Modeling Techniques
Models of Corporations

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May 9, 2023

Modeling Techniques through Models of Corporate Taxation

Session 1

A Basics of modeling
   1 Introduction—the power of simple.
   2 Foundation of corporate models, Fisher's separation theorem (Fisher, 1930)
   3 Two-period model (Modigliani and Miller, 1958)
   4 Supply and demand (Berger and Seegert, 2023)

B Incidence and welfare
   1 Who pays the tax?
   2 How does incidence intersect with market power?
   3 Overshifting (Ritz, 2014; Pless and van Benthem, 2019; Agrawal and Hoyt, 2019).
   4 Extensions: salience, evasion, and empirical estimates (Bradley and Feldman, 2020; Kopczuk, Marion, Muehlegger, and Slemrod, 2013; Mace, Patel, and Seegert, 2020)

A.1 Introduction—the power of simple

Models are simplifications of the world

“What a useful thing a pocket-map is!" I remarked.

“That’s another thing we’ve learned from your Nation," said Mein Herr, “map-making. But we’ve carried it much further than you. What do *you* consider the *largest* map that would be really useful?"

“About six inches to the mile."

“Only *six* inches!" exclaimed Mein Herr. “We very soon got to six yards to the mile. Then we tried a *hundred* yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of a *mile to the mile*!"

“Have you used it much?" I enquired.

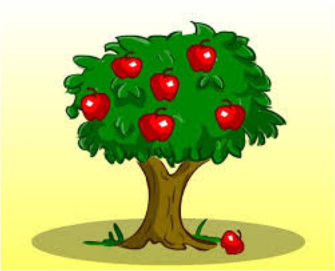
“It has never been spread out, yet," said Mein Herr: "the farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well."

from Lewis Carroll, Sylvie and Bruno Concluded, Chapter XI, London 1895

What explains why apples fall from their tree?

Different disciplines, different models

1. Physics: gravity.
2. Evolutionary biologist: trees that shot their apples upward into space did not propagate.
3. Economist: trees just responded to positive incentives to drop fruit to earth.
4. Accounting?



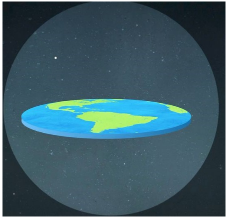
Different models and words with different meanings

WARNING: **Economist by training**

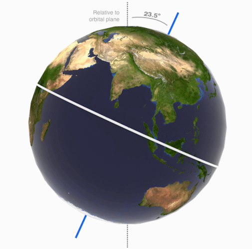
I may use words slightly differently than you are used to. Please do not hesitate to stop and ask.

Retained earnigns I mean a pile of cash.

Consider the distance between the Univ. Utah and BYU



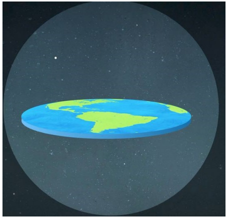
1. Flat earth model.
   - 36.41 miles



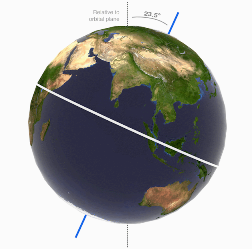
2. Spherical earth model.
   - 36.44 miles

- If you do not plan on going very far, flat earth is a fine model.

Consider the distance between the Univ. Utah and Tokyo



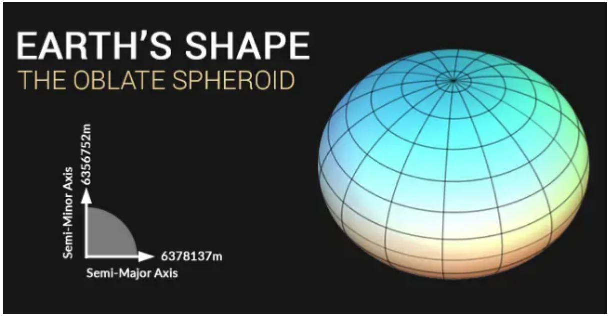
1. Flat earth model.
    - 5110.88 miles miles



2. Spherical earth model.
    - 5479.873 miles

- In far distances may need a better model.

Consider the distance between the Univ. Utah and Tokyo



1. Flat earth model.
   - 5110.88 miles miles
2. Spherical earth model.
   - 5479.873 miles
3. Vincenty oblate spheroid earth model
   - 5492.64 miles

- How detailed of a model do you need?

ORIGINAL CABLE

Technology
Buried fiber-optic cable

Completion
Mid-1980s

Path length
~ 1,000 miles

Round-trip time for data
14.5
milliseconds and up

Approach
Multiple routes followed the easiest rights-of-way—along rail lines. But that means time-sucking jogs and detours.

SPREAD NETWORKS

Technology
Buried fiber-optic cable

Completion
August 2010

Path length
825 miles

Round-trip time for data
13.1
milliseconds

Approach
Spread bought its own rights-of-way, avoiding a Philadelphia-ward dip in favor of a shorter path northwest through central Pennsylvania.

MCKAY BROTHERS

Technology
Microwave beams through air

Completion
July 4, 2012

Path length
744 miles

Round-trip time for data
9
milliseconds

Approach
Microwaves generally move faster than photons in optical fiber, and McKay's network uses just 20 towers on a nearly perfect great circle.

TRADEWORX

Technology
Microwave beams through air

Completion
Winter 2012

Path length
~ 731 miles

Round-trip time for data
8.5
milliseconds (est.)

Approach
Tradeworx is highly secretive, but the company is open about the price of a subscription: $250,000 a year.

Preliminary building blocks

Models need three things

1. Players—who is making a decision (e.g., firm, shareholder, CEO).
2. Strategies—what can the players do (e.g., choose investment levels).
3. Payoffs—what do the players receive (e.g., firm value or utility).

In my writing, I like to spell these out right away and in this order.

Preliminary building blocks

Models are used to highlight trade offs

1. Is your model about a new trade off? (e.g., dividends versus mergers).
2. Is your model about a new feature that affects the tradeoff (e.g., information revelation).
3. Make sure everything supports the novel aspect of your model.

Models start out simple and progress as we add features

1. We will start with the very basic models.
2. These models will be missing a lot of important details.
3. The hope is that these models can be the jumping off point for you to use in your own work
4. and the tools we learn can help build hypotheses from these models.

A.2 Foundation of corporate models

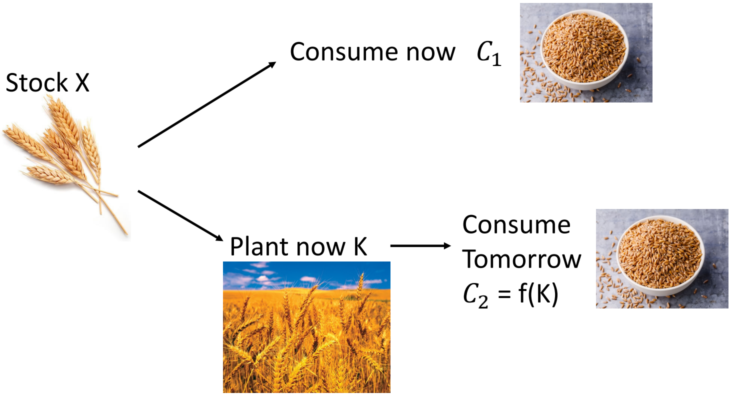
Fisher's separation theorem

Player: Robinson Crusoe

Robinson Crusoe

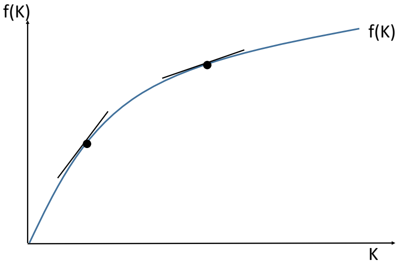
X amount of wheat

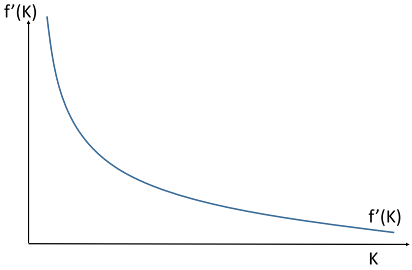
Strategies: Consume now or invest; choose C_1, K



What can be consumed or planted

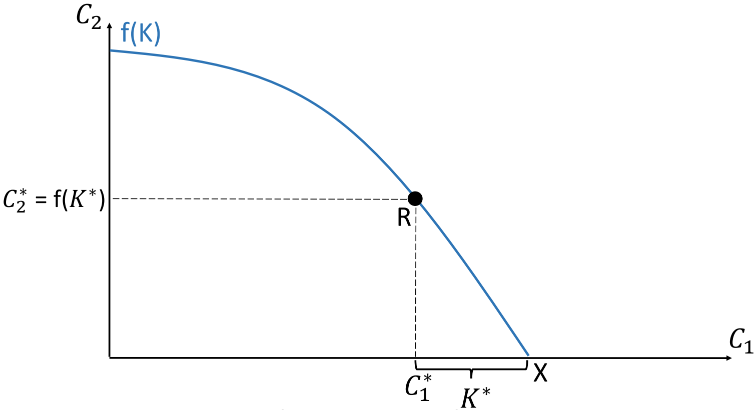
Assumptions on the production function





1. $f(0) = 0$, no production without some planting.
2. $f'() > 0$, the more you plant the more yield.
3. $f''() < 0$, the more you plant the lower the marginal yield.
   - **Diminishing returns** only so much room on the island, as you plant more use worse land or over crowd the wheat such that doubling the seed will not double the yield.

Transformation from C_1 to C_2



- Diminishing returns, get less C_2 for each unit of K as K increases.

Payoffs: utility over consumption

$$max_{C_1, C_2, K} \ U(C_1, C_2)$$

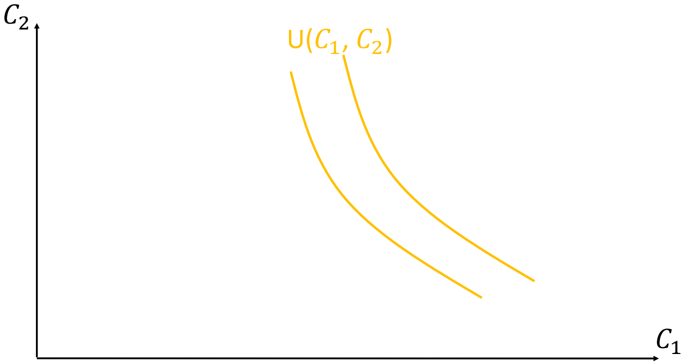
Constraints

1. Cannot consume more today than you have $0 \leq C_1 \leq X$.
2. The sum of consumption today and investment cannot be more than you have $K + C_1 \leq X$.
3. What you consume tomorrow is the yield from production $C_2 = f(K)$.

Assumptions

1. $\partial U(C_1, C_2)/\partial C_1 \equiv U_1 > 0$.
2. $\partial U(C_1, C_2)/\partial C_2 \equiv U_2 > 0$.

Indifference curves are combinations of C_1 and C_2. with the same utility



Household maximization

$$max_{C_1,C_2,K} \;\; U(C_1, C_2) \quad s.t. \;\; 0 \leq C_1 \leq X \;\; \& \;\; X = K + C_1 \;\; \& \;\; C_2 = f(K)$$

$$\mathcal{L} = U(C_1, C_2) + \lambda(X - C_1 - K) + \gamma(f(K) - C_2)$$

$$\frac{\partial \mathcal{L}}{\partial C_1} : U_1 = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial C_2} : U_2 = \gamma$$

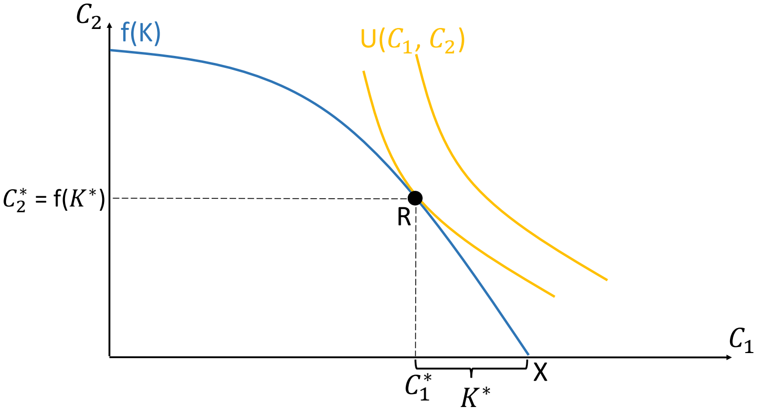
$$\frac{\partial \mathcal{L}}{\partial K} : \lambda = \gamma f'(K)$$

$$\rightarrow U_1 = \gamma f'(K) = U_2 f'(K)$$

- First-order condition: $U_1/U_2 = f'(K)$.

Household maximization

- First-order condition: $U_1/U_2 = f'(K)$.



Optimization can be interpreted in two ways:

1. Marginal change in utility between C_1 and C_2 must equal the marginal change in production. (Marginal rate of substitution equals the marginal rate of transformation MRS = MRT)
   - $U_1/U_2 = f'(K)$.
2. The rate of time preferences $\gamma(C_1, C_2) \equiv -U_1/U_2 - 1$ equals the net marginal product of capital
   - $\gamma(C_1, C_2) = f'(K) - 1$.

- This model told us about the tradeoff between consumption today and tomorrow.

The role of the capital market

- Now, lets see how capital markets change this tradeoff.

1. Player: a single household with endowment X.
2. Strategies:
   - Consumption now C_1,
   - Borrowing or saving B at interest rate r,
   - Investment K,
3. Payoffs: utility over consumption today and tomorrow $U(C_1, C_2)$

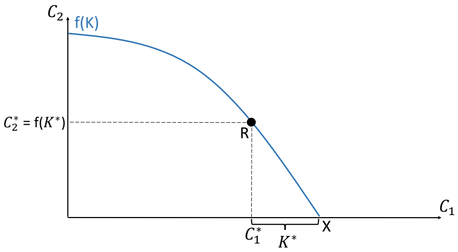
$$C_1 = X - K + B$$

$$C_2 = f(K) - (1 + r)B.$$

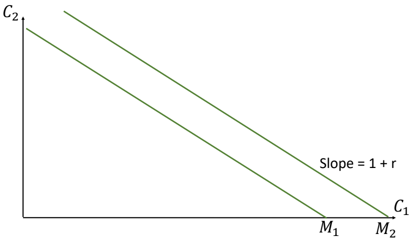
Two ways of transforming C_1 and C_2

Production/Investment:

$$C_2 = f(K) = f(X - C_1).$$

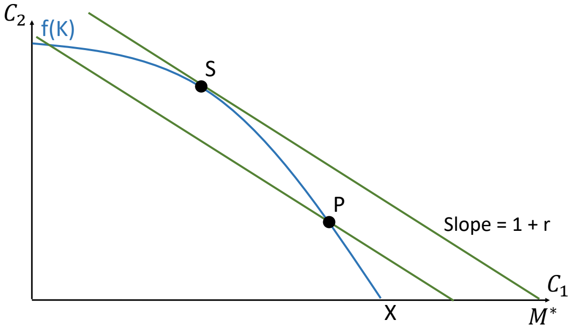


Capital markets: $C_2 = -(1 + r)C_1 + (1 + r)X.$



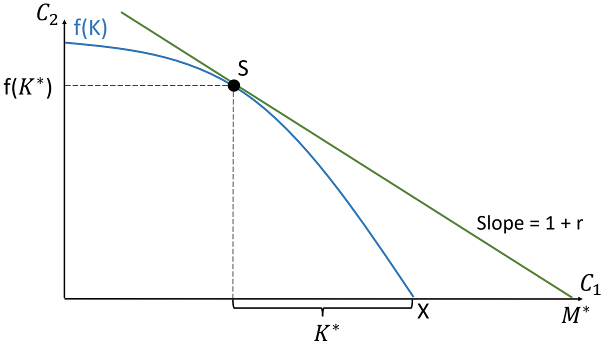
- Wealth is greater at M_2 than M_1.

First, find how much to produce (S or P)



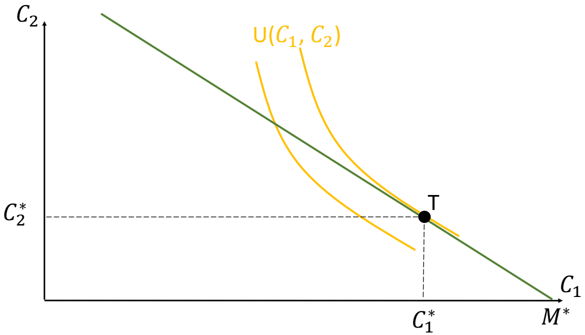
- Maximize wealth M^*, where $f'(K) = 1 + r$

First, find how much to produce



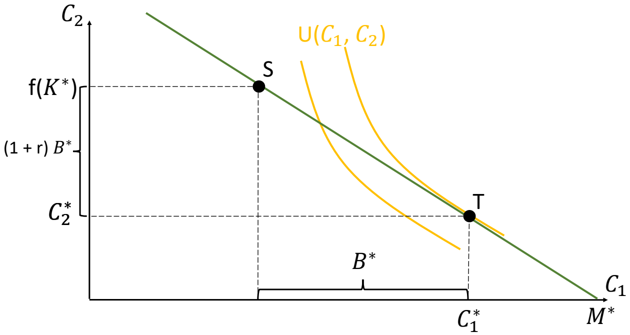
- K^* determines how much to produce $f(K^*)$.
- M^* determines wealth.

