site—that disrupted the flow of traffic on the river. Like many other portage sites, this provided a natural place to develop a city (Bleakley and Lin, 2012). The land surrounding the falls is heterogeneous both in distance from the river and suitability to build, due to excessive swampland. This suggests Louisville's opportunity function was relatively steep. In contrast, Lexington is not on a navigable river. However, Lexington is located in the center of the inner Bluegrass Region, which provides vast amounts of fertile and homogeneous land. This suggests Lexington's opportunity function was relatively flat, $|R'(M(\tau))_{Lexington}| < |R'(M(\tau))_{Louisville}|$.

[Insert Figure 5 about here.]

Given the differences in slopes, Propositions 1 and 4 predict that Lexington will experience faster growth initially, even possibly being created by a rush. The model predicts, however, that Louisville will eventually become larger, due to its transport cost advantage. These predictions are corroborated by history, as shown in Figure 5. Lexington experienced rapid growth reaching a population of 18,410 by 1790, only eight years after being chartered. In the same year, Louisville's population was 200, despite being chartered two years earlier than Lexington. It took Louisville roughly 60 years for it to surpass Lexington in population.

3.2 Model

We build on the canonical model of cities of Henderson (1974), where city size is determined by the trade-off between agglomeration and congestion externalities. The benefit of living in a city is initially increasing with population but eventually decreases when the city becomes too large. How new cities form as population in established cities grows is a thorny issue in that literature because of coordination failure in migration decisions (the so-called 'migration pathology'): no one wants to be the only one to move to a new place—which provides very low utility initially—so cities teem and become grossly oversized before new ones can form.

The inclusion of the opportunity function solves this coordination failure. In the context of cities, it is natural to think of land as providing opportunities to early residents, though other examples surely exist. In particular, our opportunity function builds on the insights from monocentric city models (Alonso, 1964; Mills, 1967; Muth, 1969).[17] Early residents can acquire parcels of land closer to the center, which they value because of commuting costs or their option value. Individuals who arrive later must acquire parcels farther from the center and incur larger commuting costs but have larger parcels—reflecting the fact that density decreases from the center of the city. The growth pattern of cities therefore depends on the trade-off between higher incomes in established cities and opportunities in the form of more centrally located land in new cities.

Income function. Following Henderson (1974), we assume the aggregate production function for some composite good in city i is $F_i(N_i(t)) = A_i(t)N_i(t)$, where $N_i(t)$ denotes the population at time t (which is also the labor input to production). Production amenities (TFP) $A_i(t) = a_i N_i(t)^{\varepsilon_i}$ in cities are captured by the parameter a_i, and the agglomeration economies are subsumed by the term $N_i(t)^{\varepsilon_i}$, where $\varepsilon_i > 0$ captures matching, sharing, and learning externalities (see, e.g., Duranton and Puga 2004, and Abdel-Rahman and Anas 2004, for extensive reviews of micro-foundations of external economies of scale).

Given perfect competition, a worker's wage w_i equals the value of that person's marginal product of labor: $w_i(N_i(t)) = A_i(t) = a_i N_i(t)^{\varepsilon_i}$. Production of the composite good also generates pollution, $P(N_i(t)) = F_i(N_i(t))^{\gamma_i}$ where $\gamma_i > 0$ is a parameter.[18] Each individual is assumed to bear the average cost of the pollution produced within the city. Hence every individual receives (net) income—'consumption-based utility'—as follows: $Y_i(N_i(t)) = w_i(N_i(t)) - P(N_i(t))/N_i(t)$. This formulation ensures income is zero with no inhabitants and then strictly rises and falls with population, consistent with the inverted-∪ shape of Henderson (1974) and with our assumptions (1).

Opportunity function. In the canonical urban model, there are no rank-dependent opportunities. Hence, since initially $Y_1(N(0)) > Y_2(0) = 0$, an individual who moves to

[17] Recent work has extended Henderson's (1974) model to consider the creation and growth of new cities. For example, Henderson and Venables (2009) build a dynamic model with durable housing capital that avoids population swings in cities that arise in other urban models. Our model can be viewed as a version that replaces the role of housing capital with opportunities that exist in cities.

[18] This parametric example of pollution is consistent with Tolley (1974)'s description on p. 334: "The nature of pollution and congestion is that extra pollutants and vehicles do not shift production functions at all at low amounts, and extra amounts have increasingly severe effects as levels are raised until ultimately fumes kill and there are so many vehicles that traffic cannot move."

the new city 2 forgoes the higher income in the established city 1. Why would anyone be willing to move to the new city 2? In the canonical urban model of Henderson (1974), the short answer is: 'They won't.'[19] For a new city to form, it must offer the same income as the existing city. This requires city 1 to become grossly oversized so that income there falls to zero; only then will a new city form in a catastrophic manner.

In our framework, individuals move because the new city provides opportunities that the established one does not, and those compensate for foregone income. These opportunities depend on how many people have already moved to the new city. Specifically, we model the opportunities in city 2 as parcels of land that individuals claim as they enter the city. The first person who claims a parcel of land defines the center of the city. All subsequent entrants claim a parcel that is adjacent and next in order to the previously claimed parcel.

[Insert Figure 4 about here.]

We assume the city grows according to an Archimedean spiral, as Figure 4 shows. The use of an Archimedean spiral to model city growth builds on the monocentric city model pioneered by Alonso (1964), Mills (1967), and Muth (1969). Cities are often assumed to grow as a disc from their centers, sometimes as concentric circles. The use of a spiral allows the parcels to differ continuously in distance and area. Formally, assume that the radius is given by the angle θ and production in the central business district uses $2\pi + 1$ parcels of land, where π is the mathematical constant. Each parcel of land is assumed to be formed by two lines radiating from the spiral's pole with an angle of one between them.[20] Parcels differ in their area and distance from the center. Those who arrive later have to progressively claim parcels that are farther away from the center but are larger (see Figure 4).

The distance and area of a parcel can be derived as a function of rank $M(\tau)$, using a discrete analog depicted in Figure 4. The distance of the $M(\tau)$th parcel from the center of the city is given by the radius, which, given our assumptions, is simply $M(\tau)$. Individuals value the distance of their parcel to the center of the city because they incur a cost

[19]Anas (1992) provides an early example of a model of city growth in the absence of opportunities. The dynamics of Anas (1992) suggest large swings in population when a new city is formed. This model sparked research by Helsley and Strange (1994), Brueckner (2000), Cuberes (2009), and Henderson and Venables (2009), which included land developers or durable housing to solve this collective action problem.

[20]The assumption that the angle between the lines radiating from the spiral's poles is one implicitly assumes there are 2 π parcels of land for a given rotation.

of commuting, $M(\tau)^{\phi_i}$, where $\phi_i > 0$ is the elasticity of commuting costs with respect to rank (and, implicitly, distance). The cost of commuting is allowed to be nonlinear with respect to distance, capturing possible fixed costs and other deteriorating factors.[21]

Let $\alpha_i(M(\tau))$ denote the area of the $M(\tau)$th parcel, which is found by integrating between the two curves θ and $\theta - 2\pi$ in polar coordinates between the angles $M(\tau)$ and $M(\tau) + 1$.[22] This yields

$$\alpha_i(M(\tau)) = \frac{1}{2}\int_{M(\tau)}^{M(\tau)+1} \left[\theta^2 - (\theta - 2\pi)^2\right] d\theta = 2\pi M(\tau) - 2\pi^2 + \pi.$$

The area of the parcel an individual receives increases with rank, $\alpha'(\cdot) > 0$. Individuals receive increasing value from the area of their parcel, according to the function $\rho_i(\alpha_i(M(\tau)))$, such that $\rho_i'(\cdot) > 0$. The opportunity an individual receives in the new city is given by the (net) value of that person's parcel:

$$R_i(M(t)) = \rho_i(\alpha_i(M(\tau))) - c_i M(\tau)^{\phi_i}. \tag{9}$$

City 2 is formed by a rush when, according to Propositions 2 and 3, the opportunity function is nonmonotonic and initially increasing. Consider an area value function of the form $\rho_i = (\delta_i(\nu_i + \alpha_i(M(t))))^{\zeta}$. Further, let $\delta_i = 1/(2\pi)$ and $\nu_i = 2\pi^2 - \pi$, such that the value of land is normalized to zero for the person of rank 0, i.e., $\rho_i(0) = 0$. In this case, the size of the rush is given by

$$\Delta M = \left[\frac{1}{c_i}\frac{\zeta}{\zeta+1}\frac{\phi_i+1}{\phi_i}\right]^{\frac{1}{\phi_i-\zeta}}. \tag{10}$$

The size of the rush increases with the value of the area of a parcel, specifically ζ, and decreases with the costs of being farther from the center, specifically c_i and ϕ_i. This result suggests that cities with more heterogeneous land, where the cost of being further from the center is greater, will experience smaller rushes. The growth patterns of cities depend on the shape of the opportunity function. In this example, the parameter c_i characterizes how heterogeneous land is in city i. Consider two potential new cities that

[21] The model is general enough to allow for a host of factors that may deteriorate from the center, where the best land is. Historically, this could be that land farther from the center is swampier or less suited for building. The presence of rivers that have to be crossed as the city expands may also play a role.

[22] The area between two curves, r_1 and r_2, in between the angles a and b is given by $\frac{1}{2}\int_b^a (r_1^2 - r_2^2) d\theta$.

differ only in that one has relatively more heterogeneous land. In the city where land is heterogeneous, the value of being close is relatively greater, given by a high value of c_i. In this case, the equilibrium size of the rush is larger, the opportunity function is steeper and, from the general results, we know that growth will be slower.

4 Gentrification

Neighborhoods rise, stagnate, and decline over either long cycles or short periods (see Rosenthal and Ross 2015 for a recent survey). The latter—fast change such as neighborhood tipping or gentrification—have in particular attracted substantial media and policy attention, because their effects are highly visible and sometimes disruptive.