Second, find how much to consume



- Maximize utility where $U_1(C_1, C_2)/U_2(C_1, C_2) = 1 + r$, where M^* is given.

Second, find how much to consume



- Start at point S.
- Borrow B^* and repay $B^*(1 + r)$ to get to T.

Optimization with capital markets

$$max_{C_1} \;\; U(C_1, C_2) \quad s.t. \;\; 0 \leq C_1 \leq X \;\; \& \;\; C_1 = X - K + B \;\; \& \;\; C_2 = f(K) - (1+r)B$$

Optimality conditions for an interior solution

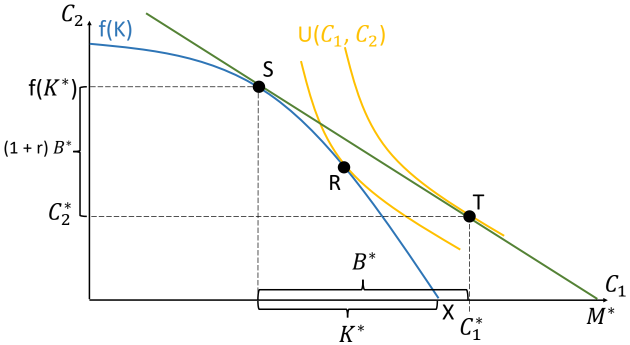
1. $U_1 / U_2 = 1 + r$

2. $f'(K) = 1 + r$

Marginal rate of substitution and marginal product of capital has to equal $1 + r$.

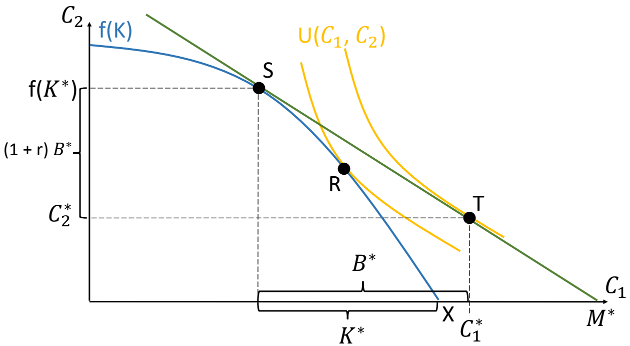
Optimization with capital markets

1. **Separation Theorem:** Point *S* defines the production decision and is independent of household preferences and initial capital endowment.



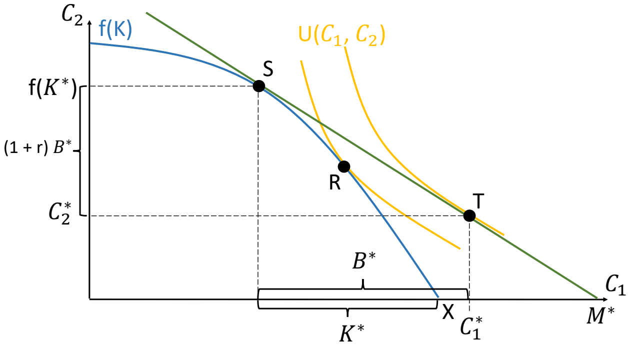
Optimization with capital markets

2. The optimal production decision maximizes wealth M^*.



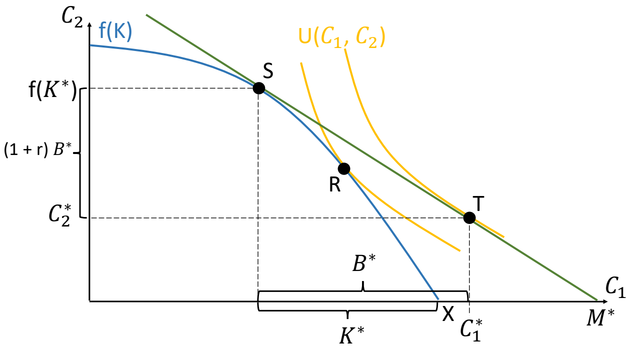
Optimization with capital markets

3. Point T defines the consumption decision and is independent of production, once we know M^*.



Capital markets expand the feasible points

4. Utility at point T is greater than at point R, and is a Pareto optimum.



Fisher's model implications

1. **Separation theorem** The production decision is independent of household preferences and initial capital endowment.
   - $f'(K) - 1 = r$.
2. The optimal production decision maximizes wealth and net present value.
   - Wealth $M^* = \frac{f(K)}{1+r} + C_1 - B$.
   - Net present value = $\frac{f(K)}{1+r} - K$.
3. The optimal consumption decision depends on wealth.
   - Production and interest rate only matter as it impacts wealth.
4. This equilibrium is a Pareto optimum
   - No two households could make a mutually beneficial trade.
   - Aggregate production is maximized.
   - No one's utility could be increased without decreasing someone elses.

What else might be important in this model?

1. How could/should this model be extended?

2. What are the limitations of this model?

Extensions of the model

All four results hold even if there are

1. More than two periods.
2. Different capital and consumption goods.
3. Joint ownership of production across households.

This analysis is partial equilibrium

1. It holds fixed r.
2. It is poorly suited to study intertemporal allocations.
3. Solow (1956) model can be incorporated to study capital accumulation.
4. Overlapping generation models Carmichael (1982), Barro (1974), Burbidge (1963), some inconsistency of laissez-faire allocation and social planner.

A.3 Two-period model example with Modigliani and Miller (1958)

We want to investigate optimal debt issuances

1. How do we build a model to investigate debt issuances?

2. What is the minimum structure needed to gain insights into this problem?

3. What is the key **tradeoff**?
   - Benefit: debt can increase capital.
   - Benefit: debt can increase dividends.
   - Cost: pay back with interest next period.

Cost and benefit of debt

B is debt (bonds, borrowing).

1. Benefit: debt can increase capital or dividends.

$$B = K + D - X$$

   - K is capital (investment) used to produce $f(K)$.
   - D is dividends (what we consume now).
   - X is initial cash on hand (exogenously given).

2. Cost: pay back with interest next period.

$$(1 + r)B/(1 + r)$$

   - Pay back $(1 + r)B$, but do so next period, r is the interest rate.

Basic model moving forward relabeled dividends and debt

A firm chooses its dividend and debt policies to maximize the value of the firm, which is consumption today plus discounted consumption tomorrow:

$$max_{B,D} \quad D + \frac{f(K) - (1+r)B}{1+r} = D + \frac{f(X+B-D) - (1+r)B}{1+r}$$

1. B is debt.
2. Capital is $K = X + B - D$.
3. D is dividends (what we consume now).
4. X is initial cash on hand (exogenously given).
5. r is the interest rate.

Marginal benefit equals marginal cost

A firm chooses its dividend and debt policies to maximize the value of the firm

$$max_{B,D} \quad V = D + \frac{f(X + B - D) - (1 + r)B}{1 + r} \tag{3}$$

First order condition with respect to debt B

$$\partial B : \quad \frac{f'(X + B - D)}{1 + r} - \frac{1 + r}{1 + r} = 0 \tag{4}$$

$$\underbrace{f'(X + B - D)}_{\text{marginal benefit}} = \underbrace{1 + r}_{\text{marginal cost}}$$

Marginal benefit equals marginal cost

Firm chooses its dividend and debt policies to maximize the value of the firm

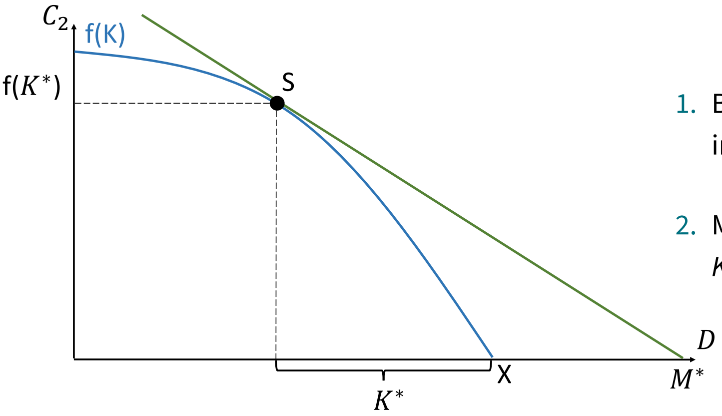
$$max_{B,D} V = D + \frac{f(X + B - D) - (1 + r)B}{1 + r} \tag{5}$$

First order condition with respect to dividends D

$$\partial D : \quad 1 - \frac{f'(X + B - D)}{1 + r} = 0 \tag{6}$$

$$\underbrace{f'(X + B - D)}_{\text{marginal benefit}} = \underbrace{1 + r}_{\text{marginal cost}}$$

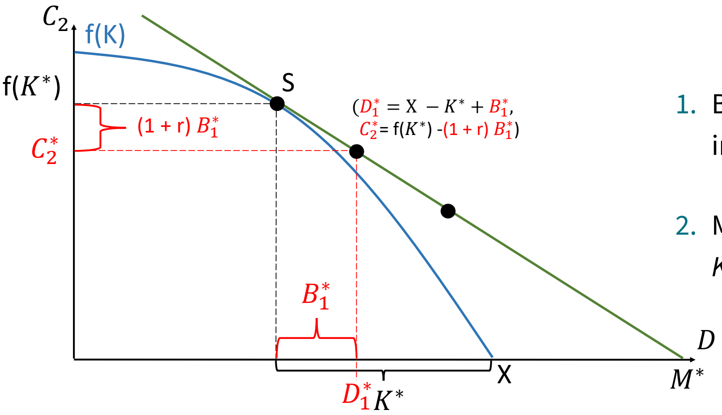
Capital is determined but not dividends or debt



1. Both first-order conditions imply $f'(K) = 1 + r$.

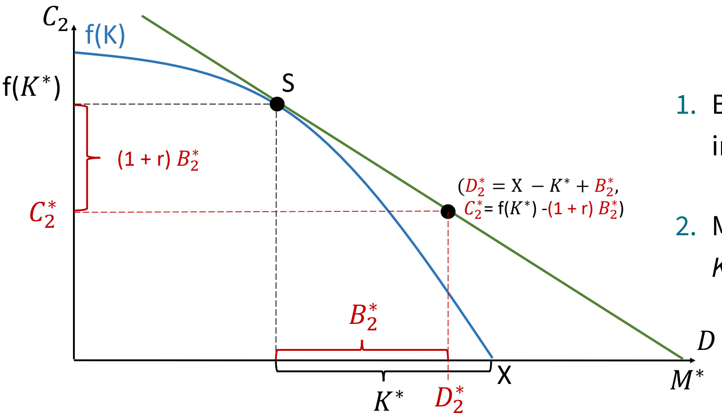
2. Many ways of getting the same $K = X + B - D$.

Borrow a little to fund modest dividends



1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.

Borrow a lot to fund a large dividend



1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.

Modligani-Miller in our basic model

1. The optimal debt and dividend policies are indeterminate!

2. Value remains constant with an increase in debt and higher dividend payments (or the reverse).

3. Of course, this is not the end of story because there are taxes.

What else might be important in this model?

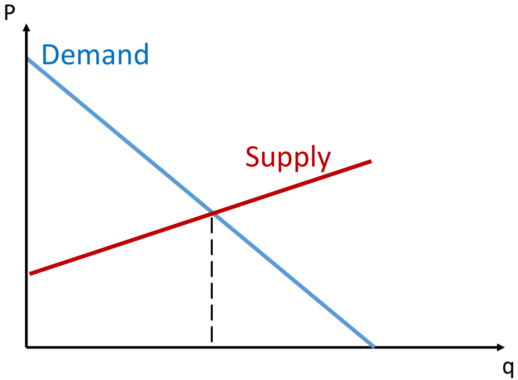
1. What interest rate matters for investment? Long-run or short-run?

2. How would depreciation be included in the model?

A.4 Supply and demand

Berger and Seegert (2023)

We all know supply and demand



1. Starting point for many models.
2. Good intuition.
   -
3. Even simple models are valued by good journals.
   - Berger and Seegert (2023) conditionally accepted at the Journal of Finance.

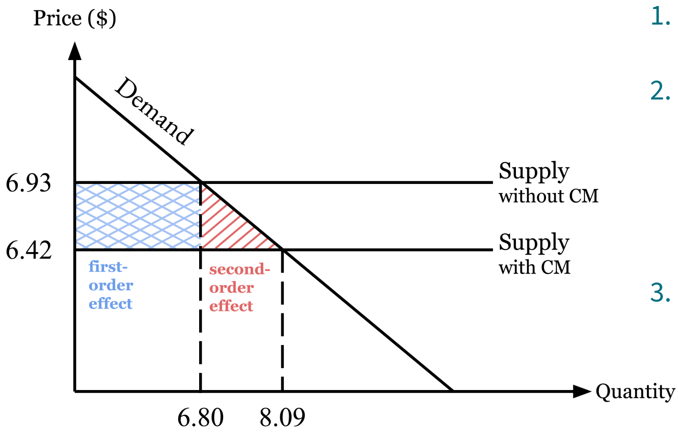
Benefits from cash management

Consider the following scenario and then draw a supply and demand graph,

1. Two markets, the retail market between retail firms and customers and the wholesale market between retail firms and wholesale firms.
2. Two types of retail firms,
   - With access to cash management
   - Without access to cash management.
3. Draw supply and demand in the retail market accounting for whether the retail firm has access to cash management.
4. Draw supply and demand in the wholesale market accounting for whether the retail firm has access to cash management.

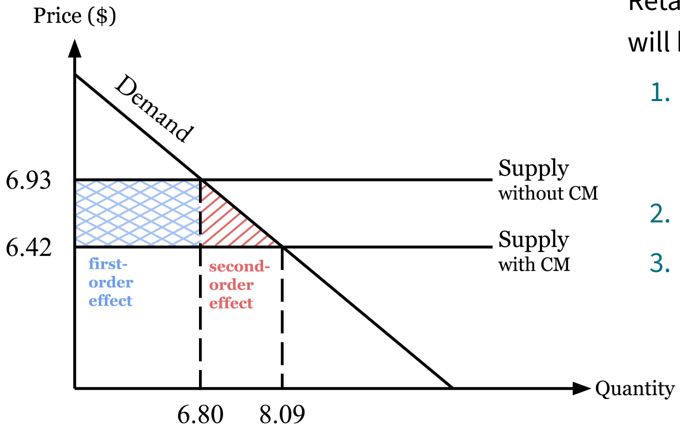
- Focus in this analysis is the retail firm.

Wholesale market supply and demand



1. Standard demand curve.
2. Perfectly elastic supply.
   - Simplifying assumption, good/bad?
   - How does it change the analysis?
3. Lack of cash management modeled as an additional marginal cost (tax)
   - Shift up in the supply curve.

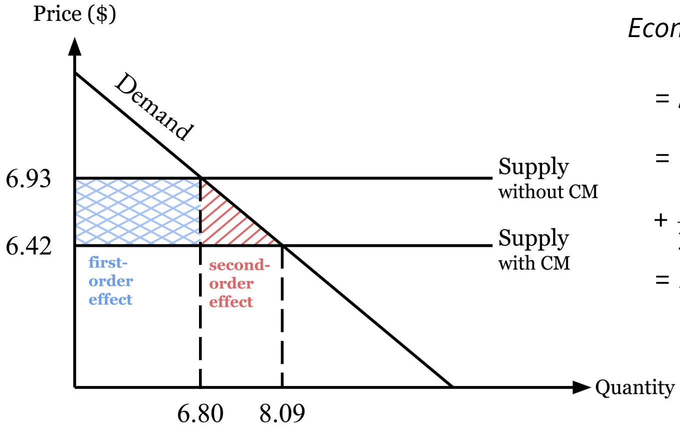
Predictions from the wholesale market



Retail firms with cash management will have

1. Lower wholesale prices for the same product.
2. Buy more product.
3. Lack of cash management leads to a cost of higher wholesale prices and fewer products bought.

Estimating costs from the wholesale market



Economic value wholesale market

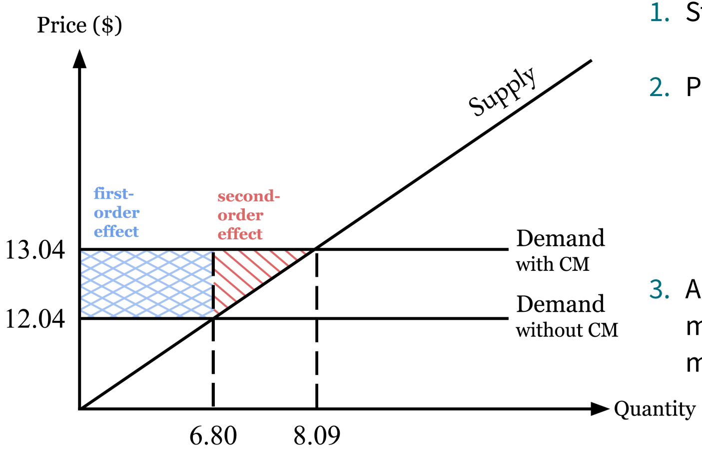
$$= \Delta P_w \times q_w(\theta_w > 0) + \frac{1}{2}\Delta P_w \Delta q_w$$

$$= (6.93 - 6.42) \times 6.8$$

$$+ \frac{1}{2}(6.93 - 6.42) \times (8.09 - 6.80)$$

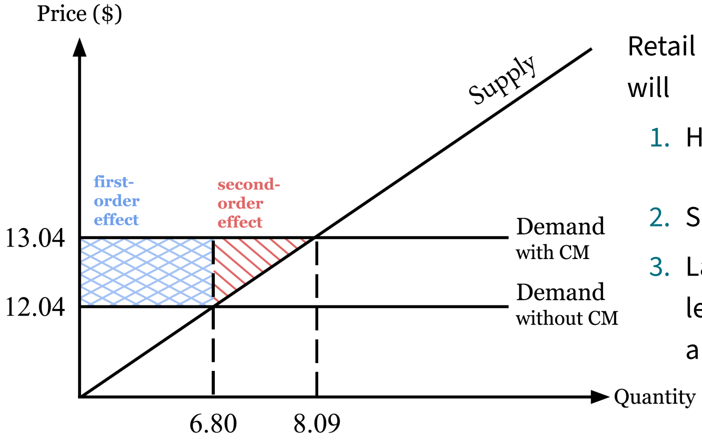
$$= \$3.80$$

Retail market supply and demand



1. Standard supply curve.
2. Perfectly elastic demand.
   - Simplifying assumption, good/bad?
   - How does it change the analysis?
3. Access to cash management modeled as an additional marginal benefit
   - Shift up in the demand curve.

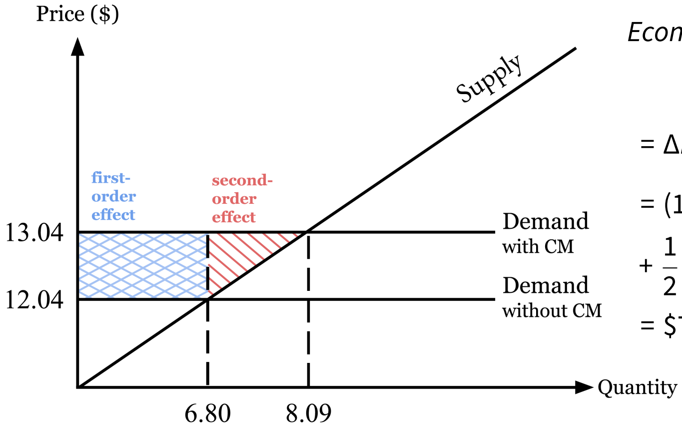
Predictions from the retail market



Retail firms with cash management will

1. Have higher retail prices.
2. Sell more products.
3. Lack of cash management leads to a cost of lower prices and lower quantity.

Estimating costs from the retail market



Economic value retail market

$$= \Delta P_r \times q_r(\theta_r > 0) + \frac{1}{2}\Delta P_r \Delta q_r$$

$$= (13.04 - 12.04) \times 6.80$$

$$+ \frac{1}{2}(13.04 - 12.04) \times (8.09 - 6.80)$$

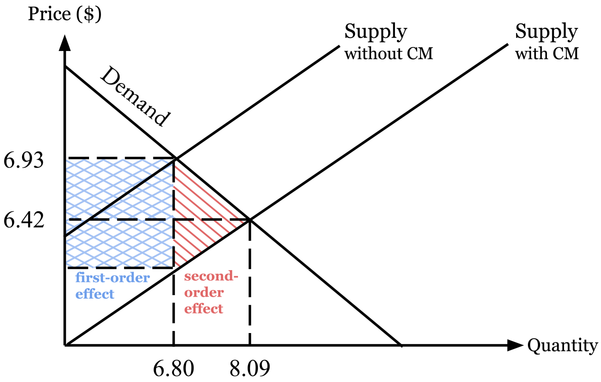
$$= \$7.45$$

The value of cash management

Berger and Seegert (2023) finds that the value of cash management in the marijuana industry in Washington is substantial,

- Total value $18,000,000 or 1.8% of total industry sales.
- $6,000,000 in the wholesale market
- $12,000,000 in the retail market.

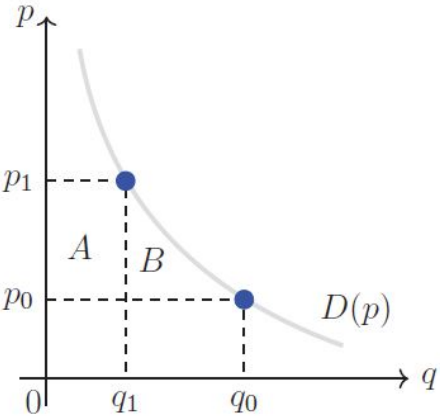
Predictions robust to other modeling choices



In the wholesale market supply can be inelastic.

1. Same predictions on price and quantity.
2. Costs from lack of cash management larger.

Relax linearity assumption

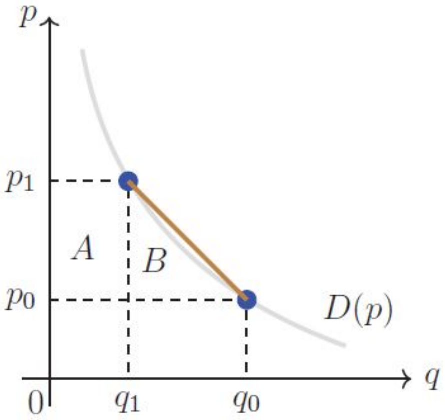


Kang and Vasserman (2022)

1. Bounds on the change in welfare.

2. Tighter bounds with different assumptions.

Relax linearity assumption



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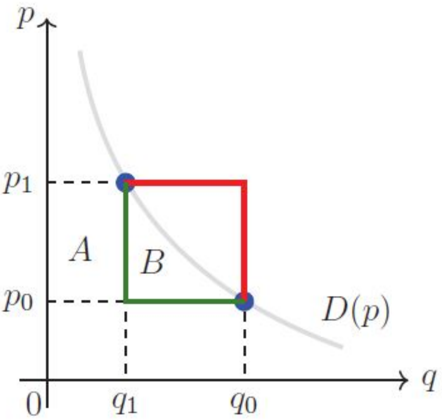


Kang and Vasserman (2022)

1. Bounds on the change in welfare.
2. Tighter bounds with different assumptions.

Reefer Cashness—example from Richard Sansing

Consider a duopoly with one banked firm (B) and one unbanked firm (U). The products are perfect substitutes, except consumers must pay with cash at the unbanked firm. For every dollar customers are willing to pay using a non-cash medium of exchange, they are willing to pay θ dollars in cash. Each firm pays the same per unit cost c for inputs. The non-cash market price p is

$$p = d - q_B - q_U,$$

where $d > c$.

Firm B solves $\quad max_{q_B} \quad q_B(p - c)$

Firm U solves $\quad max_{q_U} \quad q_U(\theta p - c)$

Solution and implications

$$q_B = \frac{\theta(d - 2c) + c}{3\theta},$$
$$q_U = \frac{\theta(d + c) - 2c}{3\theta},$$

1. Banked firm's output increases

2. Unbanked firm's decreases

3. Economic effect is complicated, firm B is better off and firm U is worse off.

4. If θ is too small, the unbanked firm drops out.

Usefulness of a model increases with its simplicity

1. What is the simplest your model can be to show the result you want to highlight?

2. Is your model robust to more realistic assumptions?

Remember the goal of the model is to *clarify* the tradeoff you are studying.

Session 1 B

Incidence and welfare—who pays the tax?

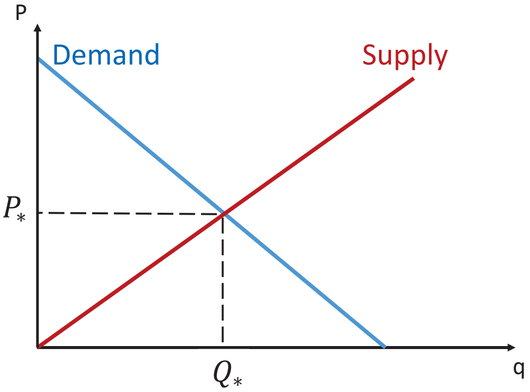
Incidence and welfare

B Incidence and welfare

1. Who pays the tax?
2. How does incidence intersect with market power?
3. Overshifting (Ritz, 2014; Pless and van Benthem, 2019; Agrawal and Hoyt, 2019).
4. Extensions: salience, evasion, and empirical estimates (Bradley and Feldman, 2020; Kopczuk, Marion, Muehlegger, and Slemrod, 2013; Mace, Patel, and Seegert, 2020)

B.1 Who pays the tax?

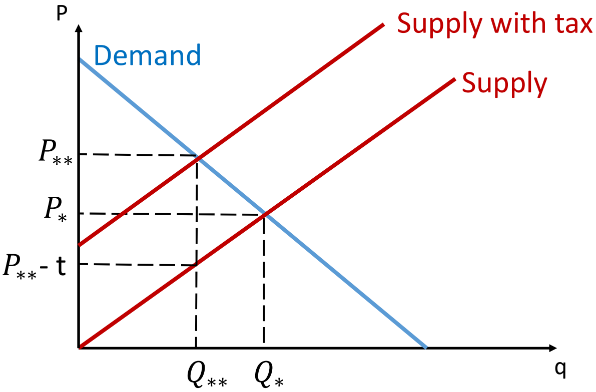
Linear supply and demand example



- Start with basic supply and demand

$$Q = D(p) = S(p)$$

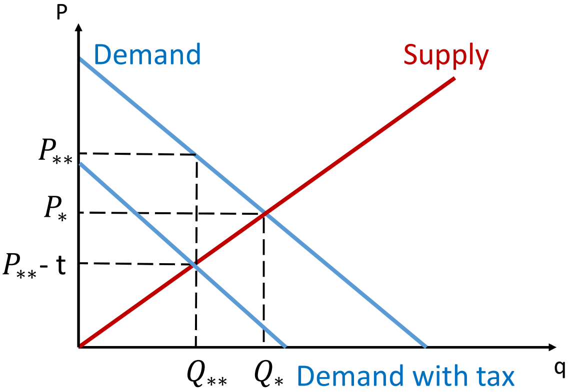
Model a tax as a shift in supply (marginal cost)



- Add a tax

$$Q = D(P) = S(P - t)$$

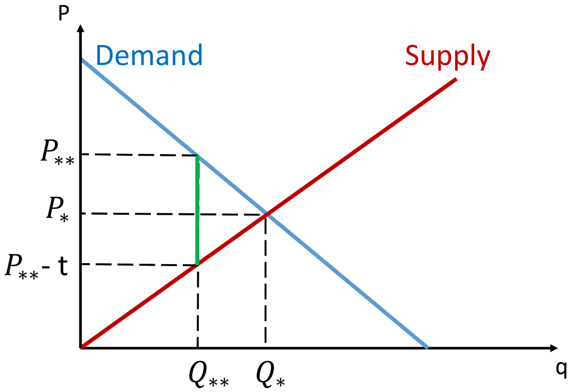
Could model a tax as a shift down in demand



- Add a tax

$$Q = D(P) = S(P - t)$$

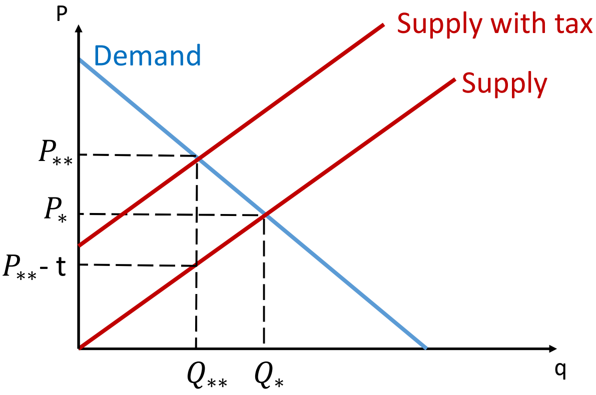
Could model a tax as a wedge between demand and supply



- Add a tax

$$Q = D(P) = S(P - t)$$

The pass-through rate is the change in consumer price



- Totally differentiate

$$Q = D(P) = S(P - t)$$

$$\frac{\partial D}{\partial P} dP = \frac{\partial S}{\partial P} dP - \frac{\partial S}{\partial P} dt$$

$$\frac{dP}{dt} = \frac{-\frac{\partial S}{\partial P}}{\frac{\partial D}{\partial P} - \frac{\partial S}{\partial P}}$$

$$\rho = \frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D} > 0$$

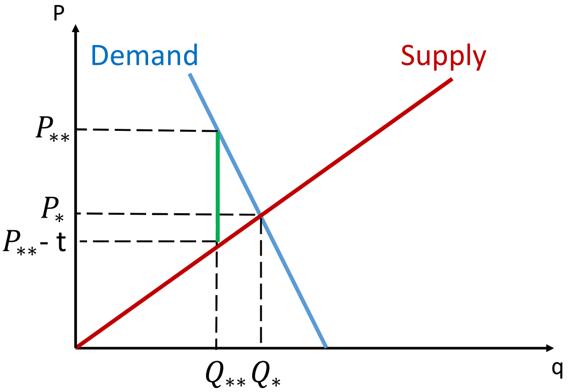
Quick reminder about elasticities

Elasticity of demand

$$\varepsilon_D = \frac{dQ}{dP}\frac{P}{Q} = \frac{dQ/Q}{dP/P} = \frac{\%\Delta Q}{\%\Delta P} = \frac{1}{slope_D}\frac{P}{Q}$$

1. Elasticity of demand is negative because the slope of the demand curve is negative.
2. With linear demand, elasticity increases in magnitude with higher P and lower Q.
3. Revenue is maximized where the elasticity of demand = -1.
4. Monopolist always in the elastic part of the demand curve $|\varepsilon_d| > 1$

The pass-through rate increases with more inelastic demand

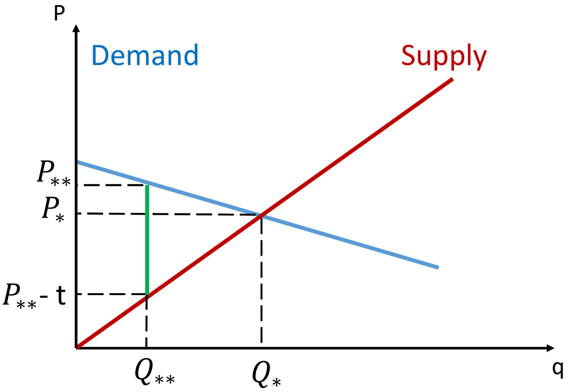


- consumer price increases more as demand becomes more inelastic relative to supply

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_D \to 0 \implies \frac{dP}{dt} = 1$

The pass-through rate decreases with more elastic demand

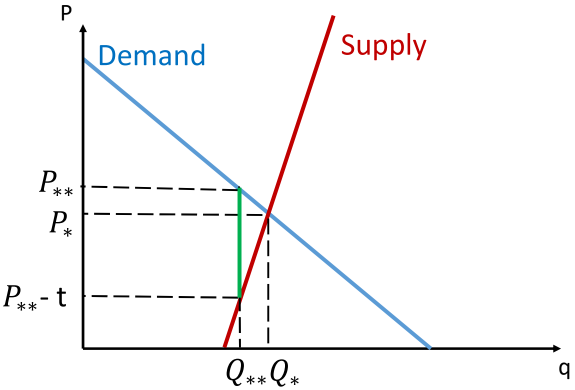


- Consumer prices increase less as demand becomes more elastic relative to supply

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_D \to \infty \implies \frac{dP}{dt} = 0$

The pass-through rate decreases with more inelastic supply

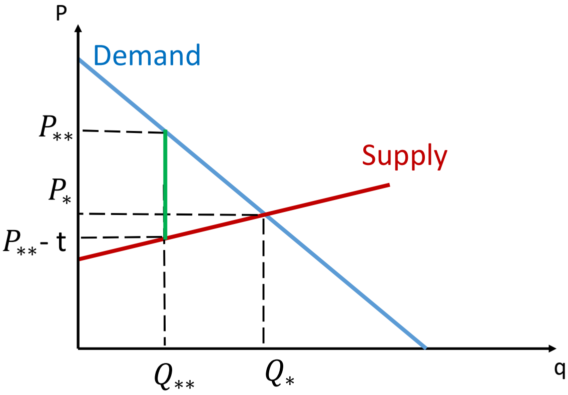


- Consumer prices increase less as supply becomes more inelastic relative to demand

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_S \to 0 \implies \frac{dP}{dt} = 0$

The pass-through rate increases with more elastic supply



- Consumer prices increase more as supply becomes more elastic relative to demand

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_S \to \infty \implies \frac{dP}{dt} = 1$

The basic takeaway—the inelastic party pays the tax

The first principle of tax incidence is that being inelastic is costly.

When you leave a party you should say, stay elastic my friends, stay elastic.

B.2 Can Monopolists push the tax onto consumers?