4.1 Motivating example: Pace of gentrification in NY, Boston, and Philadelphia

In this motivating example, we consider the tradeoff individuals face when choosing neighborhoods and how this can lead to either slow or fast gentrification. Through the lens of our model, the tradeoff is that high-priced neighborhoods offer newer housing stock, less crime, and better endogenous urban amenities, whereas low-priced neighborhoods offer opportunities in selection, price, and stronger potential asset price appreciation. Our general model shows that neighborhoods with nonmonotonic—or flatter—opportunity functions will experience faster growth, i.e., will gentrify more rapidly. As explained below, we view a flatter opportunity function as stemming from the presence of asset risk in neighborhoods. In a nutshell, our model predicts that riskier neighborhoods should experience more rapid change.

The fundamental tradeoff between high- and low-priced neighborhoods has changed in the US since the early 1990s and led to what is now generally viewed as central city revival and gentrification (Couture and Handbury, 2020). Before the 1990s, high crime rates in central cities discouraged movement there and pushed people to suburban locations. Starting in the early 1990s, crime unexpectedly and substantially fell. As a result, high-income individuals started to move to neighborhoods with high (yet decreasing) crime, older housing stock, and low average income (Ellen et al., 2019).[23]

[23]Other deep underlying factors—such as land use regulations, the elasticity of housing supply, chang-

Such moves were risky. Early movers in particular bear the risk that crime might not fall as much as expected and that other high-income earners may not follow. In case the neighborhood does not improve, early movers are then stuck with immobile assets that require substantial renovations and maintenance (recall the housing stock is old in those neighborhoods) and into which the may not want to invest more given the evolution of the neighborhood (Rosenthal and Ross, 2015). The asset risk is, however, balanced by better opportunities: early movers benefit from paying a lower price for their asset and the ability to pick their preferred item from a large choice set. Furthermore, asset price appreciation may be stronger in the future. Both faster subsequent price appreciation in riskier neighborhoods and the asset price risk itself result in a flatter opportunity function in our model, which thus predicts that we should see faster changes—rushes—in riskier neighborhoods (Proposition 1 of our model).

We operationalize these ideas in Section 4.2 below using a simple version of our model to explain rushes in gentrification. Consistent with our model we document in Appendix B.1 that—conditional on gentrifying—riskier neighborhood in Boston, New York, and Philadelphia experienced faster changes than less risky neighborhoods during the period 1990–2010. We follow recent work on the risk-return tradeoff in real estate and measure risk by the neighborhoods' distance to the central business district (CBD). Our data show the latter is strongly correlated with the age of the housing stock, with more recent housing being concentrated in the suburbs. Hence, the source of risk is mainly related to the age of the housing stock, as highlighted in our model.[24] We find that, conditional on gentrifying, neighborhoods with more risk as measured by the age of their housing stock were more likely to experience a rush. Across our three metropolitan areas, a gentrifying neighborhood with a housing stock built in 1964 (75th percentile) is 3.44 percent less likely to experience a rush than a gentrifying neighborhood with a housing stock built in 1940 (25th percentile), conditional on gentrifying and a number of

ing valuations of urban amenities, and the life-cycle of the housing stock—also drive the pace and scope of gentrification (Rosenthal, 2008; Rosenthal and Ross, 2015).

[24]Rosenthal and Maloney (2022, pp.2-4) find that neighborhoods closer to the CBD are riskier neighborhoods as measured by the volatility of year-over-year housing returns: *"home price appreciation rates decline almost monotonically with distance to the closest city center* [...] *home price volatility declines with distance from the CBD and is 13% lower for zipcodes 10 miles away from the center."* This price appreciation is the counterpart of higher non-systematic or idiosyncratic asset price risk in neighborhoods closer to the CBD. Standard asset pricing models predict that this risk must be compensated by larger price appreciation, which is precisely what Rosenthal and Maloney (2022, p.2) find: *"home prices appreciate faster in city centers, in part because of risk-return tradeoffs."*

other controls. This represents a 5.5% increase from the baseline probability of 62.7% of experiencing rapid change.

4.2 Model

Theoretically, we view gentrification as a form of a rush: slow changes in neighborhood fundamentals modify the value of rank-dependent opportunities, which can trigger rapid and profound transformations as gentrifiers rush into new neighborhoods.[25] We have shown that rapid growth occurs when opportunities are flatter. To illustrate Proposition 1, we propose a model where the opportunity function differs across neighborhoods, based on differences in housing stock and local amenities. In some cases, this leads to rapid growth (gentrification).

Income Function. Assume entities 1 and 2 are two neighborhoods and that $N_1(t)$ and $N_2(t)$ denote the population of highly educated and affluent residents living in those neighborhoods. Each resident derives utility from real income, w/P, and from neighborhood-specific amenities, A_i. Nominal income w is assumed to be the same for each resident and location—residents work somewhere else in the city and do not change jobs when moving between neighborhoods. The income function—the instantaneous utility—in neighborhood i at time t is given by $Y_i(N_i(t)) = A_i(N_i(t))\frac{w}{P_i(N_i(t))}$, where $A_i(N_i(t)) = a_i N_i(t)^{\varepsilon}$ are local 'amenities' (e.g., good schools, social networks, or specific public or private goods) that depend on the number of highly educated residents living in the neighborhood; and $P_i(N_i(t)) = \left[\overline{P}^{\gamma} + (c_i N_i(t))^{\gamma}\right]^{1/\gamma}$ is a cost-of-living index of neighborhood i.

Two comments are in order. First, observe that $A_i(0) = 0$, i.e., highly educated residents receive zero instantaneous utility from having no other highly educated in the neighborhood. Furthermore, $A_i' > 0$, i.e., amenities are increasing in N_i, with elasticity ε. This captures the idea that utility derived from social interactions, the provision of specific public and private goods, or school quality (which depends on peer effects and funding) all increase with the number affluent people in a neighborhood.[26]

[25]The slow changes in neighborhood fundamentals—the deep causes of gentrification—are not yet well identified, but they involve some combination of rising incomes, declining crime rates, deteriorating housing stock that gets torn down and renewed, change in industrial structure such as de-industrialization, and sometimes just "surprises."

[26]Clearly, there is an 'income risk' in the neighborhood, which depends on the decisions of the others.

Second, $P_i(0) = \overline{P}^{\gamma}$, where $\overline{P}$ is the baseline price level in the absence of highly educated residents in the neighborhood, and $P_i' > 0$; i.e., cost-of-living increases with the number of highly educated residents in the neighborhood. This may be because the prices of many consumption amenities increase with income and with land prices that drive up production costs. For example, the difference between drinking coffee made at home to high-end coffee shops selling pour-over coffees.[27]

To satisfy condition (1), we assume that $\gamma > \varepsilon$. When this holds, the amenity effect initially dominates, whereas the cost-of-living effect takes over when the number of highly educated residents increases enough.

Opportunity Function. As equation (3) shows, the decision to move from 1 to 2 depends on income—instantaneous utility—and rank-dependent opportunities. Without opportunities, no one wants to be the first to move since $Y_1(N(0)) > Y_2(0) = 0$.

In the context of gentrification, and in line with our motivating example, we think real estate provides opportunities of two sorts. First, there are potential monetary gains from real estate, depending on the arrival rank and subsequent price appreciation. If one arrives early prices are still relatively low, which offers opportunities for sizeable capital gains in the long run. Second, there are potential gains related to choice. Early movers to a neighborhood can pick from a larger set of properties that are up for sale, which may be especially important in areas with on old housing stock. Opportunities decrease as the choice set shrinks, so late movers have lower rank-dependent utility, as they pick from a smaller set and are potentially stuck with a less-preferred choice in the long run. To summarize, early movers can pick their preferred choice at a low price.

There is, however, a third channel that pulls in the opposite direction. Empirically, neighborhoods that may gentrify in the future have initially a relatively old housing stock (see, e.g., Rosenthal, 2008; Brueckner and Rosenthal, 2009). Since affluent residents prefer high-quality housing, moving to that neighborhood requires that agents either replace the old housing or must renovate the old place to suit their needs (Munneke and Womack, 2015). The value of that investment is highly idiosyncratic, since people have very different tastes for houses. In a nutshell, if an individual invests $1,000,000

This is at the heart of our model because the new entity must offer lower income than the established one.

[27] There are generally two effects at work. First, increasing production costs as land and other production factors become more expensive. Second, a less elastic demand when incomes are high, which allows producers to charge higher markups.

into renovating his dream loft, that dream might only be worth $500,000 to someone else. Early movers hence face more risk on the secondary market if the neighborhood ends up not gentrifying in the end. If, for whatever reason, they must move out of the neighborhood they need to resell the renovated (or new) place. If there is little or no demand from affluent agents who want to move there, they will not be able to recoup a large share of their sunk cost. In other words, the part of the cost they can recoup is increasing in the rank at which they move to the neighborhood.

Formally, let $m_i(M(\tau))$ and $\rho_i(M(\tau))$ be the matching value and the price (including renovation costs) of housing, which depend on the mass $M(\tau)$ of agents who arrived before τ in i. Naturally, $m_i' < 0$ as the choice set gets smaller. We further assume that $\rho_i' > 0$: waiting longer entails a higher price as properties appreciate and as the average quality of available housing decreases. Thus the price-adjusted matching value m_i/ρ_i is decreasing in arrival rank.

We model the direct asset price risk using a secondary-market value function. Let $\mathrm{e}^{-\zeta_i M(\tau)}$ be the expected share of investment lost as a function of arrival rank, where ζ_i is a neighborhood-specific risk parameter. A low ζ_i denotes a risky neighborhood. The expected loss is $\rho_i(M(\tau))\mathrm{e}^{-\zeta_i M(\tau)}$, and we assume that this expected loss is decreasing with arrival rank, which requires $\rho_i'(M(\tau))/\rho_i(M(\tau)) < \zeta_i$. Then the full opportunity function is given by

$$R_i(M(\tau)) = \frac{m_i(M(\tau))}{\rho_i(M(\tau))} - \rho_i(M(\tau))\mathrm{e}^{-\zeta_i M(\tau)}, \tag{11}$$

which is hump-shaped in arrival rank $M(\tau)$. Being there early is good (larger choice set and more room for appreciation of the house), but being there too early might be bad, since only a small part of the irreversible investment can be recouped in case the agent has to move somewhere else and the neighborhood has not yet taken off.