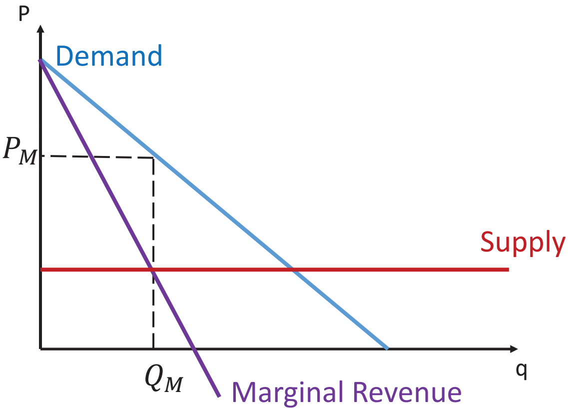
Poll

Which statement is true about incidence and market power?

a The monopolist pushes most of the tax onto consumers.
  - Monopolist has market power and is thus able to avoid the tax.

b The monopolist pays most of the tax.
  - The monopolist has all of the rents (or a lot of them) before the tax and thus when a tax is imposed it has to come out of their rents.

Linear supply and demand



Special case: linear demand

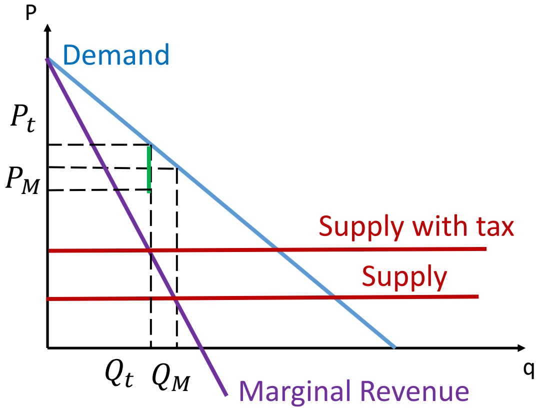
- Demand $P = A - BQ$
- Cost = CQ
- Monopolist problem

$$max_Q \quad (A - BQ)Q - CQ$$

$$A - BQ - BQ - C = 0$$

$$Q = \frac{A - C}{2B}, \quad P = \frac{A + C}{2}$$

Tax with monopolist and linear supply and demand

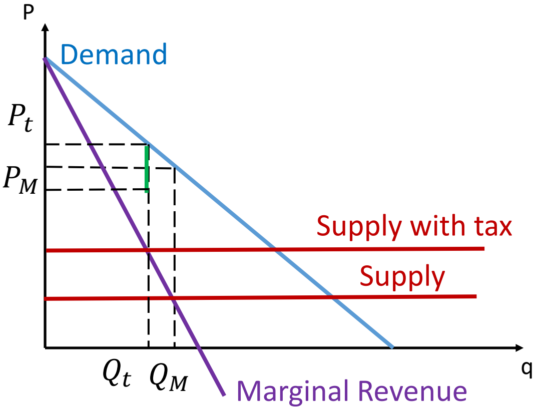


Special case: linear demand

- Demand $P = A - BQ$
- Cost = $CQ + tQ$

How much of the tax does the monopolist push onto the consumers?

Monopolist evenly splits the tax with consumers



- Monopolist problem

$$max_Q \quad (A - BQ)Q - CQ - tQ$$

$$A - BQ - BQ - C - t = 0$$

$$Q = \frac{A - C - t}{2B}$$

$$P = \frac{A + C}{2} + \frac{1}{2}t$$

The monopolist pushes *half* of the tax onto the consumers.

More complicated examples call for other tools

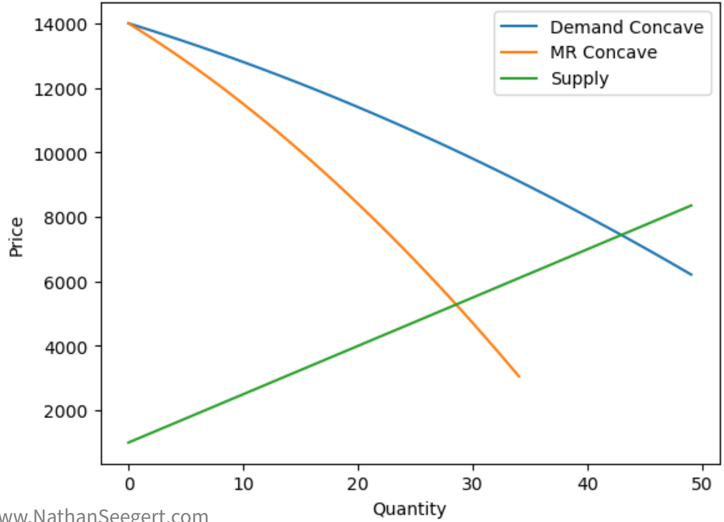
1. Linear models are nice. Easy to solve by hand.

2. Sometimes we need to go beyond linear models.

3. In these cases, there is python!
    - Or other programs like Mathematica.

4. Python code for these examples is provided on `www.nathanseegert.com/teaching`

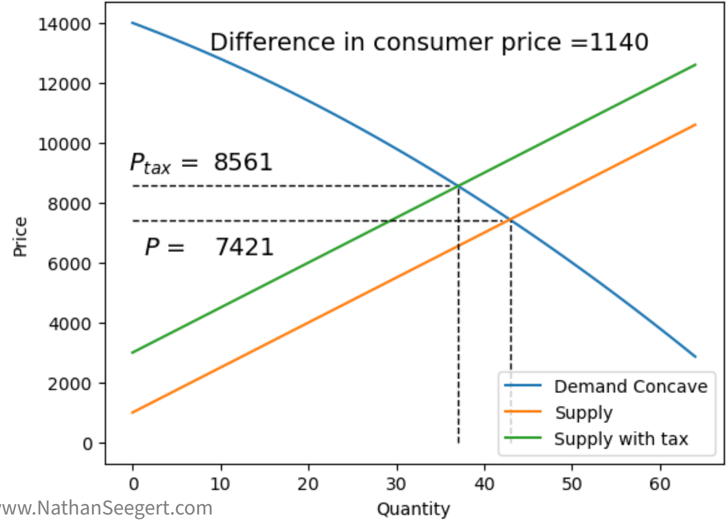
Concave inverse demand



Special case: concave demand

- Concave inverse demand $P = A + BQ + CQ^2$
- Linear MC $C'(D(P)) = Z + YQ$
- A = 14,000, B = -110, C = -1
- Z = 1,000, Y = 150

Competitive price with and without a tax

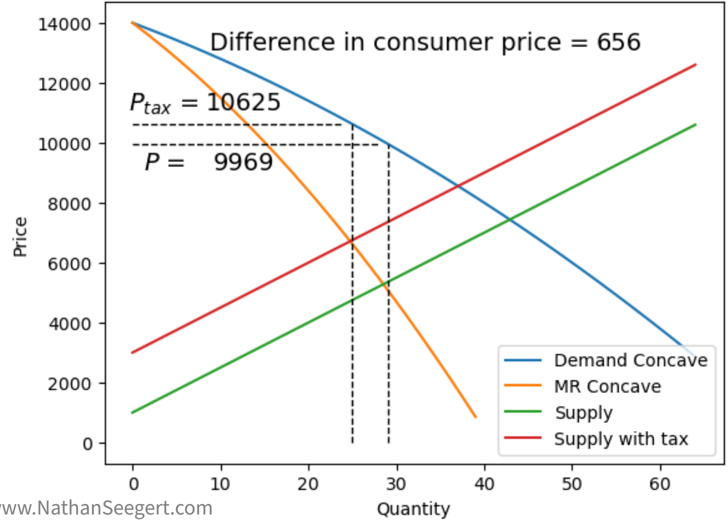


Special case: concave demand

- Concave inverse demand $P = A + BQ + CQ^2$
- Linear MC $C'(D(P)) = z_1 + y_1 Q + tQ$
- Tax t = 2000

Consumers pay 1140/2000 or 57% of the tax

Monopolist pays more of the tax with concave demand

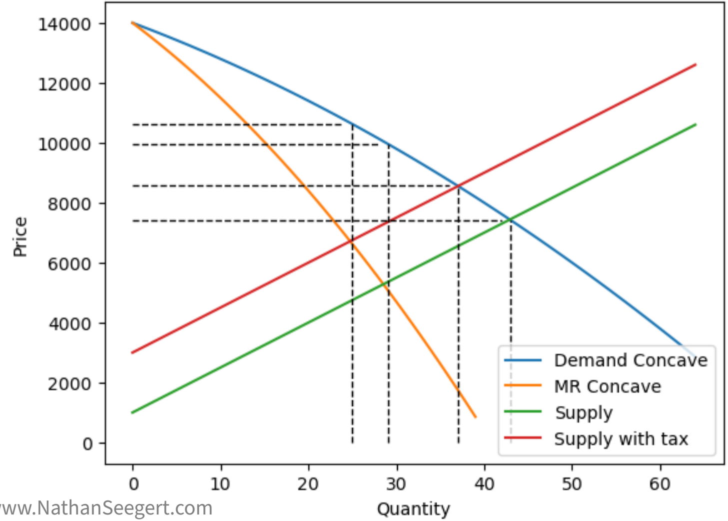


Special case: concave demand

- Concave inverse demand
  $P = A + BQ + CQ^2$
- Linear MC
  $C'(D(P)) = Z + YQ + tQ$
- Tax t = 2000

Consumers pay 656/2000 or 33% of the tax

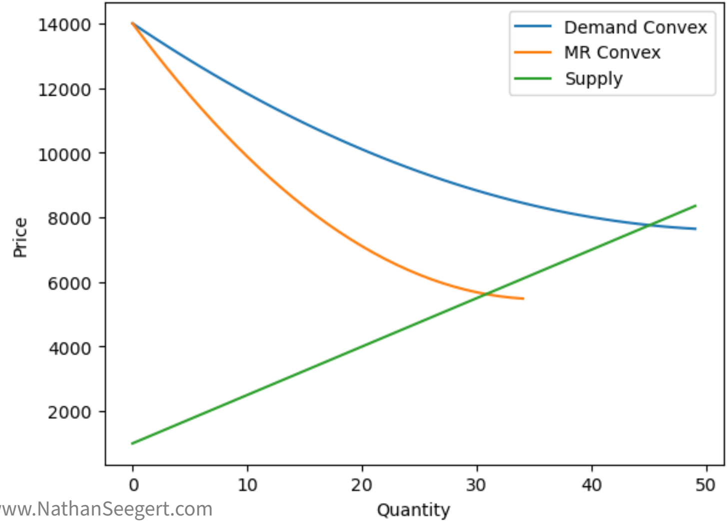
Monopolist pays more of the tax with concave demand



Special case: concave inverse demand

- Competitive: consumers pay 1140/2000 or 57%
- Monopoly: consumers pay 656/2000 or 33%

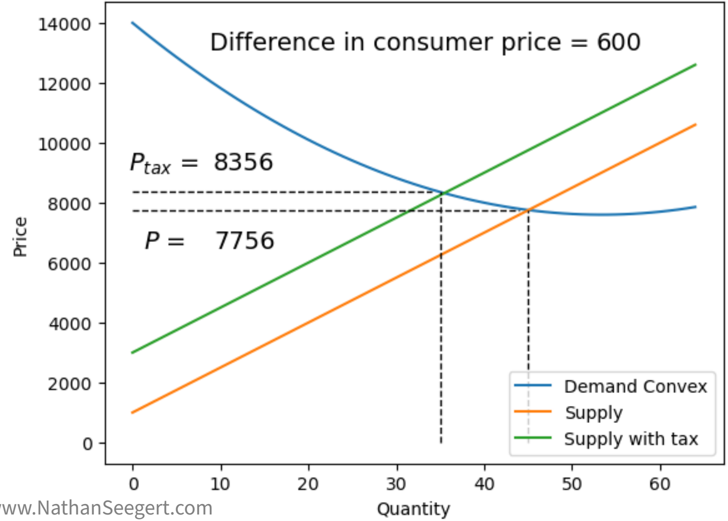
Very convex inverse demand



Special case: convex demand

- Convex inverse demand $P = A + BQ + CQ^2$
- Linear MC $C'(D(P)) = Z + YQ$
- A = 14,000, B = -240, C = 2.25
- Z = 1,000, Y = 150

Competitive price with and without a tax

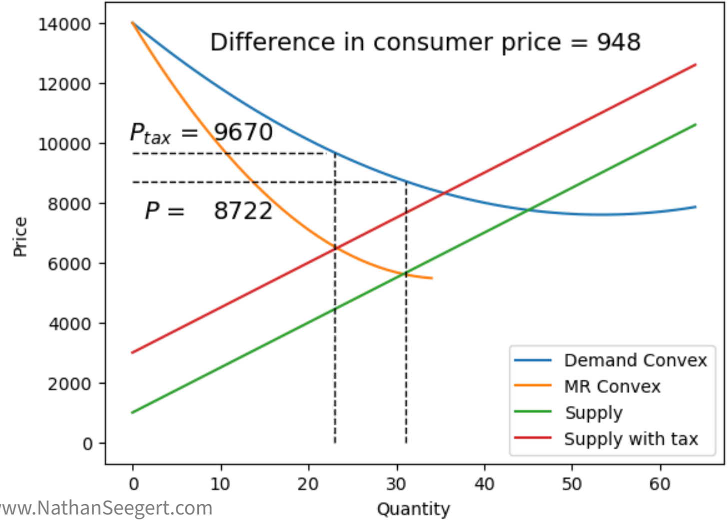


Special case: convex demand

- Convex inverse demand
  $P = A + BQ + CQ^2$
- Linear MC
  $C'(D(P)) = z_1 + y_1 Q + tQ$
- Tax t = 2000

Consumers pay 600/2000 or 30% of the tax

Monopolist pays less of the tax with convex demand

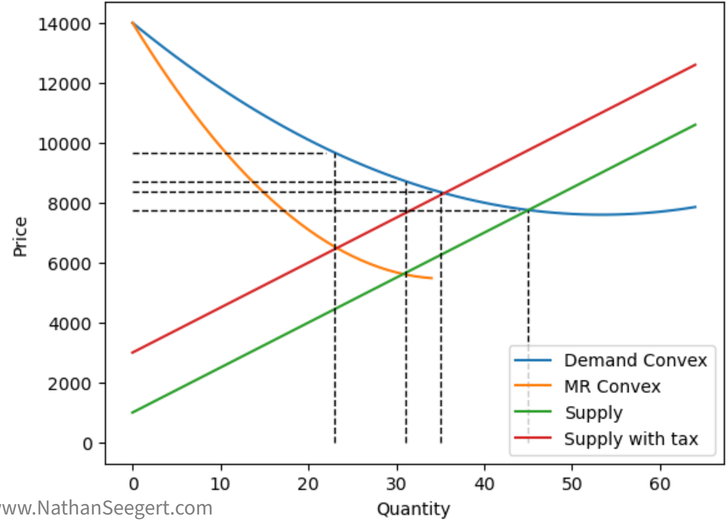


Special case: convex demand

- Convex inverse demand $P = A + BQ + CQ^2$
- Linear MC $C'(D(P)) = Z + YQ + tQ$
- Tax t = 2000

Consumers pay 948/2000 or 47% of the tax

Monopolist pays less of the tax with convex demand



Special case: convex demand

- Competitive: consumers pay 600/2000 or 30%
- Monopoly: consumers pay 948/2000 or 47%

Pass-through for a monopolist

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + 1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}}$$

- ε_{ms} is the elasticity of marginal surplus, measuring the curvature of the demand curve.
  - $ms = -(\partial p/\partial q)q$
  - Log-concave demand $\frac{1}{\varepsilon_{ms}} > 0$.
  - log-convex demand $\frac{1}{\varepsilon_{ms}} < 0$.
- Linear demand $\varepsilon_{ms} = 1$.
- Exponential demand $1/\varepsilon_{ms} \to 0$.
- Constant elasticity demand $\varepsilon_{ms} = -\varepsilon_D$.

Pass-through for a monopolist special cases

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + 1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}}$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and linear demand ($1/\varepsilon_{ms} = 1$)

$$\rho = \frac{1}{2}$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and concave demand ($1/\varepsilon_{ms} > 1$)

$$\rho \in \left[0, \frac{1}{2}\right]$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and convex demand ($1/\varepsilon_{ms} < 1$)

$$\rho \in \left[\frac{1}{2}, 1\right]$$

Generalized formula for imperfect competition

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + \theta}{\varepsilon_S} + \frac{\theta}{\varepsilon_{ms}}}$$

- $\theta \in (0, 1)$ is the conduct parameter; perfect competition $\theta = 0$ monopoly $\theta = 1$.
  - In a Cournot model with N symmetric firms $\theta = 1/N$.
- Assumes $\frac{1}{\varepsilon_\theta} = 0$. See Weyl and Fabinger (2013) for a discussion of this assumption.

Generalized formula for imperfect competition

How much do consumer prices change with the tax as market power changes?

$$\frac{\partial \rho}{\partial \theta} = \frac{1}{\left(1 - \frac{\varepsilon_D + \theta}{\varepsilon_S} + \frac{\theta}{\varepsilon_{ms}}\right)^2} \left(\frac{-1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}\right)$$

1. Sign depends on how big or small elasticity of marginal surplus, which measures the curvature of the logarithm of demand.