Observe that the opportunity function (11) can be flatter for two reasons: (i) when the asset price risk is larger (lower ζ_i); or (ii) when ρ_i' is larger, i.e., price appreciation is faster. When this is the case, our model predicts that growth is faster and rushes occur, in line with the empirical evidence summarized in our motivating example and detailed in Appendix B.1.

5 Entrepreneurship

We finally investigate the growth of entrepreneurship, where opportunities allow to acquire entrepreneurship human capital. First, we provide a motivating example looking at geographies across the United States. Second, we build a model of entrepreneurship in the context of two industries. In both cases, opportunities to gain entrepreneurship human capital depend on whether the geographic area or industry is relatively young. Younger areas and newer industries provide more opportunities than older areas and established industries.

5.1 Motivating examples: Finance and Tech. and Provo and Utah and Burlington, Vermont

In this motivating example, we posit that workers trade off pay for experiences that help them gain entrepreneurship human capital (Becker, 1962). Further, we suggest that the experiences that build entrepreneurship human capital depend on a worker's rank within an industry or firm. Early workers are given a broader portfolio of tasks to do that may more quickly build entrepreneurship human capital. In addition, early workers gain a better understanding of how the firm and industry operate and may be able to use that knowledge to start their own business. We consider differences in growth between industries and differences in entrepreneurship in cities in the US to investigate whether rank-dependent opportunities help explain these differences.

First, consider the growth between the 1980s and 2000s in finance (an established industry) and technology or 'tech' for short (a new industry). Several stylized facts match our model's predictions. First, wages are initially higher in finance than in tech. Second, an individual could be of higher rank in the tech industry than in finance—but there was risk in tech due to fewer jobs. This characterization of the opportunities suggest that they were nonmonotonic—you wanted to be early but not first. Third, as the model predicts with a nonmonotonic opportunity function, there was a tech boom (a rush), where the industry grew rapidly.[28]

Second, consider entrepreneurship in Provo, Utah and Burlington, Vermont. Provo

[28] In Appendix B.2, we provide additional evidence consistent with our mechanism of opportunities due to higher human capital accumulation in the younger industry. For example, we find the growth rate of entrepreneurs is positively correlated with the tech industry's growth and negatively correlated with the finance industry's growth.

and Burlington are an interesting comparison because they are mid-sized cities with populations of 116,886 and 163,414, respectively, with universities (Brigham Young University and University of Vermont, respectively). Despite these similarities, it is easier for someone to be of higher rank (more senior) in Provo than Burlington because of the demographics of these cities. Yournger individuals have a greater likelihood of being in a higher position in the firm they work for in areas with younger populations (Liang, Wang and Lazear, 2018). Therefore, the rank of an individual in the age distribution is particularly important for human capital accumulation, business acumen, and entrepreneurship. In this comparison, Provo has a younger population than Burlington. Specifically, Provo has a fertility rate that is relatively high at 256 birth per 10,000 people in 1990 and 250 in 2007 and a median age of 25 in 2020, compared to 38 in the United States. In contrast, Burlington has an older population with a fertility rate that is relatively low at 150 births per 10,000 people in 1990 and 105 in 2007. The median age in 2020 in Burlington is 36.5, close to the average for the U.S. despite having a university. In terms of the model, this suggests that there are more opportunities (the opportunity function is flatter) in Provo than in Burlington to gain human capital necessary to start a business.

Given the differences in opportunities, the model predicts that Provo will have a faster growth of entrepreneurs than Burlington. This prediction is corroborated in the data. In 2020, Provo had 12,411 new business applications compared to 1,749 in Burlington. Over a longer period, 2005 to 2020, Provo had 16 new business applications per 1,000 people, while Burlington had only 9.

[Insert Figure 6 about here.]

The comparison of Provo and Burlington is consistent with broader patterns across the United States. Figure 6 shows a bin scatter plot of all counties in the United States with fertility in 1990 on the horizontal axis and average business applications from 2005 to 2020 on the vertical axis. Each dot represents roughly 160 counties. The red solid line graphs the positive relationship between fertility and business applications. Said differently, younger counties have a faster growth in new businesses.

5.2 Model

We build our entrepreneurship model building on the idea that workers trade off earnings and other benefits such as human capital accumulation (Becker, 1962). There are opportunities to being early to an industry because early entrants have a higher rank or seniority and gain more business acumen. We also allow for there to be costs to entering an industry too early. For instance, there is increased displacement risk in young industries and that risk is especially high in smaller cities. In a nutshell, there are opportunities to being early but not necessarily first.

Income function. Assume that entities 1 and 2 are two industries—one established with $N_1(t)$ workers and one new with $N_2(t)$ workers. Each worker receives industry-specific earnings that combine agglomeration and rivalry externalities, $A_i(N_i)$ and $P_i(N_i)$, respectively, as follows. $Y_i(N_i(t)) = A_i(N_i(t)) + P(N_i(t))$. As an industry grows, firms can share suppliers and other fixed costs and better match with workers (Duranton and Puga, 2004). Initially, these agglomeration benefits are large. Formally, we model this as $A(N_i(t)) = -a_i[(1 + N_i(t))^{-1}) - 1]$, following Buchanan (1965).[29] As an industry grows, competition increases, which bids up the cost of inputs. This rivalry externality is likely small initially and eventually grows large. Formally, we model this as $P(N_i(t)) = (N_i(t) + 1)^{-\gamma} - 1$, where $\gamma < 1$.

With these agglomeration and rivalry externalities, income is: (i) initially increasing with the workforce in the industry and eventually decreasing; and (ii) initially higher in industry 1 (the established industry) than in industry 2 (the new industry). Note that, when industry 2 becomes sufficiently developed, it could provide higher incomes. However, until industry 2 has a critical mass, workers who choose to work there forgo the higher income in industry 1. Hence, for workers to move to industry 2, that industry must provide them opportunities.

Opportunity function. Workers are willing to forgo higher incomes in the established industry to gain entrepreneurship human capital in the new industry. The opportunities to gain this human capital depend a worker's rank within the industry.

[29]Buchanan (1965) used a similar specification to discuss the agglomeration benefits and congestion costs of public goods. We refer to the assumption that income increases with industry size as agglomeration externality. Since the model is dynamic and since industry size $N_i(t)$ reflects some accretion of physical and human capital over the past, these terms can also be viewed as capturing industry maturity.

For individuals to start a successful business they need, what is typically called, "business acumen" (Liang, Wang and Lazear, 2018). Not all job experiences, however, lead to business acumen. Individuals that are given low-level tasks likely gain less business acumen than those who have a broader portfolio of tasks and decision-making authority. The amount of human capital an individual acquires therefore depends on that person's rank within the industry. Individuals selecting an industry trade off the initially lower incomes in the newer industry with the opportunity to accumulate human capital that these industries provide by allowing them to move up more quickly.

Working in a new industry is not without risk, however. Individuals are much more likely to be displaced in a new industry, due to increased demand and cost uncertainty. The cost of displacement is not only the lost wages during transition but a loss of firm- and industry-specific human capital, which that can lead to depressed future earnings (Topel, 1990; Neal, 1995). The cost of this risk, however, decreases as an industry grows, because the probability a worker can find a job within the industry increases.

Formally, assume there exists a rank $\overline{M}$ such that ranks greater than $\overline{M}$ do not benefit from being early. Let an individual's human capital accumulation $m_i(\cdot)$ be an increasing function of the share of the workforce $s_i(\cdot)$ below their rank $M(\tau)$. The share of the workforce below rank $M(\tau)$ who benefits from being early in an industry is given by

$$s(M(\tau)) = \begin{cases} [e^{-\rho_i(\overline{M}-M(\tau))/\overline{M}} - 1]/(e^{-\rho_i} - 1), & \text{if } M(\tau) < \overline{M}. \\ 0, & \text{otherwise,} \end{cases}$$

where ρ_i is the expected steady-state growth rate of industry i. We capture the risk to workers in new industries as $-\rho_i\zeta_i M(\tau)e^{-\rho_i\zeta_i M(\tau)}$, where a high ζ_i denotes a risky industry, and we maintain that $\rho_i\zeta_i < 1$. In this formulation, the risks are decreasing and convex for industries with positive growth rates and increasing and convex for industries with negative growth rates. The opportunities a worker receives from moving to a new industry combine the benefits of human capital accumulation and the risk due to displacement as follows. $R_i(M(\tau)) = m_i(s(M(\tau)) + \rho_i\zeta_i M(\tau)e^{-\rho_i\zeta_i M(\tau)}$. If a new industry were perfectly safe, $\zeta_i = 0$, then the opportunity function would be monotonically decreasing. Riskier industries, however, may cause the opportunity function to be hump shaped—indicating the benefit of being early but not first.

6 Conclusion

Economic change in industries, neighborhoods, and cities is often characterized by rushes—the rapid and simultaneous movement of new workers and firms from established industries, cities, or neighborhoods to new ones. We have proposed a model that generates such rushes. Contrary to the literature that explains rapid change and correlations in individual decisions through what may be broadly called 'information externalities,' our model does not need the presence of such externalities.[30] In our model, rushes occur because agents trade off changes in incomes against rank-dependent opportunities. Being early is good, but being too early is not. When payoffs generated by opportunities are non-monotonic, agents are enticed to wait—nobody wants to preempt the rush and be first—but not wait too long—since opportunities become less valuable after the rush has occurred. Put simply, "I'd rather be second than first, but not third."

We have derived general results and illustrated them using simple models drawn from urban economics and industrial organization. The applicability of our ideas is broader than that because many economic phenomena offer payoffs that are a mixture of size-dependent—but rank-independent—incomes and size-independent—but rank-dependent—opportunities. The empirical motivating examples we have developed suggest that our mechanism is relevant. Devising empirical tests that identify it, disentangle it from alternative explanations—such as herding or information cascades—and quantify its magnitude goes beyond this paper and is left for future research.