B.3 Overshifting and market power

Weird case consumer price changes more than the tax

In the literature, this is called overshifting.

- Empirically, there have been cases where the estimates suggest consumer prices change more than the tax.
    1. Evidence of overshifting, or imprecise estimates.
    2. Overshifting in alcohol (Cook 1981; Young and Bielinska-Kwapisz 2002; Kenkel 2005)
    3. Overshifting clothing and personal care items (Poterba 1996; Besley and Rosen 1999).
    4. But could be due to price points (Conlon and Rao 2020).

Pass-through for a monopolist special cases

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + 1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}}$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and convex demand ($1/\varepsilon_{ms} < 0$)

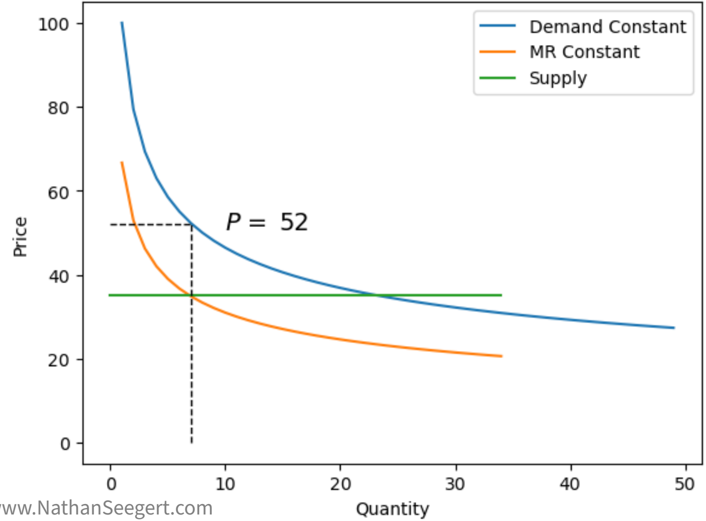
$$\rho > 1$$

- Assume constant elasticity of demand

$$Q = aP^{\varepsilon_D}$$

- The elasticity of marginal surplus $\varepsilon_{ms} = \varepsilon_D$

Monopolist pays less of the tax with convex demand

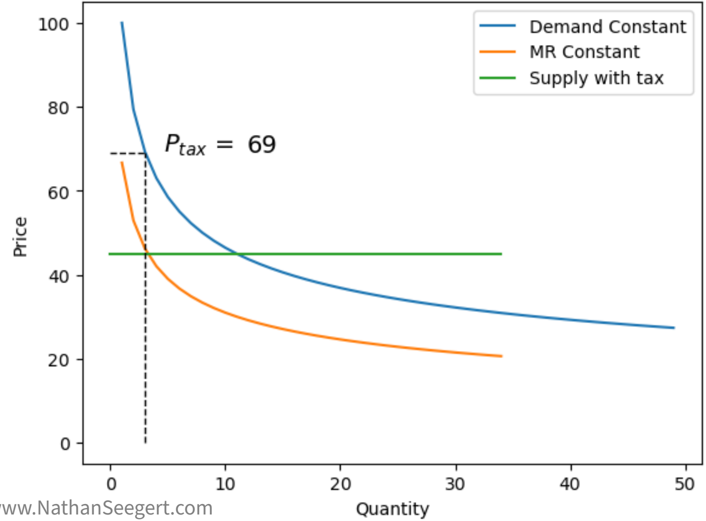


constant elasticity of demand

$$Q_D = 1000000P^{-3}$$

$$P_S = 35$$

Monopolist pays less of the tax with convex demand

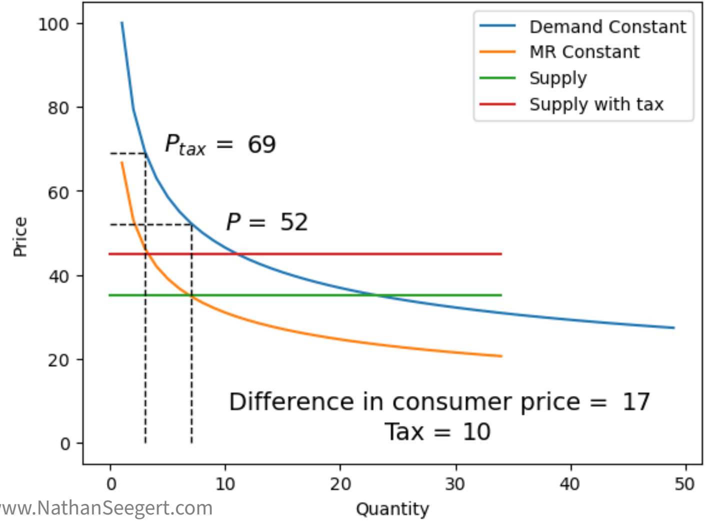


With a tax of 10

$$Q_D = 1000000P^{-3}$$

$$P_S = 35 + 10$$

Monopolist pays less of the tax with convex demand



Consumers pay 17/10 or 170% of the tax

Weird case consumer price changes more than the tax

- In simple models, overshifting is only possible with market power.
    1. Pless and van Benthem (2019) suggest using over shifting as a test for market power.
    2. Agrawal and Hoyt (2019), however, show overshifting can be found empirically with perfect competition when there are multiple produces and interdependencies.
- With multiproduct firms consumer price can *decrease* with a unit tax.
    1. Edgeworth tax paradox.
    2. Ritz (2014) show a unit tax can decrease price and industry output increases.
    3. A Pigouvian emissions unit tax can lead to an increase in industry emissions.

B.4 Extensions: salience, evasion, and empirical estimates

Including salience in the model

Consider the case in Bradley and Feldman (2020) where consumers have demand for a good with an ad valorem tax t.

- Consumers demand $x = x(p, t)$
- Consumer demand should only depend on tax-inclusive price $x = x(p(1 + t), 0)$
- Price elasticity of demand should equal gross-of-tax elasticity

$$\varepsilon_{x,p} = -\frac{\partial log x}{\partial log p} = -\frac{\partial log x}{\partial log(1 + t)} = \varepsilon_{x,1+t}$$

Including salience in the model

Bradley and Feldman (2020) conjecture that consumers under-react to less salient taxes due to inattention

$$\varepsilon_{x,p} > \varepsilon_{x,1+t}$$

- Consumers perceive a fraction $\phi > 0$ of the true tax.
- Consumer price is given by $q_{\phi} = p(1 + \phi t)$
- A 10 percent increase in 1 + t has the same effect on demand as a 1.4 percent increase in p.

Incidence on producer price attenuated by inattention

1. Incidence on producer prices is attenuated.
2. No tax neutrality: statutory incidence affects economic incidence.
3. Inattention unambiguously reduces DWL without income effects.
4. Inattention may reduce or increase DWL with income effects.

Including tax evasion in the model

1. $Q(p_r)$ quantity demanded with tax inclusive price $p_r = p - t$
2. Retailers buy product at wholesale price p_w and have costs that scale with size $c(q_r)$ and have fixed costs F_r.
3. Tax rate on retailers t_r.
4. Evasion of the tax e_r at a cost $\phi(e_r)$
5. Profits to the retailer:

$$\Pi_r(q_r, e_r) = (p_r - p_w)q_r - c_r(q_r) + t_r e_r - \phi(e_r) - F_r$$

Producer price changes depend on evasion

Totally differentiate the zero-profit condition using the envelope theorem.

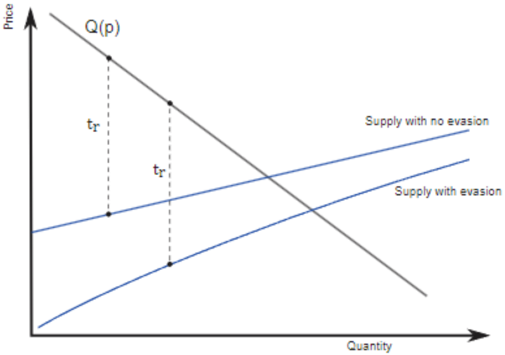
$$q\frac{dp_r}{dt_r} + e_r = 0$$

$$\frac{dp_r}{dt_r} = \frac{e_r}{q_r}$$

1. Price retailers receive falls by e_r/q_r as t_r increases.
2. Price consumers pay increases by $1 - e_r/q_r$.

Incidence in this market depends critically on the extent of tax evasion

Evasion can shift the supply curve to the right



With evasion, a tax can have a smaller impact on prices and quantities

Figure from Kopczuk et al. (2013)

Empirical estimates

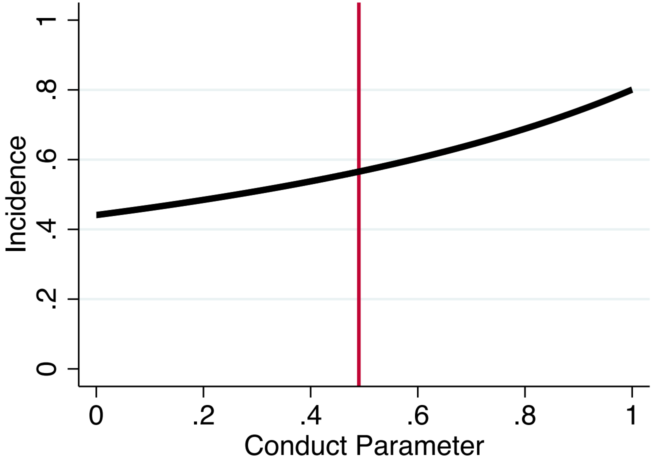
We often make simplifying assumptions when going to the data.

1. Perfect competition—simplifies incidence formula.
2. Perfectly elastic supply—often no data on this.

How important are these assumptions in practice?

- Mace, Patel, and Seegert (2020) considers these assumptions using data in the marijuana market.

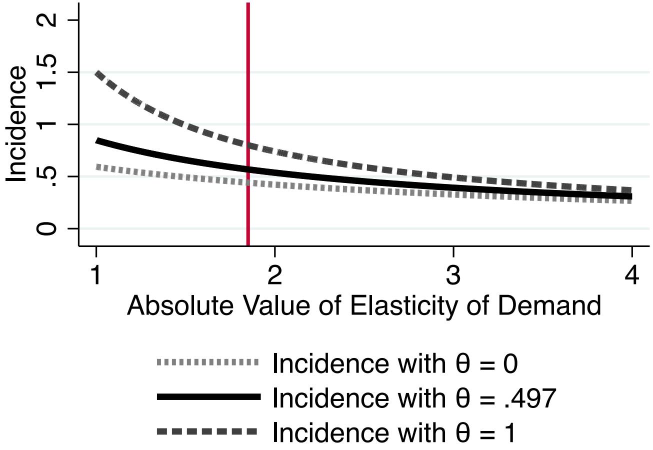
Consumers pay more of the tax with market power



The elasticity of marginal surplus is calibrated as 3.

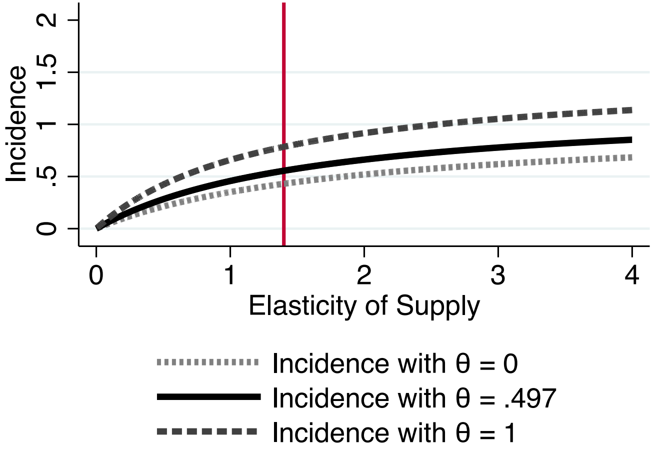
Consumers go from paying less than half to almost 80% of the tax as markets go from being competitive to monopoly.

Incidence is sensitive to the elasticity of demand



Incidence is more sensitive to the elasticity of demand in monopoly markets.

Incidence is sensitive to the elasticity of supply



Incidence is more sensitive to the elasticity of supply in monopoly markets.

Incidence—ripe for empirical estimation

The incidence of a tax depends on many factors.

1. Market power—in an ambiguous way.
2. Curvature of the demand function.
3. Presence of inattention and evasion.
4. Elasticities of supply and demand.

Session 2

Modeling Techniques through Models of Corporate Taxation

Session 2

A Connecting the model with empirical work
   1 Comparative statics
   2 Envelope theorem
   3 Sufficient statistics
   4 Structural parameter estimation

B Adding features to the basic model
   1 Expected utility with CARA utility (Bennett et al., 2020; Arnemann et al., 2022)
   2 Corporate taxes
   3 Dividend and capital gains taxes (Chetty and Saez, 2005; Ohrn and Seegert, 2019)
   4 Mergers and acquisitions (Coles, Sandvik, and Seegert, 2020)

A.1 Comparative statics

How does inflation distort investment?

Remember inflation?

Want to know whether inflation affects real investment.

1. Interest rates r should adjust for inflation π.
    - Irving Fisher 1930 noted nominal interest rates should rise one-for-one with inflation $dr/d\pi = 1$.
2. Interest rates affect real investment.
    - Interest rates are nominal while capital is a real variable (Darby, 1975; Feldstein, 1976).
3. Plausible that inflation, therefore, affects real investment.
4. Modeling tool:
    - Show comparative statics using total differentiation of equilibrium condition.

What do we need in our model?

1. What is the key question?
   - How/does inflation distort the tradeoff and therefore investment?
   - Inflation makes money borrowed today not as costly to payoff tomorrow.
2. What is the key tradeoff we are interested in?
   - Interested in investment.
   - Benefit is more production $f(K)$.
   - Cost is cost of investment (borrowing to look at inflation) rB.

Demand of capital

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \quad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B$$

1. Capital is increasing with borrowing $K = X + B$.
2. After-tax profits net interest payments $(f(K) - rB)(1 - \tau_c)$.
3. Inflationary gains on the stock of nominal borrowing. πB.
4. Capital depreciation δK and value of tax deduction for depreciation $\tau_c \delta K$.
   - Is this last piece necessary for the model?

Take the first-order condition and determine whether investment depends on the inflation.

Intuition and next steps

Does investment depend on inflation?

What assumptions went into this finding? Is that reasonable?

Partial and general equilibrium analysis

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \quad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B$$

Take the first-order condition.

$$\partial V / \partial B = (1 - \tau_c) f'(K) - (1 - \tau_c) r + \tau \delta - \delta + \pi = 0$$

$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c}$$

- At this step, you might say, inflation does lead to more investment—BUT, this is ignoring that r changes with π.
- Said differently, we need to think general equilibrium not partial equilibrium.

General equilibrium analysis

To think general equilibrium, we need to allow multiple variables to change at the same time.

First-order condition

$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c} \tag{7}$$

What variables do we think change?

1. Let capital change K.
2. Let interest rates change r.
3. Let inflation change π

Comparative statics: capital wrt inflation

Totally differentiate the first-order condition (allowing K, π, and r to change). Note $f''(K) < 0$.

$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c} \tag{8}$$

$$\text{totally differentiate} \quad f''(K)dK = dr - \frac{d\pi}{1 - \tau_c}$$

$$\frac{dK}{d\pi} = -\frac{1}{-f''(K)}\left(\frac{dr}{d\pi} - \frac{1}{1 - \tau_c}\right)$$

Comparative statics: capital wrt inflation

- How capital responds to inflation depends on how much interest rates respond to inflation.

$$\frac{dK}{d\pi} = -\frac{1}{-f''(K)}\left(\frac{dr}{d\pi} - \frac{1}{1-\tau_c}\right)$$

$$\frac{dK}{d\pi} = \begin{cases} > 0, & \text{if } \frac{dr}{d\pi} < \frac{1}{1-\tau_c} \\ = 0, & \text{if } \frac{dr}{d\pi} = \frac{1}{1-\tau_c} \\ < 0, & \text{if } \frac{dr}{d\pi} > \frac{1}{1-\tau_c} \end{cases}$$

How do interest rates change with inflation?

1. To know whether investment increases or decreases with inflation, we need to know how interest rates change with inflation.
2. Remember, Fisher 1930 noted $dr/d\pi = 1$.
3. To solve it in general equilibrium, we need to consider supply of capital (lenders).
4. Lenders receive real after-tax returns (individual tax rate t):

$$\tilde{r} = r(1 - t) - \pi \tag{9}$$

Supply of capital comparative statics

Lenders receive real after-tax returns (individual tax rate t):

$$\tilde{r} = r(1 - t) - \pi \tag{10}$$

Totally differentiate

$$d\tilde{r} = (1 - t)dr - d\pi \tag{11}$$

$$\frac{d\tilde{r}}{d\pi} = (1 - t)\frac{dr}{d\pi} - 1$$

For capital markets to clear supply = demand

Let τ = t Market supply of capital

$$\frac{d\tilde{r}}{d\pi} = \begin{cases} < 0, & \text{if } \frac{dr}{d\pi} < \frac{1}{1-t} \\ = 0, & \text{if } \frac{dr}{d\pi} = \frac{1}{1-t} \\ > 0, & \text{if } \frac{dr}{d\pi} > \frac{1}{1-t} \end{cases}$$

Market demand of capital

$$\frac{dK}{d\pi} = \begin{cases} > 0, & \text{if } \frac{dr}{d\pi} < \frac{1}{1-\tau_c} \\ = 0, & \text{if } \frac{dr}{d\pi} = \frac{1}{1-\tau_c} \\ < 0, & \text{if } \frac{dr}{d\pi} > \frac{1}{1-\tau_c} \end{cases}$$

- For capital markets to clear $\frac{dr}{d\pi} = \frac{1}{1-\tau_c}$.
- If $\frac{dr}{d\pi} > \frac{1}{1-\tau_c}$ Then the market would supply more capital but demand would go down.
- If $\frac{dr}{d\pi} < \frac{1}{1-\tau_c}$ Then the market would supply less capital but demand would go up.