[30]Rational herding in financial markets (e.g., Devenow and Welch, 1996) may lead to periods of slow movement—where investors follow the herd—followed by periods of sudden change as key investors revise their positions in light of new information. Herding in investment decisions may cause suboptimal investment delays and investment surges (see Chamley and Gale, 1994). The arrival of a new store, specific types of businesses, or affluent residents in a deprived area may reveal information as to the viability of the neighborhood, triggering an influx of other businesses or residents who waited for a signal to move (Caplin and Leahy, 1998; Behrens, Boualam, Martin and Mayneris, 2022). Furthermore, firms adopt new technology or enter into a region depending on information revealed by the decisions of others (Conley and Udry, 2010; Ossa, 2013).

References

Abdel-Rahman, Hesham M. and Alex Anas, "Theories of systems of cities," *Handbook of Regional and Urban Economics*, 2004, *4*, 2293–2339.

Alonso, William, *Location and land use: toward a general theory of land rent*, Vol. 204, Harvard University Press Cambridge, MA, 1964.

Anas, Alex, "On the birth and growth of cities: Laissez-faire and planning compared," *Regional Science and Urban Economics*, 1992, *22* (2), 243–258.

Anderson, Axel, Andreas Park, and Lones Smith, "Rushes in large timing games," *Econometrica*, 2017, *85* (3), 871–913.

Anderson, Steven T., Daniel Friedman, and Ryan Oprea, "Preemption games: Theory and experiment," *American Economic Review*, 2010, *100* (4), 1778–1803.

Becker, Gary S, "Investment in Human Capital: A Theoretical Analysis," *Journal of Political Economy*, 1962, pp. 9–49.

Behrens, Kristian, Brahim Boualam, Julien Martin, and Florian Mayneris, "Gentrification and pioneer businesses," *Review of Economics and Statistics (forthcoming)*, 2022.

Bleakley, Hoyt and Jeffrey Lin, "Portage and Path Dependence," *The Quarterly Journal of Economics*, 2012, *127* (2), 5870–644.

Boyer, Marcel, Éric Gravel, and Pierre Lasserre, "Real options and strategic competition: a survey," *Department des Sciences Economiques, Université du Quebec à Montréal Working Paper*, 2004.

Brueckner, Jan K., "Urban growth models with durable housing: An overview," *Economics of Cities: Theoretical Perspectives*, 2000, pp. 263–289.

Brueckner, Jan K and Stuart S. Rosenthal, "Gentrification and Neighborhood Housing Cycles: Will America's Future Downtowns be Rich?," *The Review of Economics and Statistics*, 2009, *91* (4), 725–743.

Buchanan, James M., "An economic theory of clubs," *Economica*, 1965, *33*, 1–14.

Caplin, Andrew and John Leahy, "Miracle on Sixth Avenue: Information Externalities and Search," *The Economic Journal*, 1998, *108* (446), 60–74.

Chamley, Christophe and Douglas Gale, "Information Revelation and Strategic Delay in a Model of Investment," *Econometrica*, 1994, *62* (5), 1065–1085.

Conley, Timothy G. and Christopher R. Udry, "Learning about a new technology: Pineapple in Ghana," *American Economic Review*, 2010, *100* (1), 35–69.

Couture, Victor and Jessie Handbury, "Urban revival in America," *Journal of Urban Economics*, 2020, *119*, 103267.

Cuberes, David, "A model of sequential city growth," *The BE Journal of Macroeconomics*, 2009, *9* (1), 18.

Devenow, Andrea and Ivo Welch, "Rational herding in financial economics," *European Economic Review*, 1996, *40* (3–5), 603–615.

Duranton, Gilles and Diego Puga, "Micro-foundations of urban agglomeration economies," *Handbook of regional and urban economics*, 2004, *4*, 2063–2117.

Ellen, Ingrid G., Keren M. Horn, and Davin Reed, "Did falling crime invite gentrification?," *Journal of Housing Economics*, 2019, *46* (101639), 1–11.

Fudenberg, Drew and Jean Tirole, "Preemption and rent equalization in the adoption of new technology," *The Review of Economic Studies*, 1985, *52* (3), 383–401.

_ , Richard Gilbert, Joseph Stiglitz, and Jean Tirole, "Preemption, leapfrogging and competition in patent races," *European Economic Review*, 1983, *22* (1), 3–31.

Glaeser, Edward L, "A nation of gamblers: Real estate speculation and American history," *American Economic Review*, 2013, *103* (3), 1–42.

Helsley, Robert W. and William C. Strange, "City formation with commitment," *Regional Science and Urban Economics*, 1994, *24* (3), 373–390.

Henderson, J. Vernon, "The sizes and types of cities," *The American Economic Review*, 1974, *64* (4), 640–656.

— **and Anthony J. Venables**, "The dynamics of city formation," *Review of Economic Dynamics*, 2009, *12* (2), 233–254.

Liang, James, Hui Wang, and Edward P Lazear, "Demographics and entrepreneurship," *Journal of Political Economy*, 2018, *126* (S1), S140–S196.

Milgrom, Paul and John Roberts, "Comparing equilibria," *The American Economic Review*, 1994, *84* (3), 441–459.

Mills, Edward S., "An aggregative model of resource allocation in a metropolitan area," *The American Economic Review*, 1967, *57* (2), 197–210.

Munneke, Henry J. and Kiplan S. Womack, "Neighborhood Renewal: The Decision to Renovate or Tear Down," *Regional Science and Urban Economics*, 2015, *54* (1), 99–115.

Muth, Richard F., "Cities and housing; the spatial pattern of urban residential land use.," 1969.

Neal, Derek, "Industry-specific Human Capital: Evidence from Displaced Workers," *Journal of Labor Economics*, 1995, *13* (4), 653–677.

Oprea, Ryan, Daniel Friedman, and Steven T Anderson, "Learning to wait: A laboratory investigation," *The Review of Economic Studies*, 2009, *76* (3), 1103–1124.

Ossa, Ralph, "A Gold Rush Theory of Economic Development," *Journal of Economic Geography*, 2013, *13* (1), 107–117.

Park, Andreas and Lones Smith, "Caller number five and related timing games," *Theoretical Economics*, 2008, *3* (2), 231–256.

Rosenthal, Stuart S., "Old Homes, Externalities, and Poor Neighborhoods. A Model of Urban Decline and Renewal," *Journal of Urban Economics*, 2008, *63* (3), 816–840.

— **and Maeve Maloney**, "Why Do Home Prices Appreciate Faster in City Centers? The Role of Risk-Return Trade-Offs in Real Estate Markets," 2022.

— **and Stephen L. Ross**, "Change and Persistence in the Economic Status of Neighborhoods and Cities," in "Handbook of Regional and Urban Economics," Vol. 5, Elsevier, 2015, pp. 1047–1120.

Seegert, Nathan, "Land Regulations and the Optimal Distribution of Cities," *Working Paper*, 2015.

Smith, J. Maynard, "The theory of games and the evolution of animal conflicts," *Journal of Theoretical Biology*, 1974, *47* (1), 209–221.

Sun, Yeneng, "The exact law of large numbers via Fubini extension and characterization of insurable risks," *Journal of Economic Theory*, 2006, *126* (1), 31–69.

Tolley, George S., "The welfare economics of city bigness," *Journal of Urban Economics*, 1974, *1* (3), 324–345.

Topel, Robert, "Specific Capital and Unemployment: Measuring the Costs and Consequences of Job Loss," in "Carnegie-Rochester Conference Series on Public Policy," Vol. 33 Elsevier 1990, pp. 181–214.

Wade, Richard C., *The Urban Frontier: The Rise of Western Cities, 1790–1830*, University of Illinois Press, 1996.

Appendix A Proofs

Appendix A.1 Proof of Theorem 1

To show existence and uniqueness of the equilibrium, we use a natural refinement of mixed strategy Nash equilibria in timing games, called ε-safe (Anderson, Park and Smith, 2017). This refinement resembles a trembling hand refinement, and, in our context, it ensures there is only one equilibrium rush size. It excludes equilibria where tiny timing mistakes $\varepsilon > 0$ on both sides at time τ cause a significant payoff loss. This refinement does not exclude equilibria if only early movement or only late movement causes a significant payoff loss in equilibrium. In these cases, an individual could guard against these losses by ensuring they were never early or never late. This provides the intuition for the ε-safe payoff:

$$U_\varepsilon(\tau, M(\tau)) = \max\left\{\inf_{\max\{t-\varepsilon,\ 0\}\leq s<t} U(s, M(s)),\ \inf_{s\in[t,t+\varepsilon)} U(s, M(s))\right\}.$$

A Nash equilibrium is safe if there exists $\bar{\varepsilon} > 0$ such that $U_\varepsilon(\tau, M(\tau)) = U(\tau, M(\tau))$ for all $t \in [\tau_1, \infty)$ and for all $\varepsilon \in (0,\ \bar{\varepsilon})$. Anderson, Park and Smith (2017) prove that this refinement excludes equilibria with periods of inaction.

The following lemma will be useful for proving the theorem.

Lemma 1 (Regularity assumption)**.** *Assume the opportunity function $R(M(\tau))$ is: (i) monotonically increasing; or (ii) initially decreasing and then increasing. Then there does not exist an equilibrium.*

Proof. (i) Suppose toward contradiction there is an equilibrium for an entity with an opportunity function that is monotonically increasing. Consider a deviation from this equilibrium where an individual moves to entity 2 at time $t+\varepsilon$ instead of their equilibrium prescribed time $t > \tau_1$, where τ_1 is the time where $M(\tau) > 0$ for the first time. This is a profitable deviation because the individual receives more income and more opportunities. The income in entity 1 is higher than in entity 2. Therefore, there is no equilibrium in which the opportunity function $R(M(\tau))$ is monotonically increasing.

(ii) Suppose toward contradiction there is an equilibrium for an entity with an opportunity function that is initially decreasing and then increasing. Consider a deviation from this equilibrium where an individual moves to entity 2 at time $t+\varepsilon$ instead of their equilibrium prescribed time $t > \tau_1$. Further assume at time t and $t+\varepsilon$, the opportunity function is increasing (we are beyond the minimum of the average opportunity). Then, by the same reasoning as above, the deviation is profitable as the individual will receive higher income and better opportunities. □

We can now prove Theorem 1.

Proof. The proof of the theorem proceeds by construction and uses the regularity assumption in equation (4), the Implicit Function Theorem, and the equilibrium refinement $\varepsilon-$safe equilibrium (Anderson, Park and Smith, 2017).

Step 1: Initial period with no growth in entity 2. By condition (5), initially moving is worse than staying (given any feasible size of movement).

Step 2: Eventual growth in entity 2. Given that population is growing and $Y_1'(N(0)) < 0$, there exists a time $\tau < \infty$ and a mass of agents ΔM such that it is strictly better to move: $Y_1(N(\tau) - \Delta M) < Y_2(\Delta M) + R(\Delta M) + \dot{R}(t)/r$.

Step 3: Unique starting time in entity 2. Proposition 3 establishes that there exists a unique rush size ΔM_1. Then, by smoothness of the income and opportunity functions, there exists a unique point τ_1 where entity 2 begins to grow, determined by $Y_1(N(\tau_1) - \Delta M_1) = Y_2(\Delta M_1) + R(\Delta M_1) + \dot{R}(\tau_1)/r$.

Step 4: Implicit Function Theorem. Since utility is continuously differentiable after τ_1 (recall from Proposition 3 that there are no more atoms after τ_1) $m(t)$ solves the equilibrium condition $\partial U/\partial\tau = 0$, for $\tau \in [\tau_1, \infty)$. The Implicit Function Theorem implies, $\partial_\tau U(\tau, M(\tau))\mathrm{d}\tau + \partial_M U(\tau, M(\tau))\mathrm{d}M = 0$ which reduces to

$$\frac{\mathrm{d}M(\tau)}{\mathrm{d}\tau} = -\frac{\partial_\tau U(\tau, M)}{\partial_M U(\tau, M)} > 0$$

by condition (6). Therefore, there exists an equilibrium with $m(t) \equiv \frac{\mathrm{d}M}{\mathrm{d}\tau} > 0$ for all $t \in [\underline{\tau}, \infty)$.

Step 5: $\varepsilon-$safe equilibria The constructed equilibrium is the only one without periods of inaction and is a $\varepsilon-$safe equilibrium. Other potential equilibria with inaction are not $\varepsilon-$safe equilibria (Anderson, Park and Smith, 2017). □

Appendix A.2 Proof of Proposition 2

Proposition 1 is proved in the main text. We now prove Proposition 2.

Proof. The proof involves four successive steps.

Step 1 (No equilibrium with slow creation). Assume (1.a) that the opportunity function is non-monotonic and initially increasing, as depicted in panel (b) of Figure 2, but that there exists an equilibrium without a rush. For this to be an equilibrium there must be no profitable deviation from the equilibrium strategy. Consider the first individual to move to entity 2. Income there is lower than in the existing entity but increases with time as more individuals move to entity 2. The opportunity function is also initially increasing by assumption (1.a). This shows that the first individual has an incentive to delay his move, thereby avoiding some time with lower income and receiving greater opportunities associated with being of a lower rank. Therefore, there does not exist an equilibrium where the entity is created with gradual migration when the opportunity function is non-monotonic and initially increasing.

Step 2 (No rush with monotonic opportunities). To demonstrate that the creation of an entity by a rush is possible only when the opportunity function is non-monotonic and initially increasing, consider the cases with a monotonic opportunity function and a non-monotonic opportunity function that is initially decreasing. Begin with a monotonic opportunity function. The size of the rush is determined by the point at which the average opportunity function intersects the (marginal) opportunity function. For monotonic opportunity functions the average opportunity function and the opportunity function intersect only for the first mover ($R_0(0) = R(0)$). For a rush of size ΔM, we have either $R_0(\Delta M) < R(\Delta M)$ (when the opportunity function is increasing) or $R_0(\Delta M) > R(\Delta M)$ (when the opportunity function is decreasing). Hence, the arbitrage condition is violated and individuals pre-empt the rush if the opportunity function is decreasing or outlast the rush if the opportunity function is increasing.

Step 3 (No rush with initially decreasing non-monotonic opportunities). Consider a non-monotonic opportunity function that is initially decreasing, as depicted in Figure 7. In this case, although there exists a ΔM such that $R(\Delta M_1) = R_0(\Delta M_1)$ it is the minimum value for $R_0(\Delta M)$, as depicted in Figure7. Hence, there is room for arbitrage and individuals pre-empt the rush because $R(0) > R_0(\Delta M_1)$. Hence, no equilibrium with a rush exists.

Step 4 (Existence of equilibrium with a rush). To demonstrate that an equilibrium exists where entity 2 is formed by a rush, consider the payoffs of individuals who create that entity and who receive the average opportunity during the rush. The entity is created where $R_0(\Delta M_1) = (1/\Delta M_1)\int_0^{\Delta M_1} R(M(t))\mathrm{d}t$, where the size of the rush is ΔM_1. This condition states that the average opportunity must equal the (marginal) opportunity at the rank equal to size of the rush. If this condition did not hold, there would be arbitrage opportunities: (i) for individuals in the rush to wait and move right after the rush (if $R_0(\Delta M_1) < R(\Delta M_1)$); or (ii) for individuals after the rush to join the rush (if $R_0(\Delta M_1) > R(\Delta M_1)$). Finally, there could be opportunities for individuals to pre-empt the rush because $R_0(\Delta M_1) > R(0)$. The latter inequality holds because the average opportunity is maximized where it intersects the marginal opportunity. Therefore, there exists an equilibrium where entity 2 is formed by a rush when the opportunity function is non-monotonic and initially increasing. □

[Insert Figure 7 about here.]

Appendix A.3 Proof of Proposition 4

Proposition 3 is proved in the main text. We now prove Proposition 4.

Proof. The size of the rush is determined by

$$\frac{1}{\Delta M}\int_0^{\Delta M} R(M(t))\mathrm{d}t = R(\Delta M),$$

Clearly, if $R_\phi(M) = \phi R(M)$ or $R_\phi(M) = R(M) + \phi$, we have

$$\frac{1}{\Delta M}\int_0^{\Delta M} R_\phi(M(t))\mathrm{d}t = R_\phi(\Delta M) \quad \Rightarrow \quad \frac{\phi}{\Delta M}\int_0^{\Delta M} R(M(t))\mathrm{d}t = \phi R(\Delta M)$$

and

$$\frac{1}{\Delta M}\int_0^{\Delta M} R_\phi(M(t))\mathrm{d}t = R_\phi(\Delta M) \quad \Rightarrow \quad \frac{1}{\Delta M}\int_0^{\Delta M} R(M(t))\mathrm{d}t + \phi = R(\Delta M) + \phi,$$

so that ΔM is the same in both cases. Any multiplicative or additive scaling does not affect the point where the average equals the marginal.

Last, if $R_\phi(M) = R(\phi M)$ we have

$$\frac{1}{\Delta M}\int_0^{\Delta M} R_\phi(M(t))\mathrm{d}t = R_\phi(\Delta M) \quad \Rightarrow \quad \frac{1}{\Delta M}\int_0^{\Delta M} R(\phi M(t))\mathrm{d}t = R(\phi \Delta M).$$

We know that the equality holds for $\phi = 1$. Assume that $\phi > 1$. The foregoing equation continues to hold true for $y = M/\phi < M$ since $\phi > 1$. Hence, the intersection between the marginal and the average shifts to the left and the size of the rush decreases. Conversely, when $0 < \phi < 1$, we let $y = M/\phi > M$ since $\phi < 1$. The intersection between the marginal and the average shifts to the right and the size of the rush increases. □

Appendix B More detailed empirical illustrations

Appendix B.1 Gentrification: NY, Boston, and Philadelphia

Data. We use block-level data from the 1990, 2000, and 2010 census waves for the New York, Boston, and Philadelphia MSAs from Behrens, Boualam, Martin and Mayneris (2022). These data come from the National Historical Geographic Information System (NHGIS) of the population center at the University of Minnesota (available online at `https://www.nhgis.org`). We use the concordance algorithm developed by Behrens, Boualam, Martin and Mayneris (2022) to construct time-consistent blocks that allow to meaningfully look at decadal changes of the variables at that geographic scale. We gather information on residents and housing units counts that are directly available at the block level. Several other variables—such as total income or the number of residents by educational attainment—are provided at a slightly higher level of aggregation, the block group. In that case, we apportion those variables to blocks using block-level population weights. Per capita and median household income, the age of the housing stock, as well as median rents and housing values are also available at the block-group level; they are directly imputed to the blocks nested within the block groups. Distance to the CBD is measured as distance from Wall Street (New York), the Prudential Center (Boston), and the Libery Bell (Philadelphia).

Results. We compare gentrification in New York City, Boston, and Philadelphia to highlight the conditions that lead to rapid growth given in Proposition 1.[31] We define a neighborhood as gentrifying if its share of highly educated residents—defined as having at least some college education—grew twice more than the city-wide share. Following Behrens et al. (2022), we measure neighborhood change at a fine geographic scale by using time-consistent block-level data from the 1990, 2000, and 2010 censuses.