Implications

1. Interest rate increases more than inflation $\frac{dr}{d\pi} = \frac{1}{1-\tau_c}$.
2. Interest rate adjusts for inflation **AND** tax implications.
3. Capital is unaffected by inflation $\frac{dK}{d\pi} = 0$.

What else might be important in this model?

1. Did modeling Capital depreciation δK and value of tax deduction for depreciation $\tau_c \delta K$ matter?

2. Could you redo the analysis abstracting from depreciation, or setting $\delta = 0$?

3. Could we now test this given current increases in inflation?

A.2 The envelope theorem

How do corporate tax rates affect total value in the economy?

Total value (welfare) in the economy

- So far, we have considered firm value solely.
- For tax policy, we may want to consider additional affects of corporate taxes.
- What do we need to include in the model to capture total value in the economy?
- How do corporate taxes distort welfare?

How do corporate taxes distort welfare?

There are several candidates

1. Change firm behavior due to changes in capital K.
2. Change tax reporting ρ of firms.
   - Let fraction μ of firm reporting be a shift in value and $1-\mu$ be a resource cost.
   - Examples of shifting are transfers to accounting firms or shifting money into a tax preferred vehicle.
   - Examples of resource costs include exerting effort in a law library figuring out credits and deductions.
   - Does it matter if it is a resource cost or shifting?
3. Change taxable income $Y(K,\rho)$ and thus tax revenues.

Consider the envelope theorem application in Coles, Patel, Seegert, and Smith (2021).

Firms maximize firm value

Write firm value in second period value

$$max_{K,\rho} \quad V = -rK + (1 - \tau_c)(f(K) - \rho) + \rho - c(\rho) \tag{12}$$

- Firms choose capital K and amount of reporting ρ.
- Taxable income $Y = f(K) - \rho$.
- Cost of reporting $c(\rho)$ and benefit of reporting $\tau_c \rho$.
- Profits $f(K)$.

Total value in the economy

Total value in the economy.

$$TV = [-rK + (1-\tau_c)(f(K)-\rho) + \rho - c(\rho)] \quad \text{Firm value} \tag{13}$$
$$+ \tau_c(f(K)-\rho) \quad \text{Tax revenue}$$
$$+ \mu c(\rho) \quad \text{Cost of reporting}$$

Cost of reporting to the extent that it shifts to accounting and law firms and is not a resource cost.

- Pure shift of value $\mu = 1$.
- Pure resource cost $\mu = 0$.

How does total value in the economy change with taxes?

Total value in the economy.

$$TV = [-rK + (1-\tau_c)(f(K)-\rho) + \rho - c(\rho)] \qquad \text{Firm value}$$
$$+ \tau_c(f(K)-\rho) \qquad \text{Tax revenue}$$
$$+ \mu c(\rho) \qquad \text{Cost of reporting}$$

Take the derivative with respect to $(1-\tau_c)$

How does total value change with the corporate tax rate?

1. We want to take the derivative $\frac{\partial TV}{\partial(1-\tau_c)}$.
2. Note, that capital and shifting are functions of the corporate tax rate.
3. Do we have to take $\partial K/\partial(1-\tau_c)$ and $\partial \rho/\partial(1-\tau_c)$ everywhere?
4. No, we can apply the envelope theorem!

Envelope theorem application

Rewrite total value in terms of taxable income $Y(K, \rho)$.

$$TV = [-rK + (1 - \tau_c)Y(K, \rho) + \rho - c(\rho)] + \tau_c Y(K, \rho) + \mu c(\rho) \tag{14}$$

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K, \rho) - Y(K, \rho) \qquad \text{direct effect}$$

$$+ \tau_c \frac{\partial Y(K, \rho)}{\partial K} \frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K, \rho)}{\partial \rho} \frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho) \frac{\partial \rho}{\partial(1-\tau_c)} \qquad \text{indirect effect}$$

Envelope theorem application

Rewrite total value in terms of taxable income $Y(K,\rho)$.

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho) \tag{15}$$

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K,\rho) - Y(K,\rho) \qquad \text{direct effect}$$

$$+\tau_c \frac{\partial Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)} \qquad \text{indirect effect}$$

Why did we not take the derivative of K and ρ inside of the square brackets but did outside?

Showing the envelope theorem

Why did we take the derivative of Y and ρ outside of the square brackets but not inside?

Consider the derivative of K and ρ in firm value

$$V = -rK + (1 - \tau_c)Y(K, \rho) + \rho - c(\rho) \tag{16}$$

Showing the envelope theorem

Consider the derivative of K and ρ in firm value

$$\frac{\partial V}{\partial(1-\tau_c)} = Y + \underbrace{\left(-r + (1-\tau_c)\frac{Y(K,\rho)}{\partial K}\right)}_{=0 \text{ bc FOC}} \frac{\partial K}{\partial(1-\tau_c)} \quad (17)$$

$$+ \underbrace{\left((1-\tau_c)\frac{Y(K,\rho)}{\partial \rho} + 1 - c'(\rho)\right)}_{=0 \text{ bc FOC}} \frac{\partial \rho}{\partial(1-\tau_c)}$$

$$= Y$$

How does total value in the economy change with tax rates?

1. Taking money from firms?
   - No, the direct effect is zero—transfer from firms to the government.
2. Firm value?
   - No, the indirect effect of firm value is zero by the envelope theorem.
3. Tax revenue changes?
   - Yes.
4. Tax reporting?
   - Yes, if reporting is shifting $\mu > 0$.

This motivates understanding the mechanisms of tax reporting.

What are other examples of the envelope theorem?

1. Shepard’s lemma: in a cost minimization problem the derivative with respect to the interest rate is capital and the derivative with respect to wages is labor.
2. Le Chatelier’s principle: labor is more responsive to a change in the wage in the long run than in the short run because in the long run the firm can adjust its capital.
3. Deadweight loss Harberger (1964) “triangle."

A.3 Sufficient statistics

How do corporate tax rates affect total value in the economy?

Is there one parameter that can tell us about distortions in the economy?

1. Feldstein (1999) argued that the elasticity of taxable income with respect to the corporate tax rate captured the welfare gain/cost from taxes.
   - The elasticity of taxable income as a sufficient statistic for welfare analysis.
   - For more on sufficient statistics see Chetty (2009).

2. Many papers have qualified this statement (Doerrenberg, Peich, and Siegloch, 2017; Coles, Patel, Seegert, and Smith, 2021).

3. Follow the analysis in Coles, Patel, Seegert, and Smith (2021) to
   - Demonstrate sufficient statistics.

Total value in the economy

Start again with total value in the economy.

$$
\begin{aligned}
TV = & [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] && \text{Firm value} \qquad (18)\\
& + \tau_c Y(K,\rho) && \text{Tax revenue}\\
& + \mu c(\rho) && \text{Cost of reporting}
\end{aligned}
$$

Is there one parameter that would be sufficient for understanding $\partial TV/\partial(1-\tau_c)$?

Derive welfare costs of corporate taxes

Take the derivative of total value with respect to the net-of-tax rate.

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K,\rho) - Y(K,\rho) + \tau_c \frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} + \mu c'(\rho) \frac{\partial \rho}{\partial(1-\tau_c)} \quad (19)$$

Rearrange to get terms that we like (note $c'(\rho) = \tau_c$).

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \left(\frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} \frac{1-\tau_c}{Y} + \mu \frac{\partial \rho}{\partial(1-\tau_c) \frac{1-\tau_c}{Y}} \right) \quad (20)$$

Rewrite in terms of elasticities

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \left(e_Y - \mu e_\tau \right) \quad (21)$$

Is the elasticity of taxable income a sufficient statistic?

We know that

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \left(e_Y - \mu e_\tau\right) \tag{22}$$

1. If the cost of tax adjustments is a resource cost ($\mu = 0$), then
   - the elasticity of taxable income is a sufficient statistic for the distortion to total value.
2. If the cost of tax adjustments is partially a transfer ($\mu > 0$), then
   - the elasticity of taxable income is an upper bound on the distortion to total value
   - the distortion to total value decreases with the tax adjustment elasticity e_τ

A.4 Structural parameter estimation

How elastic are firms?

Structural estimation

Structural estimation connects the model directly to the empirical estimation.

1. This can be as simple as running an OLS regression.
2. Alternatively, it could require estimation via general method of moments, maximum likelihood, or simulated method of moments.
3. What are the benefits?
   - Identifies exactly what your empirical estimation is telling you.
   - Allows for extrapolation out of sample for policy “experiments."

Let’s go through an example following Agostini, Bertanha, Bernier, Bilicka, He, Koumanakos, Lichard, Massenz, Palguta, Patel, Perrault, Riedel, Seegert, and Todtenhaupt (2022).

Standard model of firms with fixed cost

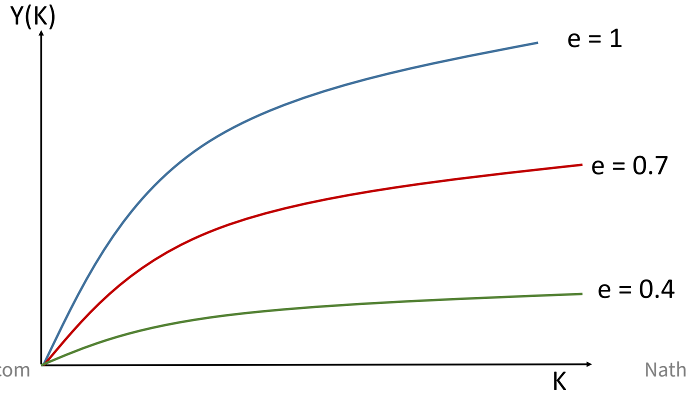
- Firm i chooses how much earnings to distribute as a dividend ($D_i \geq 0$) and how much equity to issue ($E_i \geq 0$).
- Those choices determine period 2 Capital: $K_{2,i} = K_{1,i} + E_i - D_i$.
- Profits net depreciation costs:

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$

- Fixed costs $F_i = exp(X_F' \beta_F + \nu_F)$, normally distributed.
- Productivity $A_i = exp(X_A' \beta_A + \nu_A)$, normally distributed.
- Parameter of interest e tells us how elastic firms are.

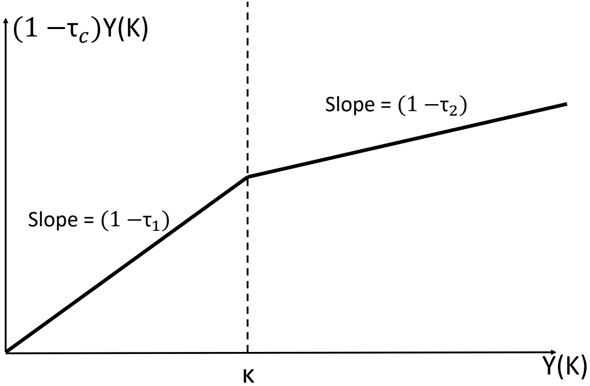
Parameter *e* tells us how elastic firms are

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$



The tax schedule with a kink in it

Profits below κ taxed at rate τ_1 and profits above κ taxed at rate τ_2, where $\tau_1 < \tau_2$.

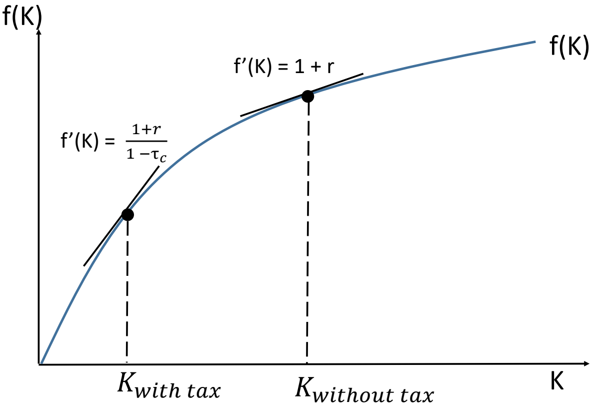


Firms maximize shareholder value subject to the corporate tax schedule

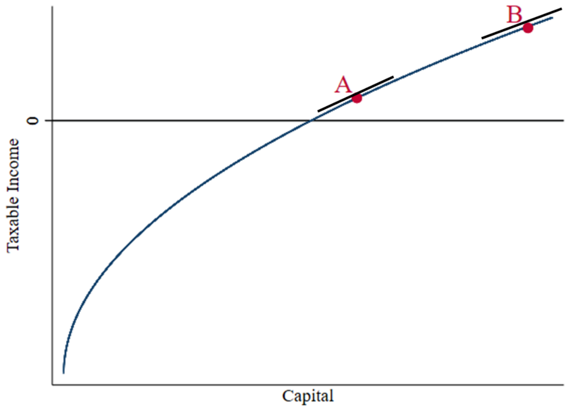
Profits below κ taxed at rate τ_1 and profits above κ taxed at rate τ_2, where $\tau_1 < \tau_2$.

$$
\begin{aligned}
max_{K_{2,i}} \quad V =& D_i - E_i + \frac{K_{2,i}}{1+r} \\
&+ \mathbb{1}(Y_i(K_{2,i}) \leq \kappa)\frac{(1-\tau_1)Y_i(K_{2,i})}{1+r} \\
&+ \mathbb{1}(Y_i(K_{2,i}) > \kappa)\frac{(1-\tau_1)\kappa + (1-\tau_2)(Y_i(K_{2,i}) - \kappa)}{1+r}
\end{aligned}
$$

Distortions to investment from corporate taxes and equity

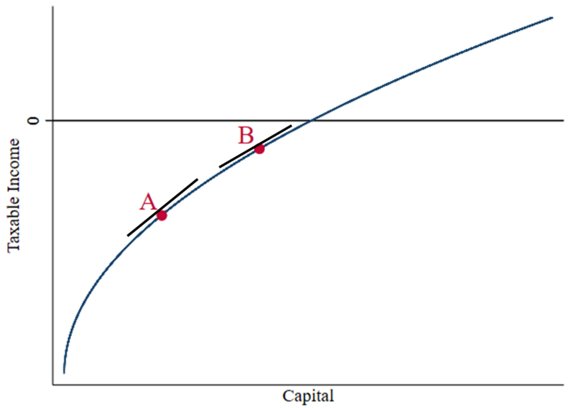


This firm reports taxable income at point A above the kink



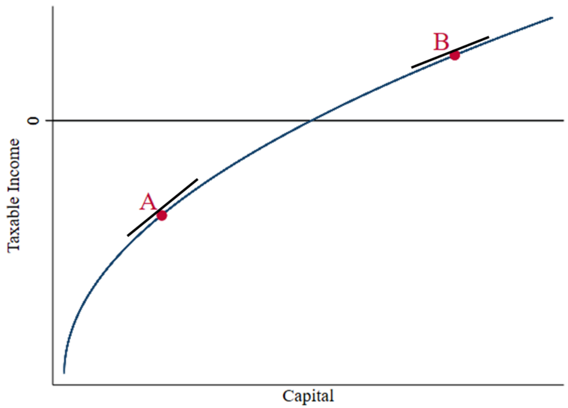
- Point A has a slope $r/(1 - t_1)$.
- Point B has a slope $r/(1 - t_0)$.
- Subject to t_1 above 0.
- Subject to t_0 below 0.

This firm reports taxable income at point B below the kink



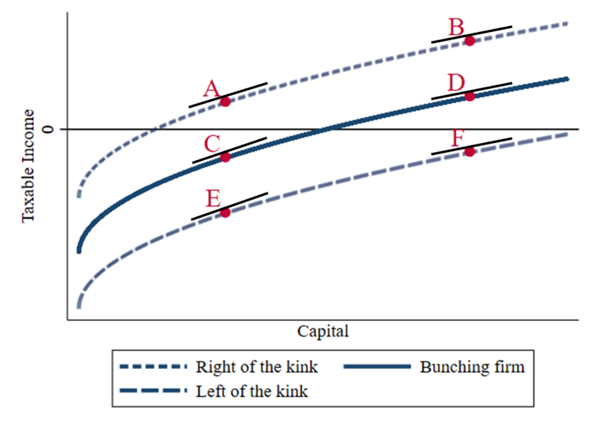
- Point A has a slope $r/(1-t_1)$.
- Point B has a slope $r/(1-t_0)$.
- Subject to t_1 above 0.
- Subject to t_0 below 0.