Let $\Delta\text{edu}_i^{t,t-1} \equiv \text{edu}_i^t - \text{edu}_i^{t-1}$ denote the change in the share of educated in block i between Census waves $t-1$ and t. Let further $\Delta\text{edu}_{city}^{t,t-1} \equiv \text{edu}_{city}^t - \text{edu}_{city}^{t-1}$ denote the corresponding city-wide change. Finally, define the *excess change in the share of educated* for block i between 1990 and 2010 as the difference compared to the city as a whole:

$$\begin{aligned}\Delta\text{xs\_edu}_i^{2010,1990} &\equiv \Delta\text{edu}_i^{2010,1990} - \Delta\text{edu}_{city}^{2010,1990} \\ &= \left[\Delta\text{edu}_i^{2010,2000} + \Delta\text{edu}_i^{2000,1990}\right] - \left[\Delta\text{edu}_{city}^{2010,2000} + \Delta\text{edu}_{city}^{2000,1990}\right].\end{aligned}$$

A positive excess change implies that the block became more educated, when compared to the city as a whole, whereas a negative excess change means that the block became less educated when compared to the city as a whole. To fix ideas, between 1990 and 2010, the city-wide shares of highly educated grew by 7.44% in New York, by 9.20% in Boston, and by 9.05% in Philadelphia.[32]

In what follows, we focus on blocks that experienced *substantial excess change* (or substantial change, for short). We define substantial change as growth twice above that of the city average. Formally, a block experiences substantial positive change if $\Delta\text{xs\_edu}_i^{2010,1990} > \Delta\text{edu}_{city}^{2010,1990}$, and substantial negative change if $\Delta\text{xs\_edu}_i^{2010,1990} < -\Delta\text{edu}_{city}^{2010,1990}$. In our data, blocks that experienced substantial change had growth rates in excess of about 15% in New York and about 18% in Boston and Philadelphia. We think this captures the idea that those are areas that experienced a lot of 'socio-demographic upgrading'—one out of five or six residents without higher education in 1990 was replaced with a highly educated counterpart by 2010 (or left in case of negative change).

[Insert Table 1 about here.]

[31] That there is a lot of neighborhood change in general, happening either over long periods or shorter cycles, has been abundantly documented elsewhere. See Rosenthal and Ross (2015) for a recent survey.

[32] The growth in the share of highly educated in New York is 3.35% between 1990 and 2000, 4.09% between 2000 and 2010, and hence 7.44% between 1990 and 2010.

Our key question of interest is whether those changes occurred slowly or rapidly. To analyze it, we decompose the time profile of these changes as follows. We compute the share θ_i of the 1990–2010 change that occurs over 1990–2000 and the share $1-\theta_i$ of that change that occurs over 2000–2010:

$$\theta_i \equiv \frac{\Delta\text{net\_edu}_i^{2000,1990}}{\Delta\text{net\_edu}_i^{2010,1990}}. \qquad \text{(Appendix B.1)}$$

A *rush* is formally defined for block i when $\theta_i < 0.25$ or $\theta_i > 0.75$, i.e., more than three-quarters of the change in the share of highly educated occurred over one of the two decades. In the reverse case, we think about this as 'gradual change' where the share of educated grew more than that of the city as a whole but not too unequally spread over the two decades.

What do our data tell us about neighborhood change? Panel (a) of Table 1 summarizes a number of descriptive statistics for rushes in the three metropolitan areas.[33] Take New York first. We have 61,205 blocks with positive changes. Out of these, we have 24,506 substantial positive changes (40.04% of the changes). How many of those substantial changes occur via rushes, and how many occur gradually? Using our definition of rushes, we find 15,691 rushes (64.03% of the substantial positive changes). In words, rushes seem pervasive and account for about three-fifths to two-thirds of substantial neighborhood change in New York over the 1990–2010 period. The results for the two other cities—as well as when examining only more central blocks, defined as being less than the MSA median distance from the CDB—are very similar, with rushes accounting for around three-fifths of the substantial positive changes in socioeconomic neighborhood composition.

Panel (b) of Table 1 summarizes simple equality-of-means tests for a number of variables usually associated with gentrification, distinguishing between blocks that experienced a rush and those that experienced slower change. We see that blocks with rushes had significantly older housing stocks in 1990, with correspondingly lower rents and housing values. However, there is no clear pattern between the two types of blocks concerning the change in that housing stock over the two decades, which suggests changes were accompanied by a mix of rebuilding and renovation (Munneke and Womack, 2015).

[33] In Table 1, we exclude the bottom decile of the smallest blocks in the 1990 population distribution in each city (i.e., blocks with less than 15 residents in New York, 10 residents in Boston, and 11 residents in Philadelphia). Small blocks have too noisy percentage changes to be meaningful.

Unsurprisingly, blocks that experienced rushes also had lower per capita income in 1990.

Which factors may explain why some blocks experience rushes while others experience slower change? Proposition 1 shows that a flatter opportunity function will lead to faster growth, while Proposition 4 shows that the size of the rush is larger if opportunities are less compressed. In our model, higher moving risk (a low ζ_i) or faster price growth (larger ρ_i') leads to a flatter opportunity function, which should correlate with faster growth. Table 2 summarizes the results from linear probability model that investigate the correlates of rushes. The dependent variable is a dummy taking value 1 if the block experienced a rush and zero otherwise.[34] The observations are all blocks that experience significant positive change.

[Insert Table 2 about here.]

Table 2 shows our results for all three MSAs pooled (columns 1-3), New York (columns 4-6), Boston (columns 7-9), and Philadelphia (columns 10-12). First, as shown rushes are more prevalent in gentrifying areas closer to the MSA CBD. This is in line with findings by Rosenthal and Maloney (2022) who show that idiosyncratic asset risk and price appreciation are higher closer to the central city. Yet, as shown this effect vanishes (or gets substantially attenuated) once we control for the age and price of the housing stock. This suggests that the risk-return relationship if driven by the age and initial price of the housing stock. Remarkably, we see from the data that fast change—rushes—are more prevalent in areas where per capita income was lower, the housing stock older, and prices cheaper. In a nutshell, the data suggest that rapid change is more frequent in more risky places. To our knowledge, this fact has not been noted until now.

To summarize, rushes are pervasive in the sense that substantial neighborhood change occurs rapidly more often than not.[35] Across three major US metropolitan areas, we find that about 60% of substantial changes between 1990 and 2010 occurred to more than 75% over one of the two decades only. Rank-dependent opportunities provided by older real estate—as emphasized in our model—may explain that type of change.

[34]Results using a probit model are very similar.

[35]If rushes occur around the year 2000 in our data, we may even underestimate the number of rushes. Assume, e.g., the 100% of the change occurs between 1995 and 2005, with 50% before 2000 and 50% after. We classify this as slow change, though it is a rush. Ideally, we would require more frequent data, but those are not available at a small geographic scale.

Appendix B.2 Entrepreneurship: Finance and technology industries

We combine the model above and census data on the growth of the finance and technology (tech) industries between 1980 and 2000 to investigate the size of rushes following Proposition 4. These two industries represent important industries with similar levels of skill but different relative ages. The finance industry is old and established, while the technology industry is relatively new (and was especially so in 1980). These two industries therefore provide an ideal comparison for growth.

The model states that initially wages in the technology industry are lower than those in finance but that there are opportunities to accumulate entrepreneurship human capital. This suggests places with faster tech growth will also have high growth in self-employed people. While there are opportunities in the technology industry, it is also risky. There are fewer jobs in the technology industry than in finance, and this is magnified in some smaller geographic areas. This risk can create a nonmonotonic opportunity function—inducing a rush into the tech industry. Further, following Proposition 4, the rush will be larger in cities with fewer initial tech jobs, where the risk of entering the tech industry is larger.

We use data from the Integrated Public Use Microdata Series (IPUMS) of the US Census and the County Business Patterns (CBP). These data include individual-level data on occupation (NAICS), worker class (e.g., self-employed or private wage earner), wages, and demographic information (e.g., age, sex, race, education). We report descriptive evidence from the finance and technology industries between 1980 and 2000 in Table 3. Columns (1) and (2) show that (mincerized) wages in 1980 are 2.6% lower in the technology industry than in finance, but by 2000, they are 2.7% higher in the technology industry. Given this discrepancy in wages, we would predict that, for individuals to be willing to enter the tech industry, it must provide entrepreneurship human capital. We report evidence in support of this in columns (3) through (6).

The growth rate of self-employed individuals in MSAs is positively correlated with the MSA's growth rate in the technology industry and negatively correlated with its growth rate in the finance industry. This correlation is robust to controlling for the total growth rate (column 3) and including state- and industry-level controls (column 4). We report placebo tests in columns (5) and (6), using the growth rate of private sector wage earners and government employees. For both of these worker classes, their growth rate is not positively correlated with the growth rate in the technology industry. This evidence is consistent with our model, where new industries provide entrepreneurship human capital in lieu of the higher wages paid in more established industries.

As a result of these opportunities, the technology industry boomed between 1980 and 2000. Column (7) shows that the growth rate in the technology industry between 1980 and 2000 was substantially larger than in finance. Consistent with Proposition 4, we find that technology growth was higher in cities with smaller initial amounts of technology employment (column 8). Entering the industry in these cities is riskier because there are fewer potential jobs in case of job displacement. This risk creates an initially flatter opportunity function and therefore faster growth or larger rushes. For example, we find the technology growth rate was higher in Provo, UT (low initial employment), than in Ann Arbor, MI (high initial employment). This evidence is supported by the resulting wage growth rates in the technology industry, relative to finance, and in small initial computer industry MSAs, relative to large initial tech industry MSAs. Columns (9) and (10) show that wages grew faster in technology than in finance and the wage grew even faster in small initial tech MSAs, though the last result is not precisely estimated.

[Insert Table 3 about here.]

To summarize, rank-dependent opportunities may help explain rushes into and growth differences between industries. We find evidence that job and wage growth were faster in technology than in finance—and even faster in MSAs with relatively low tech in 1980, which made the industry geographically riskier. We also find that growth in technology jobs, and not finance jobs, is correlated with more entrepreneurship.

Figure 1: The price of opportunities.