This firm reports taxable income at the kink



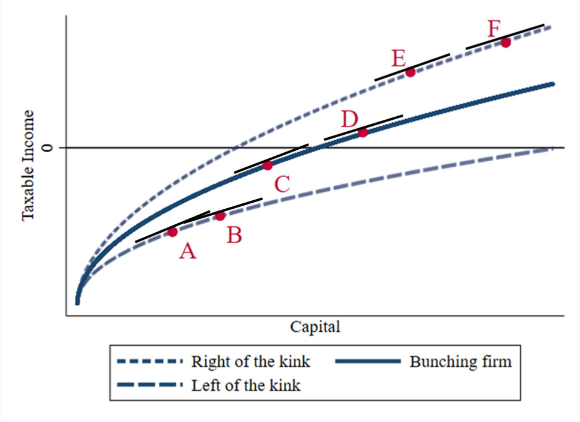
- Point A has a slope $r/(1-t_1)$.
- Point B has a slope $r/(1-t_0)$.
- Subject to t_1 above 0.
- Subject to t_0 below 0.

Varying the fixed cost F_i



- Point A, C, E have a slope $r/(1-t_1)$.
- Point B, D, F have a slope $r/(1-t_0)$.

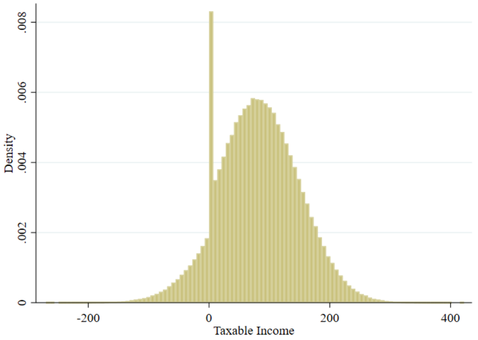
Varying productivity A_i



- Point A, C, E have a slope $r/(1-t_1)$.
- Point B, D, F have a slope $r/(1-t_0)$.

Taxable income is given by piecewise function

$$Y_i^* = \begin{cases} \frac{1+e}{e} r^{-e}(1-\tau_1)^e A_i - F_i, & A_i \leq \underline{A}(e,\kappa,\tau_1) \\ \kappa, \quad \underline{A}(e,\kappa,\tau_1) < A_i < & \bar{A}(e,\kappa,\tau_2) \\ \frac{1+e}{e} r^{-e}(1-\tau_2)^e A_i - F_i, & A_i \geq \bar{A}(e,\kappa,\tau_2) \end{cases}$$



Two step estimation

- Fixed costs $F_i = exp(X_F' \beta_F + \nu_F)$, normally distributed.
- Productivity $A_i = exp(X_A' \beta_A + \nu_A)$, normally distributed.

Case 1: $Y < \kappa$

$$
\begin{aligned}
Y &= \underbrace{\frac{1+e}{e} r^{-e} (1-\tau_1)^e}_{\lambda_1} A_i - F_i \\
&= X_A \beta_A \lambda_1 + X_F \beta_F (-1) + \lambda_1 \nu_A - \nu_F
\end{aligned}
$$

Two step estimation

- Fixed costs $F_i = exp(X_F' \beta_F + \nu_F)$, normally distributed.
- Productivity $A_i = exp(X_A' \beta_A + \nu_A)$, normally distributed.

Case 2: $Y > \kappa$

$$
\begin{aligned}
Y &= \underbrace{\frac{1+e}{e} r^{-e} (1-\tau_2)^e}_{\lambda_2} A_i - F_i \\
&= X_A \beta_A \lambda_2 + X_F \beta_F (-1) + \lambda_2 \nu_A - \nu_F
\end{aligned}
$$

Two step estimation: using variation in productivity and fixed cost

1. The conditional expectation when $Y < \kappa$ (similarly for $Y > \kappa$)

$$\mathbb{E}[Y|X_A, X_F, Y < \kappa] = X_A(\beta_A\lambda_1) + X_F(-\beta_F) - w_1 \frac{\phi\left(\frac{X_A(-\beta_A\lambda_1)+X_F(\beta_F)}{w_1}\right)}{\Phi\left(\frac{X_A(-\beta_A\lambda_1)+X_F(\beta_F)}{w_1}\right)}$$

2. Ratio of coefficients on productivity:

$$\frac{\beta_A\lambda_1}{\beta_A\lambda_2} = \frac{\frac{1+e}{e}r^{-e}(1-\tau_1)^e}{\frac{1+e}{e}r^{-e}(1-\tau_2)^e} = \frac{(1-\tau_1)^e}{(1-\tau_2)^e}$$

3. Derive the parameter e

$$e = ln\left(\frac{\beta_A\lambda_1}{\beta_A\lambda_2}\right)\frac{1}{ln(1-\tau_1) - ln(1-\tau_2)}$$

Implications, extensions, and limitations

Firms respond to tax rates

$$\varepsilon_i = e\left(1 + \frac{F_i}{Y_i}\right)$$

1. Implication: Firms with higher taxable incomes have lower elasticities.
   - Consistent with empirical evidence in Devereux et al. (2014).
2. Extension: Include profit shifting.
3. Limitation: Assume that e is a structural parameter that captures all firm responsiveness.

Package to estimate elasticities

Nathan Seegert

Home Vitae Research Teaching Media Code

The Elasticity of Taxable Income Across Countries

with Claudio Agostini, Marinho Bertanha, Govindadeva Bernier, Katarzyna Bilicka, Jaroslav Bukovina, Yuxuan He, Evangelos Koumanakos, Tomas Lichard, Jan Palguta, Elena Patel, Louis Perrault, Kristina Strohmaier, Maximilian Todtenhaupt, and Branislav Zudel.

This package is in *beta testing*. This package is not yet a complete package (ado file program) and is currently being tested with real data. Feel free to test the code or build off of it for your own work. As a courtesy, please let me know if you find any errors.

STATA - manual installation

1. download and unzip the file **"stata_crosscountry.zip"**
2. This folder contains four files.
   a) 0_simulateData.do This file lets you simulate data to be used with the other files.
   b) 1_fixedCost_2021129.do This file implements the fixed cost method described in the paper.
   c) 2_TwoStep_20220201.do This file implements the two step method described in the paper.
   d) twostep_bserrors.ado This file is used to produce standard errors.

- www.NathanSeegert.com/code

B.1 Expected Utility with CARA utility

Do firms always maximize firm value?

Do firms always maximize profits?

Most of the economics literature focuses on firm value maximization, but the reality is more complicated (Jensen and Meckling, 1976; Smith and Stulz, 1985).

1. We want to investigate agency problems between managers and stock holders (who want firm value).

2. Consider two potential agency problems
   - Different incentives (e.g., empire building) for the manager.
   - Different risk preferences for the manager (e.g., risk averse).

Do firms always maximize profits?

Managers choose investment to maximize their utility, which consists of their wealth, firm value, and their benefits from empire building.

$$u = w_0 + \alpha\mu_V(K) - \frac{1}{2}\rho\sigma^2(K) + g(K) \quad (23)$$

- w_0 external wealth.
- $\mu_V(K)$ expected value of the firm depends on investment K.
- α weight that firm value enters manager's utility.
- ρ risk aversion parameter.
- $\sigma^2(K)$ variance of firm value, which depends on investment K.
- $g(K)$ benefit from empire building, $g'(K) > 0, g''(K) < 0$.

This simple formula can be derived from CARA utility and a normal distribution of firm value or CRRA utility and a log normal distribution of firm value.

Modeling compensation packages of managers

Now, allow shareholders to compensate managers to align incentives.

1. Effective ownership δ through accumulation of stock and options net of dispositions.
   - To account for managers having other incentives (e.g., empire building).
2. Compensation convexity through vega, ν—such as option grants.
   - To account for managers being more risk averse than shareholders.
3. Together, these features update manager's utility

$$u = w_0 + (\alpha + \delta)\mu_V - \frac{1}{2}(\rho - \nu)\sigma^2 + g(K) \tag{24}$$

What else might be important in this model?

1. How would personal taxes such as dividend taxes affect this model?
2. As the dividend tax changes how would this change the incentives for compensation committees?

Agency model—objective function with dividend taxes

Let τ_d be the dividend tax rate.

$$w_0 + (1 - \tau_d)\delta\mu - \frac{1}{2}(\rho - \nu)\delta_0^2(1 - \tau_d)^2\sigma^2 \tag{25}$$

With dividend taxation, how might compensation committees might want to adjust their recommendations?

1. Hypothesis 1: Higher dividend taxes may require compensation committees to increase δ to get the same incentive alignment.
2. Hypothesis 2: Higher dividend taxes may allow compensation committees to decrease ν to get the same risk preference alignment.

Empirical evidence of personal taxes and CEO compensation

Using the previous model, or something similar, the following research investigates the role of taxes on firm behavior/compensation.

1. Arnemann, Buhlmann, Ruf, and Voget (2022) find higher income taxes on CEOs lowers firm performance.
2. Bennett, Coles, and Wang (2020) find income taxes are *not* paid by the CEO.
3. Coles, Sandvik, and Seegert (2020) find that personal taxes and different compensation incentives provide different incentives for M&A activity and ultimately performance.

B.2 Corporate taxes

Do corporate taxes distort investment decisions?

Adding corporate taxes to our basic model

We want to investigate whether/how corporate income taxes distort investment.

1. Consider investment from equity issuances E and the tradeoff between today and tomorrow:
   - Cost: $-E$ today.
   - Benefit: higher profits tomorrow $f(X+E)$, where $K = X + E$.

Does the corporate income tax τ_c distort this tradeoff for firms?

Adding corporate taxes to our basic model with equity

Shareholders choose equity E to maximize value V, by trading off less income now with higher profits tomorrow.

$$max_E \quad V = -E + \frac{(1-\tau_c)f(X+E)}{1+r} \tag{26}$$

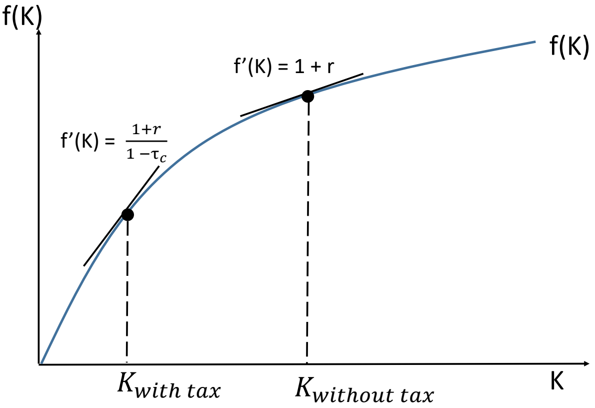
Take the first-order condition

$$\partial E : \quad -1 + \frac{(1-\tau_c)f'(K)}{1+r} = 0 \tag{27}$$

$$\rightarrow f'(K) = \frac{1+r}{1-\tau_c}$$

- Corporate taxes have a large distortion!

Distortions to investment from corporate taxes and equity



Adding corporate taxes to our basic model with debt financing

2. Investment could come from debt B that creates a tradeoff between more production tomorrow and payment with interest tomorrow:
   - Cost: $(1 + r)B$ tomorrow.
   - Benefit: higher profits tomorrow $f(X + B)$.

Does the corporate income tax τ_c distort this tradeoff for firms?

- Let $\gamma \in [0, 1]$ be the percent of debt costs that are tax deductible.

Write down this two-period model and find the first-order condition.

Corporate taxes in the model with debt investment

Shareholders choose B to maximize firm value trading off higher profits and more debt

$$max_B \quad V = \frac{(1-\tau_c)\left[f(X+B) - \gamma(1+r)B\right] - (1-\gamma)(1+r)B}{1+r} \tag{28}$$

- Let $\gamma \in [0, 1]$ be the percent of debt costs that are tax deductible.

Corporate taxes in the model with debt investment

Take the first-order condition

$$\partial B: \quad \frac{(1-\tau_c)\left[f'(K)-\gamma(1+r)\right]-(1-\gamma)(1+r)}{1+r}=0 \tag{29}$$

$$\rightarrow f'(K)=\gamma(1+r)+(1-\gamma)\frac{1+r}{1-\tau_c}$$

- If $\gamma = 1$, then there is no distortion from corporate taxes if debt is the marginal source of investment.
- If $\gamma = 0$, then there is a large distortion of corporate taxes (Hall and Jorgenson, 1967).

What else might be important in this model?

1. Depreciation schedules for tax purposes relative to economic depreciation.

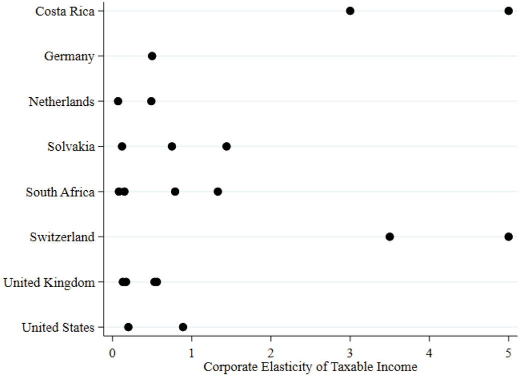
2. How would we empirically test whether corporate taxes distort investment or taxable income?

Empirical estimates—How elastic are firms?

Use changes in tax rates from tax schedules (bunching).

- Gruber and Rauh (2007); Coles, Patel, Seegert, and Smith (2021); Dwenger and Steiner (2012); Lediga, Riedel, and Strohmaier (2019); Krapf and Staubli (2020); Bukovina, Lichard, Palguta, and Zudel (2021); Bachas and Soto (2021); Massenz and Bosch (2022).

Empirical estimates of the distortions of corporate taxes



- Bunching estimates of distortion of corporate taxes.

B.3 Dividend taxes

Do dividend taxes distort investment?

Do dividend taxes distort investment behavior?

Firms choose dividends and equity policy D and E, to maximize firm value by trading off dividends or equity today and production tomorrow.

$$V = D - E + \frac{f(X - D + E) + X - D + E}{1 + r} \tag{30}$$

- Today firms can pay D dividends or ask for equity E.
- Tomorrow capital $K = X - D + E$ produces $f(K)$ and the firm liquidates and gives back K.[1]

[1] This is important because of rules on dividend taxes between equity and retained earnings (Chetty and Saez, 2010).

Do dividend taxes distort investment behavior?

Dividend taxes make dividends less valuable today, maybe firms will **over-invest**. But maybe not?

$$V = (1-\tau_d)D - E + \frac{(1-\tau_d)[(1-\tau_c)f(X-D+E)+X-D]+E}{1+r} \tag{31}$$

- Dividend taxes τ_d are paid on dividends today, but not rebated to equity.
- Dividend taxes paid on production and retained earnings tomorrow, but not equity.
- For comparison, model corporate income tax τ_c.

Consider a model with equity and dividend taxes

Let $D = 0$, and firms choose equity E to maximize firm value.

$$max_E \quad V = -E + \frac{(1-\tau_d)[(1-\tau_c)f(X+E)+X]+E}{1+r} \tag{32}$$

Take the first-order condition

$$\partial V/\partial E = -1 + \frac{(1-\tau_d)[(1-\tau_c)f'(X+E)]+1}{1+r} = 0 \tag{33}$$

$$f'(X+E) = \frac{r}{(1-\tau_d)(1-\tau_c)}$$

- Dividend tax rate distorts investment similar to corporate taxes.

Consider a model with dividends and dividend taxes

Let $E = 0$, and firms choose dividends D to maximize firm value.

$$max_D \quad V = (1-\tau_d)D + \frac{(1-\tau_d)[(1-\tau_c)f(X-D) + X - D]}{1+r} \tag{34}$$

Take the first-order condition and determine how big the distortion from the dividend tax is.

Consider a model with dividends and dividend taxes

$$\partial V / \partial D = (1 - \tau_d) - \frac{(1 - \tau_d)[(1 - \tau_c) f'(X - D) + 1]}{1 + r} = 0 \tag{35}$$

$$(1 - \tau_c) f'(X - D) + 1 = \frac{(1 - \tau_d)(1 + r)}{(1 - \tau_d)}$$

$$f'(X - D) = \frac{1 + r}{(1 - \tau_c)}$$

- Dividend tax rate drops out—no distortion.

New view vs old view—matter of firm type

Whether dividend taxes distort investment decisions seem to depend on whether the firms are issuing equity or paying dividends.

1. Old view: **distortion**. Cash constrained firms; $D = 0$ and $E > 0$,
2. New view: **no distortion**. Cash rich firms; $D > 0$ and $E = 0$,
3. Cash intermediate firms; $D = 0$ and $E = 0$.
   - Ignore because not that interesting.

New view vs old view—empirical evidence

- Chetty and Saez (2005) document
  1. Dividends increased after the dividend tax cut of 2003.
     - Seems at odds with new view.
  2. The adjustment was rapid.
     - Seems at odds with old view, because supply mechanism would take longer.
- Gordon and Dietz (2008) and Chetty and Saez (2010) propose an agency model based on Jensen and Meckling (1976).
- Yagan (2015) finds that despite increased dividend payments there was no change to corporate investment or employee compensation.
  - Consistent with the new view—but a puzzle, where did the money come from?
- Ohrn and Seegert (2019) include M&A into the model and show it reconciles all of the empirical findings.
  - The model is also consistent with evidence on M&A behavior around 2003.