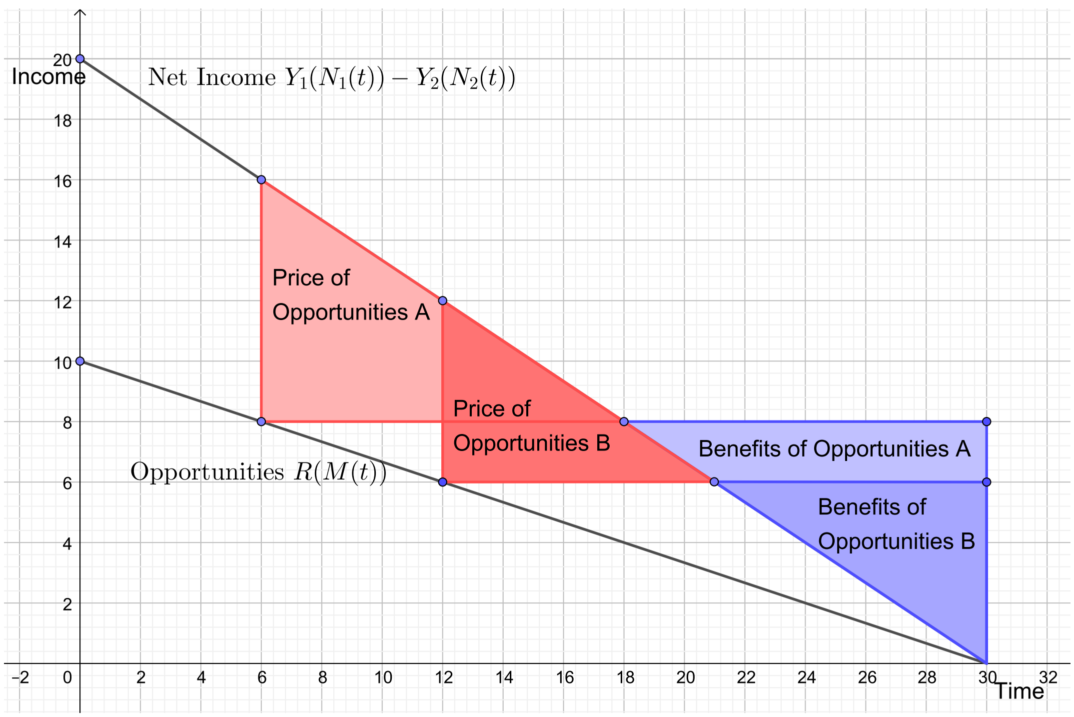


Figure 2: Conditions for rushes to occur in equilibrium.

(a) No rush in equilibrum. (b) Rush in equilibrium.

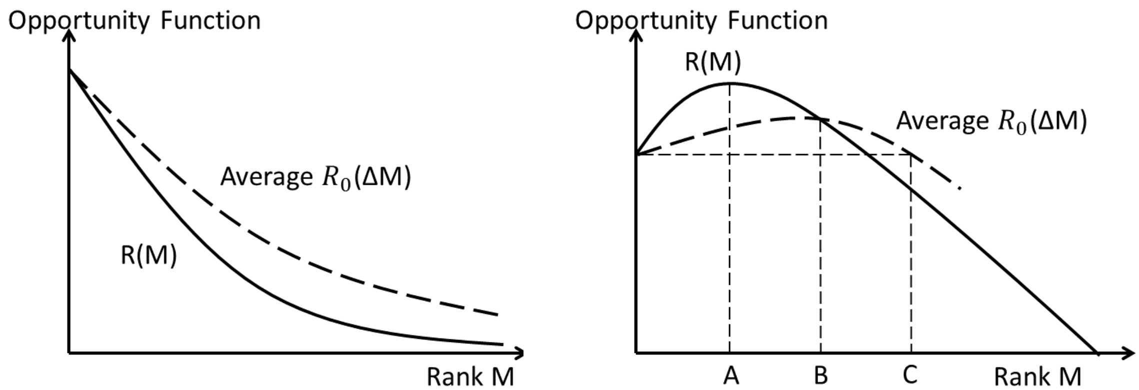


Figure 3: Effects of changes in the opportunity function.

(a) Proportional shift. (b) Skewed and compressed.

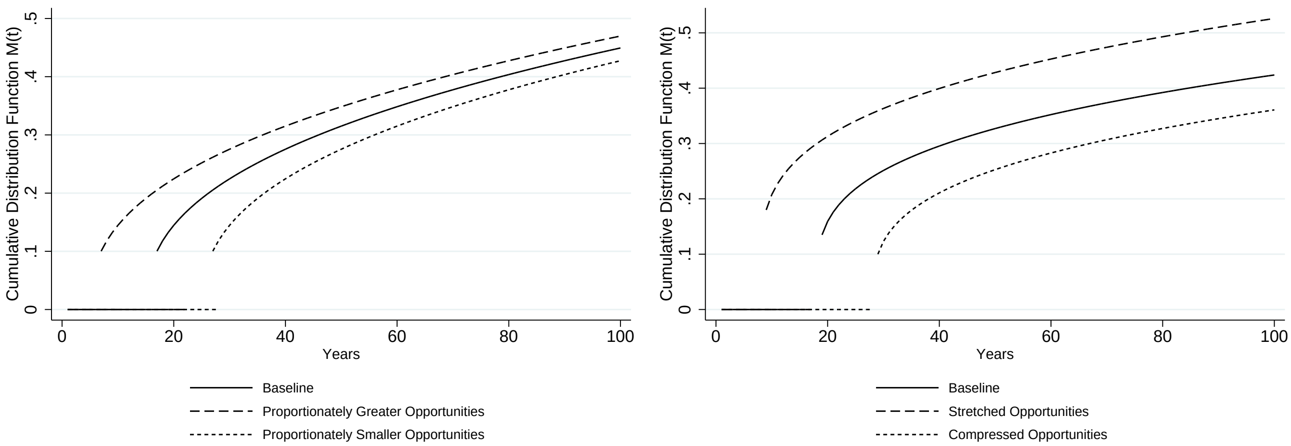


Figure 4: Parcels of land in a city.

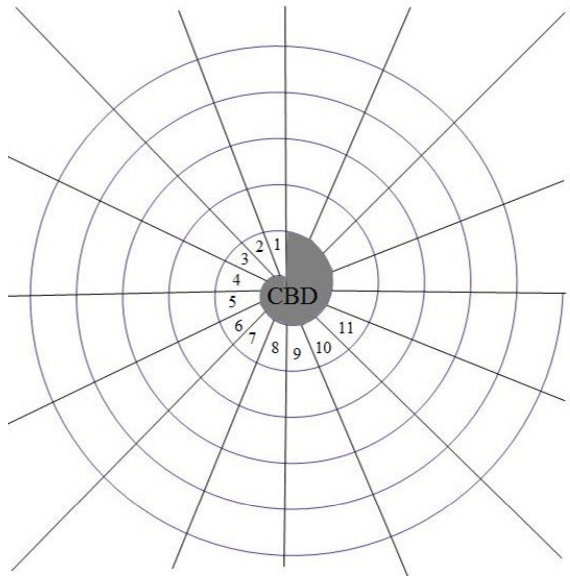
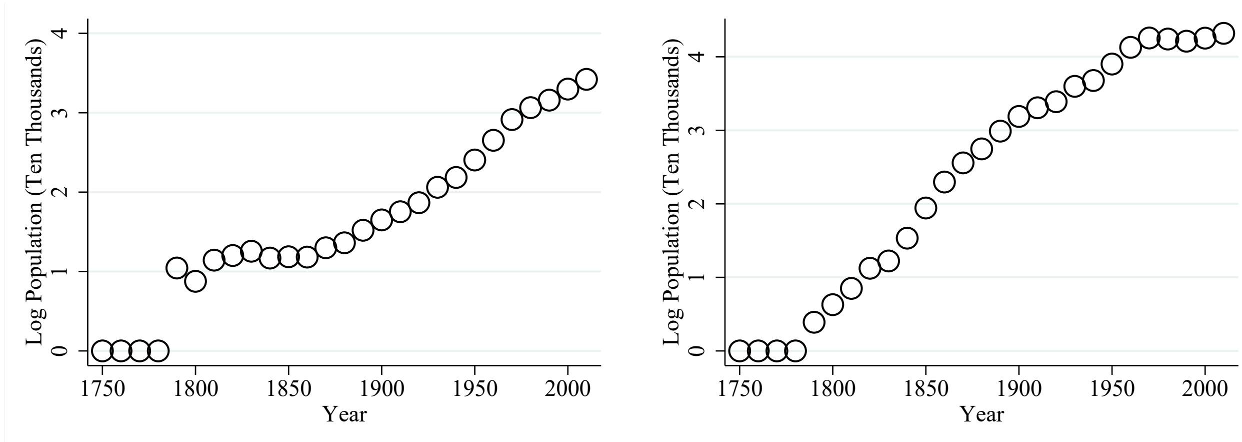


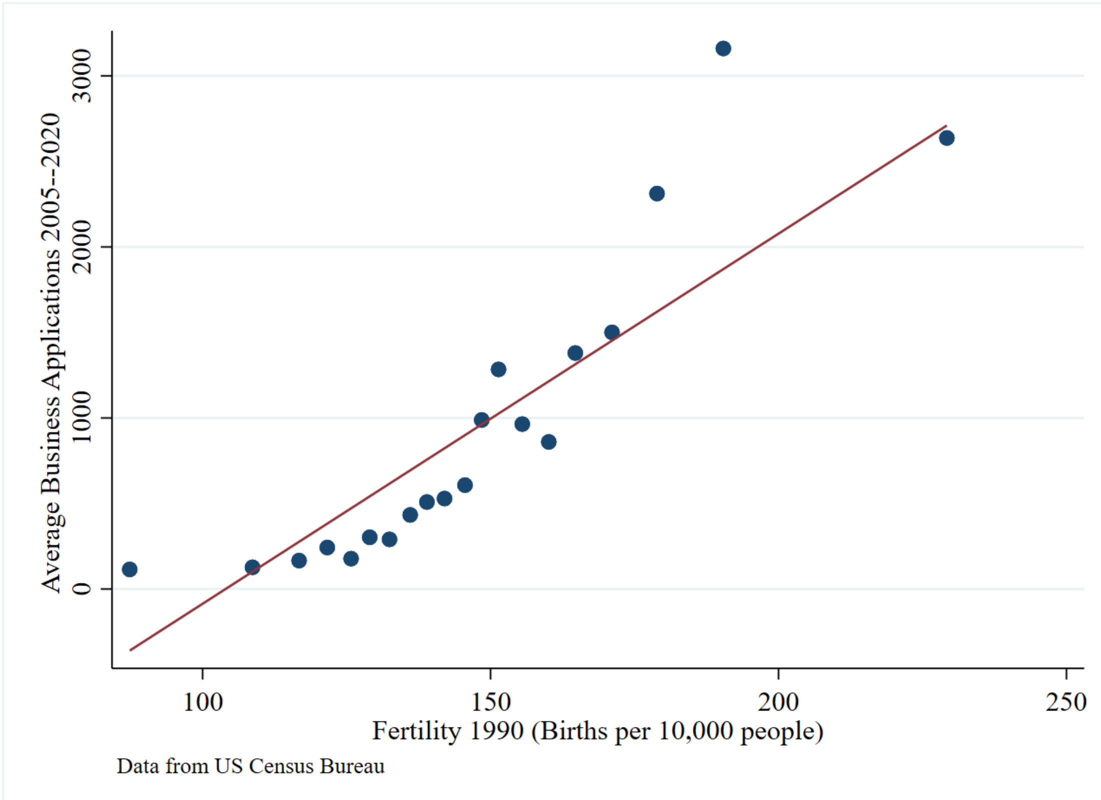
Figure 5: Population growth in Lexington and Louisville.

(a) Lexington. (b) Louisville.



Notes: This figure uses data from the decennial census. The model predicts rapid growth or a rush in Lexington and slow initial growth in Louisville, based on the differences in the heterogeneity of land in both cities.

Figure 6: Business Applications and Fertility Rates.



Notes: This figure uses data from the US Census. The model predicts rapid growth of entrepreneurs in younger areas.

Figure 7: Potential Opportunity Function

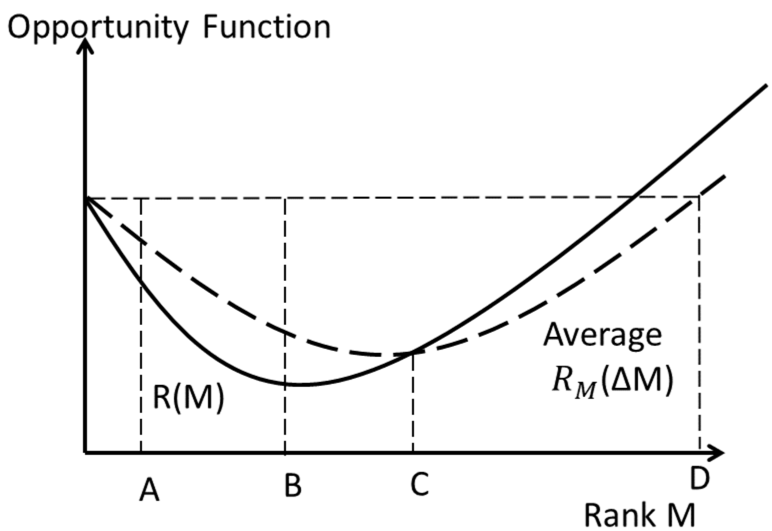


Table 1: Rushes and block-level characteristics, 1990–2010.

	(1) New York	(2) Boston	(3) Philadelphia
(a) Prevalence of rushes in CBSAs			
# stable blocks in CBSA	122,021	71,330	63,487
# blocks with substantial positive change	24,506	10,381	8,232
# blocks with rushes	15,691	6,407	4,967
Share of rushes	64.03%	61.72%	60.34%
# stable blocks, less than median distance to CBD	60,939	35,672	31,717
# blocks with substantial positive change	12,944	5,241	4,093
# blocks with rushes	8,483	3,199	2,490
Share of rushes	65.54%	61.04%	60.84%
(b) Equality-of-means tests, conditional on substantial positive change			
Median construction year of buildings in 1990 (rush = 1)	1952.60	1951.21	1952.47
Median construction year of buildings in 1990 (rush = 0)	1953.95	1952.25	1953.74
T-test	8.8254^a	3.9352^a	4.4043^a
Δ median construction year of buildings, 1990–2010 (rush = 1)	3.81	3.09	6.57
Δ median construction year of buildings, 1990–2010 (rush = 0)	4.07	3.00	7.14
T-test	1.9294^b	-0.4540	2.1497^b
Median gross rent in 1990 (rush = 1)	719.61	620.25	522.88
Median gross rent in 1990 (rush = 0)	738.37	635.17	545.51
T-test	6.8767^a	4.1325^a	6.1751^a
Median housing value 1990 (rush = 1)	207,908.50	155,409.60	97,030.38
Median housing value 1990 (rush = 0)	215,745.00	162,033.10	105,912.30
T-test	7.3929^a	7.1727^a	8.6740^a
Median per-capita income 1990 (rush = 1)	20,028.37	15,855.49	14,314.29
Median per-capita income 1990 (rush = 0)	20,721.76	16,326.36	15,468.74
T-test	5.7024^a	5.2088^a	9.6987^a

Notes: See Appendix B.1 for details on the data sources, variables, and their construction. The bottom panel of the table provides equality-of-means tests that compare blocks with rushes and without rushes, conditional on substantial positive change. Variables pertaining to building years and changes are expressed in years. Rent, housing values, and income variables are expressed in current USD. ^{a} $p < 0.01$; ^{b} $p < 0.05$; ^{c} $p < 0.1$.

Table 2: Correlates of rushes in the New York, Boston, and Philadelphia CBSAs, 1990–2010.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		Pooled MSAs			New York			Boston			Philadelphia	
log distance to CBD	-0.015^{a}	-0.002	0.005^{c}	-0.027^{a}	-0.013^{a}	-0.008^{c}	0.005	0.017^{a}	-0.009	-0.018^{a}	-0.008	0.033^{a}
	(0.003)	(0.003)	(0.003)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.006)
log med. year built		-3.752^{a}	-2.801^{a}		-3.645^{a}	-3.183^{a}		-4.264^{a}	-2.352^{a}		-2.970^{a}	-0.949
		(0.429)	(0.445)		(0.599)	(0.629)		(0.805)	(0.843)		(0.955)	(1.023)
log med. value			-0.115^{a}			-0.133^{a}			-0.189^{a}			-0.141^{a}
			(0.009)			(0.014)			(0.028)			(0.020)
log med. rent			-0.027^{a}			-0.021^{c}			-0.019			-0.019
			(0.009)			(0.012)			(0.016)			(0.019)
log per capita income			-0.137^{a}			-0.108^{a}			-0.092^{a}			-0.262^{a}
			(0.012)			(0.016)			(0.028)			(0.030)
share college educated			0.005^{a}			0.006^{a}			0.003^{a}			0.010^{a}
			(0.000)			(0.000)			(0.001)			(0.001)
MSA fixed effects	✓	✓	✓									
Observations	43,119	43,061	40,395	24,506	24,476	22,364	10,381	10,366	10,066	8,232	8,219	7,965
R-squared	0.002	0.004	0.016	0.002	0.004	0.014	0.001	0.004	0.011	0.002	0.003	0.038

Notes: The dependent variable is a dummy that takes value 1 if there was a rush in the block, as defined in the text, and 0 otherwise. Observations are all blocks with substantial positive change. p.c.= per capita values. All reported results are from linear probability models. Variables are in logs and measured in the base year 1990. All regressions include 1990 population (not reported) as control. See the Appendix for more details on the data sources, variables, and their construction. Huber-White robust standard errors in parentheses. ^{a} $p < 0.01$; ^{b} $p < 0.05$; ^{c} $p < 0.1$.

Table 3: Finance and tech industries growth, 1980 to 2000.

Dependent variable	Wage		Growth rate worker class				Growth rate industries		Growth rate wages	
	1980	2000	Self-employed		Private	Gov.	Both	Tech	Both	Tech
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Tech industry	-0.026^{a}	0.027^{a}					1.584^{a}		1.228^{a}	
	(0.000)	(0.000)					(0.569)		(0.227)	
MSA small initial tech industry								2.154^{b}		0.476
								(1.028)		(0.469)
Growth rate tech industry			0.411^{a}	0.474^{a}	-0.010^{c}	-0.036				
			(0.077)	(0.087)	(0.005)	(0.055)				
Growth rate finance industry			-1.227^{b}	-0.909	0.165^{a}	0.585				
			(0.501)	(0.553)	(0.034)	(0.357)				
Total growth rate			2.044^{a}	1.629^{b}	0.875^{a}	0.310				
			(0.581)	(0.642)	(0.039)	(0.414)				
MSA fixed effects	Yes	Yes	No	No	No	No	No	No	No	No
State fixed effects	No	No	No	Yes	No	No	No	No	No	No
Level controls	No	No	No	Yes	No	No	No	No	No	No
Adj. R-Square	0.601	0.714	0.898	0.901	0.999	0.804	0.015	0.015	0.060	0.000
Observations	270,096	384,092	223	223	223	223	446	223	446	223

Notes: The dependent variable in columns (1) and (2) are the mincerized wages, where the sample is restricted to 1980 or 2000 respectively. The dependent variable in columns (3)–(6) is the growth rate in worker class; self-employed incorporated in (3) and (4), private wage earners in (5), and government in (6). The dependent variable in columns (7) and (8) is the growth rate in the finance and tech industries in (7) and the tech industry in (8). The dependent variable in columns (9) and (10) is the wage growth rate in the finance and technology industries in (9) and the technology industry in (10). An observation in columns (1) and (2) is a person from the IPUMS sample. An observation in columns (3)–(6), (8), and (10) is a MSA. An observation in columns (7) and (9) is an MSA industry. The independent variable *Tech industry* is an indicator for wage earner or growth rate being in the technology industry. The independent variable *MSA small initial tech industry* is an indicator variable equal to one if a MSA initially has fewer technology employees than the median in 1980 and zero otherwise. Additional controls in column (4) are the aggregate number of workers in the technology and finance industries. Standard errors in parentheses. ^{a} $p < 0.01$; ^{b} $p < 0.05$; ^{c} $p < 0.1$