B.4 Mergers

Do dividend taxes distort acquisitions?

How do investor-level taxes distort investment?

To the basic model we want to study the interaction between

1. Acquisitions
2. Dividend taxes
3. Manager incentives

Are there distortions from the dividend tax on mergers?

Internal investment vs acquisitions

1. Start with a basic two-period model of corporate decision-making.
2. Firms, maximize shareholder value V and choose their level of dividend D, such that capital in period 2 is given by their retained earnings minus dividends $I = X - D$.
3. Profits, net depreciation, from investment is given by $f(I)$ and discounted by the interest rate r.

$$max_D \qquad V = D + \frac{f(X-D) + X - D}{1+r}.$$

Add dividend taxes

Dividend taxes are paid in both periods (new view model).

$$max_D \qquad V = (1 - \tau_d)D + (1 - \tau_d)\frac{f(X - D) + X - D}{1 + r}.$$

Manager's exogenous compensation package (δ, ν)

Compensation package

1. $\delta \in [0, 1]$ effective ownership through accumulation of stock.
2. CEO wealth $w = w_0 + \delta V_i$, where w_0 is initial wealth and V is value of the firm.
3. $\nu \in [0, \rho]$ add convexity through CEO vega.
4. Effective risk aversion $\tilde{\rho} = \rho - \nu$, where ρ is manager's risk aversion.

Manager payoffs

Assume the manager has CARA utility over wealth,

$$U = -e^{-(\rho-\nu)(w_0+(1-\tau_d)\delta V)}. \tag{36}$$

The manager maximizes their expected utility

$$u_0 = w_0 + (1-\tau_d)\delta\mu - \frac{1}{2}(\rho-\nu)\delta^2(1-\tau_d)^2\sigma^2, \tag{37}$$

where μ is the expected value of the firm and σ^2 is its variance.

What is an acquisition?

The firm makes an acquisition $Y = 1$, otherwise $Y = 0$.

1. The firm acquires some amount of capital C, production technology $g(\cdot)$, and potential synergies θ.

$$\theta(g(C) + C)$$

2. The firm pays the target firm their reservation payment*

$$M = (1 - \tau_d)\frac{g(C) + C}{1 + r}.$$

Value with an acquisition

$$(1-\tau_d)V_1 = (1-\tau_d)D_1 + (1-\tau_d)\frac{f(X-(D_1+M)) + \theta(g(C)+C) + I_1}{1+r}.$$

- Internal investment with an acquisition $I_1 = X - D_1 - M$.

Manager payoff taking into account the increase in volatility with an acquisition.

$$u_1 = (1-\tau_d)\delta V_1 - \frac{1}{2}(\rho-\nu)\delta^2(1-\tau_d)^2\sigma^2(1-\gamma M^2).$$

When does an acquisition get done?

Managers make an acquisition when $u_1 > u_0$, or when synergies are greater than some threshold

$$\theta > \theta * \equiv (1 - \tau_d) - \frac{1}{2}(\rho - \nu)\delta(1 - \tau_d)^2\sigma^2\gamma M.$$

What can we learn/test from this equilibrium condition?

Comparative Statics

PROPOSITION 1 *An increase in the dividend tax rate has an ambiguous effect on the threshold for the acquisitions firms undertake.*

$$\frac{\partial \theta^*}{\partial \tau_d} = -1 + (\rho - \nu)\delta(1 - \tau_d)\sigma^2\gamma M \gtrless 0.$$

How does this derivative change with the compensation package parameters?

Comparative Statics

PROPOSITION 2 *The effect of changes in the dividend tax rate is smaller for managers with less effective risk aversion.*

$$\frac{\partial(\partial\theta^*/\partial\tau_d)}{\partial\nu} = -(1-\tau_d)\delta\sigma^2\gamma M < 0.$$

PROPOSITION 3 *The effect of changes in the dividend tax rate is smaller for managers with less effective ownership.*

$$\frac{\partial(\partial\theta^*/\partial\tau_d)}{\partial\delta} = (\rho-\nu)(1-\tau_d)\sigma^2\gamma M > 0.$$

Conclusion

Models focus the reader on the tradeoff in your work

Modeling takeaways:

1. Begin with the tradeoff you are interested in studying.
   - You can start from many models that already exist.
   - Define the players, strategies, and payoffs.
2. Add in features of interest.
   - Depreciation schedules
   - Mergers and acquisitions
   - Tax reporting
   - Inflation
3. Let your model ebb and flow.
   - Add features to test whether conclusions are robust.
   - Delete features that are robust.
4. Have fun and be creative.

BONUS: User cost of capital and effective tax rate ETR

How do tax depreciation methods distort investment?

Tax rules on investment

We want to understand how tax rules impact investment.

1. Firms have depreciation allowance a_t at time t on a dollar of investment.
   - Accelerated depreciation or any other schedule.
   - $\int a_t dt = 1$, and $z \equiv \int e^{-\rho t} a_t dt$.
   - Capital depreciates exponentially at rate δ; $K_t = Ee^{-\delta t}$.
   - Firms may receive a contemporaneous investment tax credit of κ per dollar invested.
2. We could do this in continuous time (and most of the literature does), but we can get a lot from just a two period model.
3. Modeling goals: explore how to use the user cost of capital and effective tax rate ETR to investigate tax distortions.

Investment with depreciation and discount rate

To follow the continuous time literature, we can update the model as below:

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E \tag{38}$$

1. E is equity.
2. c is after-tax cost of putting a dollar into the firm.
3. $K = X - D + E$ is capital in period 2.
4. δ is the capital depreciation rate.
5. ρ is the rate at which owners discount after-tax flows.
6. z is the depreciation allowance.
7. κ is the investment tax credit.

Tax adjusted user cost of capital

First order condition

$$\partial E : -c + \frac{(1-\tau_c)f'(K)}{\delta+\rho} + \tau_c z + \kappa = 0 \tag{39}$$

$$f'(K) = \frac{c-\kappa-\tau_c z}{1-\tau_c}(\rho+\delta)$$

- The right side is the user cost of capital.

Tax adjusted user cost of capital

User cost of capital

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{40}$$

1. If $c = 1$, then this is the Hall-Jorgenson tax-adjusted user cost of capital.
2. If $\tau_c = 0$, the rental cost of capital is $c(\rho + \delta)$, which reflects the time value of money and cost of depreciation interacted with the expenditure level.
3. Everything else, is the impact of taxation.

Consider different depreciation methods

1. Let investments be depreciated at economic depreciation, then $z = \delta/(\rho + \delta)$.
2. Let investments be expensed immediately, then $z = 1$.
   - If $\kappa = 0$ and $c = 1$, then we can see that immediate expensing returns us to the cost of capital without taxes.

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{41}$$
$$= \frac{1 - \tau_c}{1 - \tau_c}(\rho + \delta)$$
$$= \rho + \delta$$

- Obviously, depreciation is more complicated than either of these scenarios in practice.

Effective tax rates (ETR)

Consider the investment level induced by the condition:

$$\rho \equiv [f'(K) - \delta](1 - ETR). \tag{42}$$

that defines the effective tax rate

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta}. \tag{43}$$

The ETR provides the “single" tax rate that produces the same investment level given by a combination of tax parameters.

Combinations of tax parameters

User cost of capital

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{44}$$

Now, we can consider different combinations of tax parameters and find the effective tax rate.

- Economic deprecation $z = \delta/(\rho + \delta)$.
- Immediate expensing $z = 1$.
- Equity financed investment $c = 1$.
- Debt financed investment $c < 1$.
- investment tax credit κ.

Effective tax rates (ETR) scenario 1

General model

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E \tag{45}$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{46}$$

Scenario 1: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Immediate expensing of investment $z = 1$.

Effective tax rates (ETR) scenario 1

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{47}$$

Scenario 1: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Immediate expensing of investment $z = 1$.

Scenario 1 user cost of capital:

$$f'(K) = \rho + \delta \tag{48}$$

Effective tax rates (ETR) scenario 1

Scenario 1 user cost of capital $c = 1, \kappa = 0,$ and $z = 1$:

$$f'(K) = \rho + \delta$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho + \delta - \delta - \rho}{\rho + \delta - \delta} = 0$$

- In this scenario immediate expensing leads to no distortions!

Effective tax rates (ETR) scenario 2

General model

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E \tag{49}$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 2: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Effective tax rates (ETR) scenario 2

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \tag{50}$$

Scenario 2: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Scenario 2 user cost of capital:

$$f'(K) = \rho/(1 - \tau_c) + \delta \tag{51}$$

Effective tax rates (ETR) scenario 2

Scenario 2 user cost of capital $c = 1, \kappa = 0, z = \delta/(\rho + \delta)$:

$$f'(K) = \rho/(1 - \tau_c) + \delta$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho/(1 - \tau_c) + \delta - \delta - \rho}{\rho/(1 - \tau_c) + \delta - \delta} = \tau_c$$

- In this scenario economic depreciation leads to a distortion that increases with the corporate tax rate.

Effective tax rates (ETR) scenario 3

General model

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta+\rho} + \tau_c zE + \kappa E$$

General user cost of capital:

$$f'(K) = \frac{c-\kappa-\tau_c z}{1-\tau_c}(\rho+\delta)$$

Scenario 3: Consider a firm with

1. Debt-financed investment $c = 1 - \tau_c$.
   - $c = (r(1-\tau_c)+\delta)/(\rho+\delta) = 1-\tau_c$
   - $c = 1-\tau_c$ with the simplification, $\delta = 0$, $\rho = r$.
2. No investment tax credit $\kappa = 0$
3. depreciation allowances equal to economic depreciation $z = \delta/(\rho+\delta)$.

Effective tax rates (ETR) scenario 3

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 3: Consider a firm with

1. Debt-financed investment $c = 1 - \tau_c$.
   - $c = (r(1 - \tau_c) + \delta)/(\rho + \delta) = 1 - \tau_c$
   - $c = 1 - \tau_c$ with the simplification, $\delta = 0$, $\rho = r$.
2. No investment tax credit $\kappa = 0$
3. depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Scenario 3 user cost of capital:

$$f'(K) = \rho$$

Effective tax rates (ETR) scenario 3

Scenario 3 user cost of capital $c = 1 - \tau_c$, $\delta = 0$, $\kappa = 0$ and $z = \delta/(\rho + \delta)$:

$$f'(K) = \rho$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho - \rho}{\rho} = 0$$

- If there is debt finance and tax depreciation is economic deprecation there is no distortion.
- If there is debt finance and tax depreciation that is more rapid than economic depreciation, then the ETR is **negative**.

ETR can be used to measure/investigate distortions

ETR = 0 implies no distortion from taxation. This occurs when

1. Equity financing of investment and immediate expensing.
2. Debt financing of investment and depreciation is allowed at economic depreciation.
   - In both cases, all investment costs are deductible.

ETR and user cost of capital are helpful to understand when and how taxes distort investment.

What else might be important in this model?

1. What other depreciation schedules might we want to model and how would they change investment behavior?

2. What other behavior may depreciation schedules change?

Empirical evidence

Use changes in depreciation (via bonus depreciation) to look at effect on investment.

1. Early literature found large investment responses (House and Shapiro, 2008; Zwick and Mahon, 2017).
   - Use differences across industries in investment.
   - Manufacturing longer lived capital than software developers and thus have more benefits from bonus depreciation.
2. These estimates might be too large though if competition is not taken into account (Patel and Seegert, 2020).
   - Investment is a strategic variable and responses to tax incentives depend on how competitive or concentrated the market is.
   - Industries with longer lived capital likely also more concentrated due to large fixed costs.

References

Agostini, Claudio, Marinho Bertanha, Govindadeva Bernier, Katarzyna Bilicka, Yuxuan He, Evangelos Koumanakos, Tomas Lichard, Gabriella Massenz, Jan Palguta, Elena Patel, Louis Perrault, Nadine Riedel, Nathan Seegert, Maximilian Todtenhaupt. 2022. The elasticity of corporate taxable income across countries. *Working paper* .

Agrawal, David R., William H. Hoyt. 2019. Pass-through in a multiproduct world. Available at SSRN 3173180.

Arnemann, Laura, Florian Buhlmann, Martin Ruf, Johannes Voget. 2022. The effect of taxes on ceo performance. *Working paper* .

Bennett, Benjamin, Jeffrey L. Coles, Zexi Wang. 2020. How executive compensation changes in response to personal income tax shocks (who pays the ceos income taxes?). *Working paper* .

Berger, Elizabeth, Nathan Seegert. 2023. Half banked: The real effects of financial exclusion on firms. Conditionally Accepted, Journal of Finance.

Bradley, Sebastien, Naomi E. Feldman. 2020. Hidden baggage: Behavioral responses to changes in airline ticket tax disclosure. *American Economic Journal: Economic Policy* **12**(4) 58–87.

Chetty, Raj. 2009. Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annual Review of Economics* **1**(1) 451–488.

Chetty, Raj, Emmanuel Saez. 2005. Dividend taxes and corporate behavior: Evidence from the 2003 dividend tax cut. *Quarterly Journal of Economics* **120(3)** 791–833.

Chetty, Raj, Emmanuel Saez. 2010. An agency theory of dividend taxation. *American Economic Journal: Economic Policy* **2(3)** 1–21.

Coles, Jeffrey L., Elena Patel, Nathan Seegert, Matthew Smith. 2021. How do firms respond to corporate taxes? *Journal of Accounting Research* **n/a**(n/a).

Coles, Jeffrey L., Jason Sandvik, Nathan Seegert. 2020. Taxes, managerial compensation, and the market for corporate control. *Working paper* .

Darby, Michael R. 1975. The financial and tax effects of monetary policy on interest rates. *Economic Inquiry* **13**(2) 266–276.

Devereux, Michael P, Li Liu, Simon Loretz. 2014. The elasticity of corporate taxable income: New evidence from uk tax records. *American Economic Journal: Economic Policy* **6**(2) 19–53.

Doerrenberg, Philipp, Andreas Peich, Sebastian Siegloch. 2017. The elasticity of taxable income in the presence of deduction possibilities. *Journal of Public Economics* **151** 41–55.

Feldstein, Martin. 1999. Tax avoidance and the deadweight loss of the income tax. *Review of Economics and Statistics* **81**(4) 674–680.

Feldstein, Martin S. 1976. Inflation, income taxes, and the rate of interest: A theoretical analysis. *American Economic Review* **66**(5) 809–820.

Fisher, Irving. 1930. The theory of interest. *New York* **43** 1–19.

Gordon, Roger, Martin Dietz. 2008. Dividends and taxes. Alan J. Auerbach, Daniel N. Shaviro, eds., *Institutional Foundations of Public Finance: Economic and Legal Perspectives*. Cambridge, MA: Harvard University Press, 203–224.

Hall, Robert E, Dale W Jorgenson. 1967. Tax policy and investment behavior. *The American economic review* **57**(3) 391–414.

Harberger, Arnold C. 1964. The measurement of waste. *The American Economic Review* **54**(3) 58–76.

House, Christopher L., Matthew D. Shapiro. 2008. Temporary investment tax incentives: Theory with evidence from bonus depreciation. *American Economic Review* **98**(3) 737–68.

Jensen, Michael C., William H. Meckling. 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* **3**(4) 305–360.

Kang, Zi Yang, Shoshana Vasserman. 2022. Robust bounds for welfare analysis. National Bureau of Economic Research.

Kopczuk, Wojciech, Justin Marion, Erich Muehlegger, Joel Slemrod. 2013. Do the laws of tax incidence hold? point of collection and the pass-through of state diesel taxes. National Bureau of Economic Research.

Mace, Christopher, Elena Patel, Nathan Seegert. 2020. Marijuana taxation and imperfect competition. *National Tax Journal* **73**(2) 545–592.

Modigliani, Franco, Merton H Miller. 1958. The cost of capital, corporation finance and the theory of investment. *The American economic review* **48**(3) 261–297.

Ohrn, Eric, Nathan Seegert. 2019. The impact of investor-level taxation on mergers and acquisitions. *Journal of Public Economics* **177** 104038.

Patel, Elena, Nathan Seegert. 2020. Does market power encourage or discourage investment? evidence from the hospital market. *The Journal of Law and Economics* **63**(4) 667–698.

Pless, Jacquelyn, Arthur A van Benthem. 2019. Pass-through as a test for market power: An application to solar subsidies. *American Economic Journal: Applied Economics* **11**(4) 367–401.

Ritz, Robert A. 2014. A new version of edgeworth's taxation paradox. *Oxford Economic Papers* **66**(1) 209–226.

Smith, Clifford W., Rene M. Stulz. 1985. The determinants of firms' hedging policies. *Journal of Financial and Quantitative Analysis* **20**(4) 391–405.

Weyl, E. Glen, Michal Fabinger. 2013. Pass-through as an economic tool: Principles of incidence under imperfect competition. *Journal of Political Economy* **121**(3) 528–583.

Yagan, Danny. 2015. Capital tax reform and the real economy: The effects of the 2003 dividend tax cut. *American Economic Review* **105**(12) 3531–63.

Zwick, Eric, James Mahon. 2017. Tax policy and heterogeneous investment behavior. *American Economic Review* **107**(1) 217–48.